9. HYPERBOLIC FUNCTIONS

1. If $\sec(x)=\cosh(\theta)$, *then find* $\tanh^2\left(\frac{\theta}{2}\right)=$

[AP EAMCET 18-09-20_Shift-1]

1. $\sec^2\left(\frac{x}{2}\right)$ 2. $\tan^2\left(\frac{x}{2}\right)$

3. $\tanh^2\left(\frac{x}{2}\right)$ 4. $\operatorname{sech}^2\left(\frac{x}{2}\right)$

2. If $\tanh^{-1}(x+iy)=\frac{1}{2}\tanh^{-1}\left(\frac{2x}{1+x^2+y^2}\right)+\frac{i}{2}\tan^{-1}\left(\frac{2y}{1-x^2-y^2}\right)$,

where $x, y \in R$, then $Tanh^{-1}(iy)=$

[AP EAMCET 18-09-20_Shift-2]

1. $i\tanh^{-1}(y)$ 2. $-i\tanh^{-1}(y)$

3. $i\tan^{-1}(y)$ 4. $-i\tan^{-1}(y)$

3. Which of the following is true about $f(x)=3\sinh(x)-2\cosh(x), \forall x \in R$

[AP EAMCET 22-09-20_Shift-1]

1. f is an odd function
2. f is a periodic function
3. f is a strictly increasing function on R
4. f is a strictly decreasing function on R

4. $\frac{1+\tanh(x/2)}{1-\tanh(x/2)}=$

[AP EAMCET 23-09-20_Shift-1]

1. e^{-x} 2. e^{x}

3. $2e^{x/2}$ 4. $2e^{-x/2}$

5. $\sinh^{-1}(-2)+\operatorname{cosech}^{-1}(-2)+\coth^{-1}(-2)=$

[TS EAMCET 09-09-20_Shift-1]

1. $\log\left(\frac{7-3\sqrt{5}}{2\sqrt{3}}\right)$ 2. $\log\left(\frac{3-\sqrt{5}}{2\sqrt{3}}\right)$

3. $\log\left(\frac{7+3\sqrt{5}}{2\sqrt{3}}\right)$ 4. $\log\left(\frac{3+\sqrt{5}}{2\sqrt{3}}\right)$

6. If $x=\operatorname{sech}^{-1}\frac{1}{2}+\tanh^{-1}\frac{1}{2}$, then $\cosh x=$

[TS EAMCET 09-09-20_Shift-2]

1. $\frac{5\sqrt{3}+4}{3}$ 2. $\frac{2\sqrt{3}+3}{2}$

3. $\frac{4\sqrt{3}+3}{3}$ 4. $\frac{4\sqrt{3}-3}{3}$

7. Assertion(A): $\operatorname{cosech}^{-1}(3)=\log\left(\frac{1+\sqrt{10}}{3}\right)$

Reason(R): $e^{\cosh^{-1}x}$ is a root of the quadratic equation $xp^2-2p-x=0$

[TS EAMCET 10-09-20_Shift-1]

1. (A) is true,(R) is true and(R) is the correct explanation for (A)
2. (A) is true,(R) is true but(R) is not the correct explanation for (A)
3. (A) is true but (R) is false
4. (A) is false but (R) is true

8. $\log\left(9+3\sqrt{2}\left(2+\sqrt{5}\right)+4\sqrt{5}\right)=$

[TS EAMCET 10-09-20_Shift-2]

1. $\sinh^{-1}3+\cosh^{-1}\left(\frac{1}{3}\right)$ 2. $\cosh^{-1}3+\sinh^{-1}3$

3. $\tanh^{-1}3+\sinh^{-1}3$ 4. $\cosh^{-1}3+\tanh^{-1}3$

9. If $\operatorname{sech}^{-1}\left(\frac{1}{2}\right)-\operatorname{cosech}^{-1}\left(\frac{3}{4}\right)=\log_e k$, then

[TS EAMCET 11-09-20_Shift-1]

1. $3k^2-12k-1=0$ 2. $3k^2-12k+1=0$

3. $9k^2-12k+1=0$ 4. $9k^2-12k-1=0$

10. If for $|x|>1$ $\operatorname{Tanh}^{-1}\left(\frac{1}{x}\right)+\operatorname{Coth}^{-1}(x)=\log_e(f(x))$,

then $f(-5)=$ **[TS EAMCET 11-09-20_Shift-2]**

1. $\frac{3}{2}$ 2. $\frac{-2}{3}$

3. $\frac{2}{3}$ 4. $\frac{1}{3}$

11. If $\sinh u = \tan\theta$, then $\cosh u =$

[AP EAMCET 19-08-2021_Shift-1]

1. $-\sec\theta$
2. $\sec\theta$
3. $\sin\theta$
4. $\cot\theta$

12. $\sinh(x+y)\cosh(x-y)$ is equal to

[AP EAMCET 20-08-2021_Shift-2]

1. $\frac{1}{2}(\sinh 2x + \sinh 2y)$
2. $(\sinh 2x + \sinh 2y)$
3. $\frac{1}{2}(\sinh 2x - \sinh 2y)$
4. $(\sinh 2x - \sinh 2y)$

13. What is the following expression is always true? **[AP EAMCET 25-08-2021_Shift-1]**

1. $\cosh\frac{x}{2} = \sqrt{\frac{\cosh x - 1}{2}}$
2. $\tanh\left(\frac{x}{2}\right) = \frac{2\sinh x}{\cosh x + 1}$
3. $\sinh\left(\frac{x}{2}\right) = \frac{\sinh x}{\sqrt{2(\cosh x + 1)}}$
4. $\sinh\left(\frac{x}{2}\right) = \frac{\sinh x}{\sqrt{2(\cosh x - 1)}}$

