40. HYPERBOLA

1. The value of b^2 in order that the foci of the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ and the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ coincide is

[TS EAMCET 09-09-20_Shift-1]

1. 1 2. 5
3. 7 4. 9

2. If the pole of the line 3x-16y+48=0 with respect to the hyperbola $9x^2-16y^2=144$ is (α,β), then $\alpha-\beta=$

[TS EAMCET 09-09-20_Shift-1]

1. 0 2. -3
3. 2 4. -7

3. Let $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\phi, b\tan\phi)$ be two points such that $\theta+\phi=\frac{\pi}{2}$ on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If (h,k) is the point intersection of the normals at P and Q, then $k=$ **[TS EAMCET 09-09-20_Shift-2]**

1. $\frac{a^2+b^2}{a}$ 2. $-\left(\frac{a^2+b^2}{a}\right)$
3. $\frac{a^2+b^2}{b}$ 4. $-\left(\frac{a^2+b^2}{b}\right)$

4. The equation of the hyperbola, whose eccentricity is $\sqrt{2}$ and whose foci are 16 units apart, is

[TS EAMCET 10-09-20_Shift-1]

1. $9x^2-4y^2=36$ 2. $2x^2-3y^2=7$
3. $x^2-y^2=16$ 4. $x^2-y^2=32$

5. If the circle $x^2+y^2=a^2$ intersects the hyperbola $xy=b^2$ at four points $(x_1,y_1),(x_2,y_2),(x_3,y_3),(x_4,y_4)$, then $y_1y_2y_3y_4=$

[TS EAMCET 10-09-20_Shift-2]

1. a^4 2. 0
3. b^4 4. b^2

6. A rectangular hyperbola passing through (3,2) has its asymptotes parallel to the coordinate axes. If (1,1) is the point of intersection of the two perpendicular tangents of that hyperbola, then its equation is

[TS EAMCET 11-09-20_Shift-1]

1. $xy=x+\frac{1}{y}$ 2. $x\left(y+1+\frac{1}{x}\right)=1$
3. $x(1-y)=y-1$ 4. $xy=x+y+1$

7. If (8,2) is a point on the hyperbola whose length of the transverse axis is 12 and conjugate axis is x=0, then the eccentricity of that hyperbola is

[TS EAMCET 11-09-20_Shift-2]

1. $\frac{2\sqrt{2}}{7}$ 2. $\frac{8}{5}$
3. $\frac{2\sqrt{2}}{\sqrt{7}}$ 4. $\frac{\sqrt{8}}{5}$

8. If p,q are the eccentricities of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and its conjugate hyperbola respectively, then the area of the square (in sq. units) formed by the points of intersection of the ellipse $\frac{x^2}{p^2}+\frac{y^2}{q^2}=1$ and the pair of lines $x^2-y^2=0$

[TS EAMCET 14-09-20_Shift-2]

1. 4 2. $\sqrt{2}$
3. $\frac{\sqrt{3}}{2}$ 4. 16

9. The asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, with any tangent to the hyperbola form a triangle whose area is $a^2\tan(\alpha)$. Then its eccentricity equals__

[AP EAMCET 19-08-2021_Shift-1]

1. $\sec(\alpha)$ 2. $\operatorname{cosec}(\alpha)$
3. $\sec^2(\alpha)$ 4. $\operatorname{cosec}^2(\alpha)$

10. If one focus of a hyperbola is $(3,0)$, then equation of its directrix is $4x-3y-3=0$ and its eccentricity $e=\frac{5}{4}$, then the co-ordinate of its vertex is ______

[AP EAMCET 20-08-2021_Shift-1]

1. $\left(\frac{3}{5},\frac{11}{5}\right)$
2. $\left(\frac{11}{5},\frac{3}{5}\right)$
3. $\left(\frac{7}{5},\frac{4}{5}\right)$
4. $\left(\frac{4}{5},\frac{7}{5}\right)$

11. If the focal chord of the hyperbola subtends a right angle at the centre, then its eccentricity is

[AP EAMCET 20-08-2021_Shift-2]

1. $e=\frac{\sqrt{3}-1}{2}$
2. $e=\frac{\sqrt{5}-1}{2}$
3. $e=\frac{\sqrt{5}+1}{2}$
4. $e=\frac{\sqrt{3}+1}{2}$

12. If the transverse and conjugate axes of hyperbola are equal, then its eccentricity is__

[AP EAMCET 23-08-2021_Shift-1]

1. $\sqrt{3}$
2. $\sqrt{2}$
3. $\frac{1}{\sqrt{2}}$
4. 2

13. In a hyperbola, if the length of transverse axis