14. $\tanh^{-1}\left(\frac{1}{3}\right) + \coth^{-1}(2) =$

[TS EAMCET 04-08-2021_Shift-2]

1. $\log\sqrt{6}$
2. $\log 6$
3. $-\log\sqrt{6}$
4. $-\log 6$

15. Let $k > 0$ and $t = \sec h^{-1}\left(\frac{1}{2}\right) - \operatorname{cosec} h^{-1}\left(\frac{3}{k}\right)$.

If $3e^t = 2 + \sqrt{3}$, then $k =$

[TS EAMCET 05-08-2021_Shift-1]

1. 2
2. 4
3. $3\sqrt{3}$
4. $3\sqrt{2}$

16. If $y = \log_2 \sin x$, then the minimum value of $\cosh y$ is **[TS EAMCET 05-08-2021_Shift-2]**

1. 2
2. $\frac{2}{e}$
3. $\frac{e}{2}$
4. 1

17. If the range of $\operatorname{Sec}^{-1} hx + \operatorname{Cos} ec^{-1} hx$ is $[a,b)$, then **[TS EAMCET 06-08-2021_Shift-2]**

1. $a = 0, b = 1$
2. $a = \sqrt{2}, b = \infty$
3. $a = \log\left(1+\sqrt{2}\right), b = \infty$
4. $a = 0, b = \log\left(1+\sqrt{2}\right)$

18. If $\cosh 2x = 241$, then $\coth x =$

[TS EAMCET 06-08-2021_Shift-1]

1. $\frac{7}{\sqrt{30}}$
2. $\frac{11}{\sqrt{30}}$
3. $\frac{7}{2\sqrt{30}}$
4. $\frac{11}{2\sqrt{30}}$

19. If $\operatorname{cos} ech\, x = \frac{4}{5}$, *then* $\sinh x =$

[AP EAMCET 04-07-2022_Shift-1]

1. $\frac{4}{5}$
2. $\frac{5}{4}$
3. $\frac{2}{3}$
4. $\frac{2}{5}$

20. The value of $\frac{1+\tanh x}{1-\tanh x}$ is

[AP EAMCET 04-07-2022_Shift-1]

1. e^x
2. e^{-2x}
3. e^{2x}
4. e^{-x}

21. *If* $4 + 6(e^{2x}+1)\tanh x = 11\cosh x + 11\sinh x$, *then* $x =$

[AP EAMCET 04-07-2022_Shift-2]

1. $\log 10$
2. $\log 4$
3. $\log 5$
4. $\log 2$

22. If $2\cosh 2x + 10\sinh 2x = 5$, then x=

[AP EAMCET 05-07-2022_Shift-1]

1. $\frac{1}{2}\log\frac{4}{3}$
2. $\frac{1}{2}\log\frac{2}{3}$
3. $\frac{1}{2}\log\frac{3}{2}$
4. $\frac{1}{2}\log\frac{3}{4}$

23. If $\operatorname{cos} ech\, x = \frac{4}{5}$, *then* $\cosh x =$

[AP EAMCET 05-07-2022_Shift-2]

1. $\sqrt{\frac{41}{21}}$ 2. $\sqrt{\frac{41}{19}}$

3. $\sqrt{\frac{41}{25}}$ 4. $\sqrt{\frac{41}{16}}$

24. Let $x \in R$ *and* $|x|<1$. Then $\tanh^{-1} x =$

[AP EAMCET 05-07-2022_Shift-2]

1. $\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$ 2. $\frac{1}{2}\log\left(\frac{1-x}{1+x}\right)$

3. $\frac{1}{2}\log\left(x+\sqrt{1-x^2}\right)$ 4. $\frac{1}{2}\log\left(x-\sqrt{1-x^2}\right)$

25. For any real number $n \in R, (\cosh x+\sinh x)^n =$

[AP EAMCET 06-07-2022_Shift-1]

1. $\cosh nx-\sinh nx$
2. $\cosh nx+\sinh nx$
3. $\cosh^2 nx+2\sinh nx$
4. $\cosh nx-\sinh nx$

26. Let $\theta \in$ R such that $3\sinh(2\theta)=13-3e^{2\theta}$, *then* $\theta =$

[AP EAMCET 06-07-2022_Shift-2]

1. $\frac{1}{2}\log 3$ 2. $\frac{1}{3}\log 3$

3. $\log 3$ 4. $\frac{1}{2}\log 5$

27. If cosh(x-log3)=sinh x, then x=

[AP EAMCET 07-07-2022_Shift-1]

1. $\frac{1}{2}\log 3$ 2. $\frac{1}{2}\log 6$

3. $\frac{1}{2}\log 5$ 4. $\log 3$

28. $\tanh(\log x)=$

[AP EAMCET 07-07-2022_Shift-2]

1. $\frac{x+1}{x-1}$ 2. $\frac{x^2+1}{x^2-1}$

3. $\frac{x^2-1}{x^2+1}$ 4. $2x$

29. If one solution of the equation $\cosh x-\frac{4}{5}\sinh x=1$ *is* $x=0$, then the other solution is x =

[AP EAMCET 08-07-2022_Shift-1]

1. $2\log 2$ 2. $2\log 5$

3. $\log\left(\frac{4}{3}\right)$ 4. $2\log 3$

30. If $k \in R$ is such that the equation $2\cosh^2 x=3\sinh x+k$ has no real solution, then which of the following is correct?

[AP EAMCET 08-07-2022_Shift-2]

1. $k<\frac{1}{2}$ 2. $k<\frac{3}{8}$

3. $k<\frac{7}{8}$ 4. $k<\frac{5}{8}$

31. If $\theta=\frac{\pi}{12}$ and $x=\log\left(\cot\left(\frac{\pi}{4}+\theta\right)\right)$, then coshx = **[TS EAMCET 18-07-2022_Shift-1]**

1. $\frac{2}{\sqrt{3}}$ 2. $\frac{-2}{\sqrt{3}}$

3. $\frac{\sqrt{3}}{2}$ 4. $\frac{-\sqrt{3}}{2}$

32. $2\cosh(x+y)\sinh(x-y)+\sinh 2y =$

[TS EAMCET 18-07-2022_Shift-1]

1. $\sinh 2x$ 2. $\frac{\sinh 2x+\sinh 2y}{2}$

3. $\frac{\sinh 2x-\sinh 2y}{2}$ 4. $\cosh 2x$

33. $\frac{e^{4x}+e^{-4x}+14}{4\left(e^x-e^{-x}\right)^2}=$

[TS EAMCET 18-07-2022_Shift-2]

1. $\sinh^2 x+\coth^2 x$ 2. $\sinh^2 x+\operatorname{sech}^2 x$

3. $\cosh^2 x+\operatorname{sech}^2 x$ 4. $\cosh^2 x+\tanh^2 x$

34. If $\tan h\, x=\frac{1}{2}$ then $\sin h2x-\sec h2x =$

[TS EAMCET 18-07-2022_Shift-2]

1. $\frac{29}{15}$ 2. $\frac{11}{15}$

3. 3 4. $\frac{-13}{15}$

35. If the minimum value of $\cos(\sinh(\log x)+\cosh(\log x))$ is k, then $\cosh(k+1)=$

[TS EAMCET 19-07-2022_Shift-1]

1. $\frac{e+e^{-1}}{2}$ 2. $\frac{e^2+e^{-2}}{2}$

3. e 4. 1

36. If $\sinh x = \frac{-1}{2}$ then $\tanh 2x =$

[TS EAMCET 19-07-2022_Shift-I]

1. $\frac{-\sqrt{5}}{2}$ 2. $-\sqrt{3}$

3. $\frac{-\sqrt{5}}{3}$ 4. $\frac{-\sqrt{3}}{2}$

37. If $x = \log_e\left(\cot\left(\frac{\pi}{4}+\theta\right)\right)$, *then* $\lim_{\theta\to 0}\frac{\theta}{(\sinh x)(\cosh x)}$

[TS EAMCET 19-07-2022_Shift-2]

1. 0 2. $-\frac{1}{2}$

3. -2 4. 1

38. *If* $e^{it} = \cos t + i\sin t$ *and* $e^{-it} = \cos t - i\sin t$ then $\cos h(x+iy) - \cosh(x-iy)$

[TS EAMCET 19-07-2022_Shift-2]

1. $2\sin h\, x \sin h\, y$
2. $2i\sin h\, x \cosh y$
3. $2\cos h\, x \cosh y$
4. $2i\sin h\, x \sinh y$

39. If $\sinh x = \tan A$, then $|\tan h\, x| =$

[TS EAMCET 20-07-2022_Shift-1]

1. $|\sin A|$ 2. $|\cos A|$

3. $|\sec A|$ 4. $|\text{cosec}A|$

40. $\frac{\sinh(x+y)+\sinh(x-y)}{\cosh(x+y)-\cosh(x-y)} =$

[TS EAMCET 20-07-2022_Shift-1]

1. tan hy 2. cot hy

3. tan hx cot hy 4. tan hy cot hx

41. Assertion(A) : $\coth x = \frac{1-k}{1+k}$ $k \in (0,2)$.

Reason (R) : The graph of $y = \tan h\, x$ always lies between the lines $y = -1$ and $y = 1$.

The correct option among the following is

[TS EAMCET 20-07-2022_Shift-2]

1. (A) is true, (R) is true and (R) is the correct explanation for (A)
2. (A) is true, (R) is true but (R) is not the correct explanation for (A)
3. (A) is true but (R) is false
4. (A) is false but (R) is true

42. If $\frac{5\sinh 2x}{7+6\cosh 2x} = \frac{3}{2}$, then $3\tan h^2 x + 20 \tan h\, x$

[TS EAMCET 20-07-2022_Shift-2]

1. 13 2. 26

3. 39 4. $\frac{13}{2}$

43. If $\alpha = \log_e\left(2+\sqrt{3}\right)$ then,

$\frac{\cosh\alpha}{1-\tanh\alpha} + \frac{\sinh\alpha}{1-\coth\alpha} =$

[15th May 2023 Shift 1]

1. $4+2\sqrt{3}$ 2. $7+4\sqrt{3}$

3. $\frac{\sqrt{3}+1}{2}$ 4. $2+\sqrt{3}$

44. If $\cosh x = \frac{5}{4}$, then $\tanh 3x =$

[15th May 2023 Shift 2]

1. $\frac{63}{65}$ 2. $\frac{25}{26}$

3. $\frac{65}{67}$ 4. $\frac{252}{265}$

45. $\cosh\alpha + \sinh\alpha = e^3$ *and* $\sinh\alpha = \frac{\alpha}{\alpha+1}$

then $\tanh\alpha$ = **[16th May 2023 Shift 1]**

1. $\frac{\alpha}{\alpha+2}$ 2. $\frac{\alpha}{\alpha-2}$

3. $\frac{\alpha}{\alpha+4}$ 4. $\frac{2\alpha}{\alpha-2}$

46. If $2\sinh x = \cosh x$, *then* $x =$

[16th May 2023 Shift 2]

1. $\frac{1}{3}\log 2$ 2. $2\log 3$

3. $\frac{1}{2}\log 3$ 4. $\log 6$

47. If $\sinh x = \frac{5}{12}$, then $\cosh\frac{x}{2} =$

[17th May 2023 Shift 1]

1. $\frac{3}{2\sqrt{5}}$
2. $\frac{2}{3\sqrt{3}}$
3. $\frac{5}{\sqrt{6}}$
4. $\frac{5}{2\sqrt{6}}$

48. If $\tan A = \tan\alpha \coth x = \cot\beta \tanh x$, then $\tan(\alpha+\beta) =$ **[17th May 2023 Shift 2]**

1. cosh2x cosec 2A
2. cosh 2x sec 2A
3. sinh 2x cos 2A
4. sinh 2x cosec 2A

49. log(cosh3+sinh3)+log(cosh3-sinh3)=

[18th May2023 shift -1]

1. 1
2. 2
3. 3
4. 0

50. If $\cosh x = \operatorname{cosec}\theta$, then $\coth^2\frac{x}{2} =$

[18th May 2023 Shift 2]

1. $\tan^2\frac{\theta}{2}$
2. $\tan^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$
3. $\cot^2\frac{\theta}{2}$
4. $\cot^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

51. If $\sin x\cosh y = \cos\theta$ *and* $\cos x\sinh y = \sin\theta$, *then* $\sin^2 x + \cosh^2 y =$

[19th May 2023 Shift 1]

1. 1
2. 2
3. $\sin 2\theta$
4. $\cos 2\theta$

52. If $Sinh(\log x) = -2$ then $x =$

[12TH MAY 2023 SHIFT-1]

1. $\sqrt{5}-2$
2. $2+\sqrt{5}$
3. $-(2+\sqrt{5})$
4. $2-\sqrt{5}$

53. $Sinh\left(\log\left(3+\sqrt{8}\right)\right) =$

[12TH MAY 2023 SHIFT -2]

1. $3^{\frac{3}{2}}$
2. $2^{\frac{3}{2}}$
3. $8^{\frac{2}{3}}$
4. $3^{\frac{1}{2}}$

54. If $\cosh x = \frac{4}{3}$, *then* $3\cosh x + 3^2\cosh 2x + 3^3\cosh 3x =$

[13TH MAY 2023 SHIFT-1]

1. 175
2. 81
3. 64
4. 27

55. If $x = \log\left(y+\sqrt{y^2+1}\right)$ then y =

[EAPCET 14-05-23 SHIFT-1]

1. $Tanhx$
2. $Cothx$
3. $Sinhx$
4. $Coshx$

56. If sinh$x$= -4/3 then sinh2x+cosh $2x$=

[EAPCET 13-05-23 SHIFT-2]

1. $-\frac{31}{41}$
2. $-\frac{20}{9}$
3. $\frac{49}{41}$
4. $\frac{1}{9}$

KEY

1)	2	**2)**	3	**3)**	3	**4)**	2	**5)**	1
6)	3	**7)**	1	**8)**	2	**9)**	3	**10)**	3
11)	2	**12)**	1	**13)**	3	**14)**	1	**15)**	2
16)	4	**17)**	3	**18)**	4	**19)**	2	**20)**	3
21)	4	**22)**	1	**23)**	4	**24)**	1	**25)**	2
26)	1	**27)**	2	**28)**	3	**29)**	4	**30)**	3
31)	1	**32)**	1	**33)**	1	**34)**	2	**35)**	4
36)	3	**37)**	2	**38)**	4	**39)**	1	**40)**	2
41)	4	**42)**	3	**43)**	4	**44)**	1	**45)**	1
46)	3	**47)**	4	**48)**	4	**49)**	4	**50)**	4
51)	2	**52)**	1	**53)**	2	**54)**	1	**55)**	3
56)	4								

SOLUTIONS

1. $\tanh^2\left(\frac{\theta}{2}\right) = \frac{\cosh\theta-1}{\cosh\theta+1} = \frac{\sec x-1}{\sec x+1}$

$= \frac{1-\cos x}{1+\cos x} = \tan^2\frac{x}{2}$

2. Put x=0

Then $Tanh^{-1}(iy) = 0+\frac{i}{2}\left(2\tan^{-1}y\right) = i\tan^{-1}y$

3. $f(x) = 3\sinh(x)-2\cosh(x)$

$= 3\left(\frac{e^x-e^{-x}}{2}\right)-2\left(\frac{e^x+e^{-x}}{2}\right)$

$f(x) = \left(\frac{e^x-5e^{-x}}{2}\right)$

$$f'(x)=\left(\frac{e^x+5e^{-x}}{2}\right)>0$$

f(x) is increasing function on 'R'

4. $\text{Tanh}\,\frac{x}{2}=\frac{e^{x/2}-e^{-x/2}}{e^{x/2}+e^{-x/2}}$

5. $\sinh^{-1}(-2)+\text{cosec}h^{-1}(-2)+\coth^{-1}(-2)$

$$=\log\left(-2+\sqrt{5}\right)+\log\left(\frac{\sqrt{5}-1}{2}\right)+\frac{1}{2}\log\left(\frac{1}{3}\right)$$

$$=\log\frac{\left(\sqrt{5}-2\right)\left(\sqrt{5}-1\right)}{2\sqrt{3}}$$

$$=\log\left(\frac{7-3\sqrt{5}}{2\sqrt{3}}\right)$$

6. $\text{sech}^{-1}x=\log\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

$$\text{Tanh}^{-1}x=\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$$

$$x=\log\left[\frac{1+\sqrt{1-\frac{1}{4}}}{1/2}\right]+\frac{1}{2}\log\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)$$

$$=\log(2\sqrt{3}+3)$$

$$\cosh x=\frac{e^x+e^{-x}}{2}=\frac{4\sqrt{3}+3}{3}$$

7. $\text{cosec}h^{-1}(3)=\log\left(\frac{1+\sqrt{1+x^2}}{3}\right)=\log\left(\frac{1+\sqrt{10}}{3}\right)$

$$R:P=\frac{2\pm\sqrt{4+4x^2}}{2x}=\frac{1\pm\sqrt{1+x^2}}{x}$$

$$e^{\cosh^{-1}x}=\frac{1\pm\sqrt{1+x^2}}{x}$$

$$^{\cosh^{-1}x}=\log\left(\frac{1+\sqrt{1+x^2}}{x}\right)$$

8. $\log\left(9+3\sqrt{2}\left(\sqrt{5}+2\right)+\sqrt{54}\right)$

$$\log\left(\left(3+\sqrt{8}\right)\left(3+\sqrt{10}\right)\right)$$

$$\log\left(3+\sqrt{9-1}\right)+\log\left(3+\sqrt{9+1}\right)$$

$\cosh^{-1}3+\sinh^{-1}3$

9. $\text{sec}\,h^{-1}\left(\frac{1}{2}\right)-\text{cos}\,ech^{-1}\left(\frac{3}{4}\right)=\log_e k$

$$\ln\left(2+\sqrt{3}\right)-\ln(3)=\ln_e k$$

$$\Rightarrow\ln_e\left(\frac{2+\sqrt{3}}{3}\right)=\ln_e k$$

$$\therefore k=\frac{2+\sqrt{3}}{3}$$

On verifying option (3) $\Rightarrow 9k^2-12k+1=0$

10. $\text{Tanh}^{-1}\left(\frac{1}{x}\right)+\text{Coth}^{-1}(x)=\log_e(f(x))$

$$\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)+\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)=\log_e f(x)$$

$$\log\left(\frac{x+1}{x-1}\right)=\log_e f(x)\Rightarrow f(x)=\frac{x+1}{x-1}$$

11. $\sinh u=\tan\theta>0$

$$\cosh u=\sqrt{1+\sinh^2 u}=\sqrt{1+\tan^2\theta}=\sec\theta$$

12. $\sinh(x+y)\cosh(x-y)$

$$=\frac{1}{2}\left[2.\sinh(x+y).\cosh(x-y)\right]$$

$$=\frac{1}{2}\left[\sinh(2x)+\sinh(2y)\right]$$

13. Formulae

14. $\tanh^{-1}\left(\frac{1}{3}\right)+\coth^{-1}(2)=$

$$=\frac{1}{2}\log\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right)+\frac{1}{2}\log\left(\frac{2+1}{2-1}\right)$$

$$=\frac{1}{2}\log\left(\frac{4}{2}\right)+\frac{1}{2}\log 3$$

$$=\frac{1}{2}(\log 2+\log 3)$$

$$=\frac{1}{2}\log 6=\log\sqrt{6}$$

15. $x\in(0,1),\text{sec}\,h^{-1}(x)=\log\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

$$\cos ech^{-1}(x)=\log\left(\frac{1+\sqrt{1+x^2}}{x}\right)$$

$$\sec h^{-1}\left(\frac{1}{2}\right)=\log\left(\frac{1+\sqrt{1-\frac{1}{4}}}{\frac{1}{2}}\right)=\log_e\left(2+\sqrt{3}\right)..(1)$$

$$3e^t=2+\sqrt{3}\Rightarrow e^t=\frac{2+\sqrt{3}}{3}$$

$$t=\log\left(\frac{2+\sqrt{3}}{3}\right)$$

$$t=\log_e\left(2+\sqrt{3}\right)-\log\left(\frac{2+\sqrt{3}}{3}\right)$$

$$t=\log_e\left(2+\sqrt{3}\right)-\log_e 3$$

$$\sec h^{-1}\left(\frac{1}{2}\right)-\cos ech^{-1}\left(\frac{3}{k}\right)$$

$$=\log_e\left(2+\sqrt{3}\right)-\log_e 3$$

$$\log\left(\frac{1+\sqrt{1+\frac{9}{k^2}}}{\frac{3}{k}}\right)=\log_e 3$$

$$\frac{k+\sqrt{k^2+9}}{3}=3$$

$$k+\sqrt{k^2+9}=9\Rightarrow \boxed{k=4}$$

16. $\cosh y=\dfrac{e^y+e^{-y}}{2}$

$$e^y+\frac{1}{e^y}\geq 2$$

$$\therefore \frac{e^y+\frac{1}{e^y}}{2}\geq 1$$

$\therefore$ min value = 1

17. Conceptual

18. $\cosh 2x=241$

$$\Rightarrow\frac{1+\tanh^2 x}{1-\tanh^2 x}=241$$

$$\Rightarrow\tanh^2 x=\frac{240}{242}$$

$$\Rightarrow\coth x=\frac{11}{2\sqrt{30}}$$

19. $\cos echx=4/5$

$$\text{Sinh } x=\frac{1}{\cos echx}=\frac{1}{\frac{4}{5}}=5/4$$

20. $\dfrac{1+\tanh x}{1-\tanh x}$

$$\tanh x=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

$$\frac{1+\frac{e^x-e^{-x}}{e^x+e^{-x}}}{1-\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)}$$

$$=\frac{e^x+\cancel{e^{-x}}+e^x-\cancel{e^{-x}}}{\cancel{e^x}+e^{-x}-\cancel{e^x}+e^{-x}}$$

$$=\frac{2^{e^x}}{2e^{-x}}=e^{2x}\ .$$

21. $4+6\left(e^{2x}+1\right)\tanh x=11\cosh x+11\sinh x$

$$4+6\left(e^{2x}+1\right)\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)=11\left(\frac{e^x+e^{-x}}{2}+\frac{e^x-e^{-x}}{2}\right)$$

$$4+6\frac{\cancel{\left(e^{2x}+1\right)}\left(e^{2x}-1\right)}{\cancel{\left(e^{2x}+1\right)}}=\frac{11}{\cancel{2}}.\cancel{2}.e^x.$$

$$4+6e^{2x}-6=11e^x.$$

$$6\left(e^x\right)^2-11e^x-2=0.$$

$$6\left(e^x\right)^2-12e^x+e^x-2=0$$

$$6e^x\left(e^x-2\right)+1\left(e^x-2\right)=0$$

$$\left(e^x-2\right)\left(6e^x+1\right)=0$$

$$e^x\neq -1/6$$

$$e^x-2=0$$

$$e^x=2$$

$$x=\log_e 2$$

$$\therefore x=\log_e 2$$

22. $2\cosh 2x+10\sinh 2x=5$

$$2.\left(\frac{e^{2x}+e^{-2x}}{2}\right)+10\left(\frac{e^{2x}-e^{-2x}}{2}\right)=5$$

$$e^{2x}+\frac{1}{e^{2x}}+5.e^{2x}-\frac{5}{e^{2x}}=5$$

$$6e^{2x}-\frac{4}{e^{2x}}=5$$

$$6\left(e^{2x}\right)^2-4=5\left(e^{2x}\right)$$

$$6\left(e^{2x}\right)^2-5\left(e^{2x}\right)-4=0$$

$$6\left(e^{2x}\right)^2-8e^{2x}+3e^{2x}-4=0$$

$$2e^{2x}\left(3e^{2x}-4\right)+\left(3e^{2x}-4\right)=0$$

$$\left(3e^{2x}-4\right)\left(2e^{2x}+1\right)=0$$

$$e^{2x}=\frac{4}{3}\Rightarrow 2x=\log\frac{4}{3}$$

$$x=\frac{1}{2}\log 4/3$$

23. $\cos echx=\frac{4}{5}$

$$\sinh x=5/4$$

$$\cosh^2 x-\sinh^2 x=1$$

$$\cosh^2 x=1+\left(\frac{5}{4}\right)^2$$

$$=\frac{16+25}{16}=\frac{41}{16}$$

$$\cosh x=\pm\sqrt{\frac{41}{16.}}$$

24. $\tanh^{-1}x=\frac{1}{2}\log\left[\frac{1+x}{1-x}\right]$

25. $(\cosh x+\sinh x)^n$

$$\left(\frac{e^x+e^{-x}}{2}+\frac{e^x-e^{-x}}{2}\right)^{n.}$$

$$\left(\frac{e^x+e^{-x}}{2}+\frac{e^x-e^{-x}}{2}\right)^{n.}$$

$$\left(\frac{e^x+\cancel{e^{-x}}+e^x-\cancel{e^{-x}}}{2}\right)^n=\left(\frac{2e^x}{2}\right)^n=e^{nx.}$$

$\cosh nx+\sinh nx.$

$$\Rightarrow\frac{e^{nx}+e^{-hx}}{2}+\frac{e^{nx}-e^{-nx}}{2}$$

$$\frac{1}{2}\left(e^{nx}+e^{-nx}+e^{nx}-e^{-nx}\right)$$

$$\frac{2.e^{nx.}}{2}=e^{nx}$$

26. $3\sinh 2\theta=13-3e^{2\theta}$

$$3\left(\frac{e^{2\theta}-e^{-2\theta}}{2}\right)=13-3e^{2\theta}$$

$$3e^{2\theta}-3e^{-2\theta}=26-6e^{2\theta.}$$

$$3e^{2\theta}-\frac{3}{e^{2\theta}}=26-6e^{2\theta.}$$

$$9\left(e^{2\theta}\right)^2-26\left(e^{2\theta}\right)-3=0.$$

$$9\left(e^{2\theta}\right)^2-27e^{2\theta}+e^{2\theta}-3=0$$

$$9e^{2\theta}\left(e^{2\theta}-3\right)+1\left(e^{2\theta}-3\right)=0$$

$$\left(e^{2\theta}-3\right)\left(9e^{2\theta}+1\right)=0$$

$$e^{2\theta}=3\Rightarrow 2\theta=\log 3$$

$$\theta=1/2\log 3$$

27. $\cosh\left(x-\log 3\right)=\sinh x$

$$\frac{e^{x-\log 3}+e^{-x+\log 3}}{2}=\frac{e^x-e^{-x}}{2}$$

$$\frac{e^x}{3}+\frac{3}{e^x}=\frac{\left(e^x\right)^2-1}{e^x}$$

$$\frac{\left(e^x\right)^2+9}{3e^x}=\frac{\left(e^x\right)^2-1}{e^x}$$

$$\left(e^x\right)^2+9=3\left(e^x\right)^2-3$$

$$6=e^{2x}\Rightarrow x=\frac{1}{2}\log 6$$

28. $\tanh(\log x)$

$$\log x=t\Rightarrow x=e^t$$

$$\Rightarrow\tanh t=\frac{e^t-e^t}{e^t+e^{-t}}=\frac{e^t-\frac{1}{e^t}}{e^t+\frac{1}{e^t}}=\frac{\left(e^t\right)^2-1}{\left(e^t\right)^2+1}=\frac{x^2-1}{x^2+1}$$

29. $\cosh x-\frac{4}{5}\sinh x=1$

$$\frac{e^x+e^{-x}}{2}-\frac{4}{5}\left(\frac{e^x-e^{-x}}{2}\right)=1$$

$$\frac{1}{2}\left[\frac{5e^x+5e^{-x}-4e^x+4e^{-x}}{5}\right]=1$$

$$e^x+9e^{-x}=10$$

$$\left(e^x\right)^2+9=10e^x$$

$$\left(e^x\right)^2-10e^x+9=0$$

$$\left(e^x-1\right)\left(e^x-9\right)=0\Rightarrow x=0\qquad x=2\log 3$$

30. $2\cosh^2 x = 3\sinh x + k$

$2(1+\sinh^2 x) = 3\sinh x + k$

$2 + 2\sinh^2 x - 3\sinh x - k = 0$

$2\sinh^2 x - 3\sinh x + (2-k) = 0$

no real solution

$b^2 - 4ac < 0$

$9 - 4.2(2-k) < 0$

$9 - 16 + 4k < 0$

$-7 + 4k < 0$

$4k < 7 \Rightarrow k < \frac{7}{8}$

31. $\theta = \frac{\pi}{12}$

$x = \log\left(\cot\left(\frac{\pi}{4} + \theta\right)\right)$

$e^x = \cot\left(\frac{\pi}{4} + \theta\right)$

$\theta = \frac{\pi}{12} = \cot\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = \cot\left(\frac{4\pi}{12}\right) = \cot\frac{\pi}{3}$

$e^x = \frac{1}{\sqrt{3}} \Rightarrow e^{-x} = \sqrt{3}$

$\frac{e^x + e^{-x}}{2} = \frac{\frac{1}{\sqrt{3}} + \sqrt{3}}{2} = \frac{2}{\sqrt{3}}$

32. $2\cosh(x+y)\sin(x-y) + \sinh 2y$

$\sinh\left(x + \cancel{y} + x - \cancel{y}\right) - \sinh\left(\cancel{x} + y - \cancel{x} + y\right) + \sinh 2y$

$\sinh 2x - \sinh 2y + \sinh 2y$

$= \sinh 2x$

33. $\frac{e^{4x} + e^{-4x} + 14}{4\left(e^x - e^{-x}\right)^2}$

$\sinh^2 x + \coth^2 x$

$= \left(\frac{e^x - e^{-x}}{2}\right)^2 + \left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right)^2$

$= \frac{\left(e^x - e^{-x}\right)^2\left(e^x - e^{-x}\right)^2 + 4\left(e^x + e^{-x}\right)^2}{4\left(e^x - e^{-x}\right)^2}$

$= \frac{e^{4x} + e^{-4x} + 14}{4\left(e^x - e^{-x}\right)^2}$

34. $\tanh x = \frac{1}{2}$

$\sinh 2x - \sec h2x$

$= \left(\frac{2\tanh x}{1 - \tanh^2 x}\right) - \left(\frac{1 - \tanh^2 x}{1 + \tan^2 hx}\right)$

$= \frac{20-9}{15} = \frac{11}{15}$

35. $\sinh(\log x) + \cosh(\log x) = \frac{e^{\log x} - e^{-\log x}}{2} + \frac{e^{\log x} + e^{-\log x}}{2} = x$

$\cos\left[\sinh(\log x) + \cosh(\log x)\right] = \cos x$

Mimimum value of $cosx = -1 = k$

$\therefore \cosh(k+1) = \frac{e^{k+1} + e^{-(k+1)}}{2} = 1$

36. $\sinh x = -\frac{1}{2}$

$\tanh 2\,\mathrm{x} = ?$

$\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x = 1 + \frac{1}{4} = \frac{5}{4}$

$\cosh x = \frac{\sqrt{5}}{2}$

$\tanh x = \frac{-1}{\sqrt{5}}$

$\tanh 2\,\mathrm{x} = \frac{2\tanh x}{1 + \tanh x} = \frac{-\sqrt{5}}{3}$

37. $e^x = \cot\left(\frac{\pi}{4} + \theta\right) = \frac{\cot\theta - 1}{\cot\theta + 1}$

$e^{-x} = \frac{\cot\theta + 1}{\cot\theta - 1}$

$\cosh x = \frac{e^x + e^{-x}}{2} = \sec 2\theta$

$\sinh x = \frac{e^x - e^{-x}}{2} = -\tan 2\theta$

$\lim_{\theta \to 0} \frac{\theta}{\sinh x \cosh x} = \lim_{\theta \to 0} \frac{-\theta\cos 2\theta}{\tan 2\theta} = \frac{-1}{2}$

38. $\cosh x \cosh iy + \sinh x \sinh iy - \cosh x \cosh iy$

$+ \sinh x \sinh iy$

$= 2\sinh x \sinhi y$

$= 2i\sinh x \sinh y$

39. $\sinh = \tan A$

$\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x = \sec^2 A \Rightarrow \cosh x = \sec A$

$\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{\tan A}{\sec A} = |\sin A|$

40. $\dfrac{\sinh(x+y)+\sinh(x-y)}{\cosh(x+y)-\cosh(x-y)}$

$\dfrac{2\cancel{\sinh x}\cosh y}{2\cancel{\sinh x}\sinh y} = \coth y$

41. We know that range of *cothx* is $(-\infty,-1)\cup(1,\infty)$

Case(i):	**Case(ii):**
$\dfrac{1-k}{1+k} > 1$	$\dfrac{k-1}{k+1} < -1$
$1-k > 1+k$	$k-1 < -1-k$
$k < 0$	$k < 0$

42. $\dfrac{5\sinh 2x}{7+6\cosh 2x} = \dfrac{3}{2}$

$10\sinh 2x = 21+18\cosh 2x$

$\dfrac{20\tanh x}{1-\tanh^2 x} = 21+18\left(\dfrac{1+\tanh^2 x}{1-\tan^2 x}\right)$

$20\tanh x = 39-3\tanh^2 x$

$3\tanh^2 x + 20\tanh x = 39$

43. $\dfrac{\cosh\alpha}{1-\dfrac{\sinh\alpha}{\cosh\alpha}} + \dfrac{\sinh\alpha}{1-\dfrac{\cosh\alpha}{\sinh\alpha}}$

$= \dfrac{\cosh^2\alpha}{\cosh\alpha-\sinh\alpha} - \dfrac{\sinh^2\alpha}{\cosh\alpha-\sinh\alpha}$

$= \dfrac{1}{\cosh\alpha-\sinh\alpha} = \dfrac{1}{\dfrac{e^{\alpha}+e^{-\alpha}}{2} - \left(\dfrac{e^{\alpha}-e^{-\alpha}}{2}\right)}$

$= \dfrac{2}{2e^{-\alpha}} = e^{\alpha} = 2+\sqrt{3}$

44. $\cosh x = \dfrac{5}{4}$ $\quad \sinh x = \dfrac{3}{4}$

$\cosh 3x = \dfrac{65}{16}$ $\quad \cosh 3x = 4\cosh^3 x - 3\cos x$

$\sinh 3x = \dfrac{63}{16}$ $\quad \sinh 3x = 3\sinh x + 4\sinh^3 x$

$Tanh\, 3x = \dfrac{63}{65}$

45. $\cosh\alpha + \sinh\alpha = e^3$

$\dfrac{e^{\alpha}+e^{-\alpha}}{2} + \dfrac{e^{\alpha}-e^{-\alpha}}{2} = e^3$

$e^{\alpha} = e^3 \Rightarrow \alpha = 3$

$\sinh\alpha = \dfrac{2}{3+1} = \dfrac{3}{4}$

$\cosh^2\alpha - \sinh^2\alpha = 1 \Rightarrow \cosh^2\alpha = 1+\dfrac{9}{16} = \dfrac{25}{16}$

$\cosh\alpha = \dfrac{5}{4}$

$\tanh\alpha = \dfrac{3}{5} \Rightarrow \dfrac{\alpha}{\alpha+2} = \tanh\alpha$

46. $2\sinh x = \cosh x \Rightarrow Tanhx = \dfrac{1}{2}$

$\Rightarrow x = Tanh^{-1}\left(\dfrac{1}{2}\right) \Rightarrow x = \dfrac{1}{2}\log 3$

47. CONCEPTUAL

48. $TanA = \tan\alpha \cot hx \Rightarrow \tan\alpha = \dfrac{\tan A}{\cot hx}$

$TanA = \cot\beta \tan hx \Rightarrow \tan\beta = \dfrac{\tan hx}{\tan A}$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$= \dfrac{\tan^2 A + \cot hx.\tanh x}{\dfrac{\cot hx\tan A}{\cot hx - \tan hx}}$

$= \dfrac{\tan^2 A+1}{\tan A(\cot hx - \tan hx)}$

$= \dfrac{\sec^2 A}{\tan A\left(\dfrac{\cosh^2 x-\sinh^2 x}{\cos hx\sin hx}\right)}$

$= \dfrac{\sin hx\cos hx}{\sin A\cos A} \times \dfrac{2}{2}$

$= \sinh(2x).\cos ec2A$

49. $\log(\cosh 3+\sinh 3)+\log(\cosh 3-\sinh 3)$

$=\log(\cosh^2 3-\sinh^2 3)$

$=\log(1)=0$

50. W.K.T.

$$\cot h^2\frac{x}{2}=\frac{\cos hx+1}{\cos hx-1}$$

$$=\frac{1+\sin\theta}{1-\sin\theta}=\frac{1}{\left[\dfrac{\sin\frac{\theta}{2}-\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}+\cos\frac{\theta}{2}}\right]^2}$$

$$=\cot^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

51. $\sin x.\cosh y=\cos\theta\rightarrow(1)$

$\cos x.\sinh y=\sin\theta\rightarrow(2)$

squaring and addaing

$\sin^2 x.\cosh^2 y+\cos^2 x.\sinh^2 y=\cos^2\theta+\sin^2\theta$

$\sin^2 x(1+\sinh^2 y)+\cos^2 x.\sinh^2 y=1$

$\sin^2 x+\sin^2 x.\sinh^2 y+\cos^2 x.\sinh^2 y=1$

$\sin^2 x+\sinh^2 y=1$

$\sin^2 x+\cosh^2 y-1=1$

52. Given sinh(logx)= -2

$$\frac{e^{\log x}-e^{-\log x}}{2}=-2$$

$$x-\frac{1}{x}=-4\Rightarrow x^2-1=-4x$$

$$x^2+4x-1=0\Rightarrow x=\frac{-4\pm\sqrt{16+4}}{2}$$

$$x=\sqrt{5}-2\quad(\because x>0)$$

53. $\text{let},\sin h\left(\log(3+\sqrt{8}\right)=x$

$\log\left(3+\sqrt{8}\right)=\sin h^{-1}x$

$\log\left(3+\sqrt{8}\right)=\log\left(x+\sqrt{x^2+1}\right)$

$3+\sqrt{8}=x+\sqrt{x^2+1}$

put $x=2^{3/2}$ satisfied

54. $\cos hx=4/3,\sin hx=\frac{\sqrt{7}}{3}$

$\cos h2x=\cosh^2 x+\sinh^2 x=\frac{23}{9}$

$\cos h3x=4\cosh^3 x-3\cosh x=148$

$\therefore 3\cosh x+3^2\cosh 2x+3^3\cosh 3x=175$

55. Given

$x=\log(y+\sqrt{y^2+1}$

$x=\sinh^{-1}y\Rightarrow y=sin\ h(x)$

56. $\sinh x=\frac{-4}{3}\Rightarrow\cosh x=\frac{5}{3}$

Now $\sin h\,2x+\cos h\,2x$

$=2\sin hx\cos hx+\cos h^2x+\sin h^2x$

$=\frac{-40}{9}+\frac{25}{9}+\frac{16}{9}=\frac{1}{9}$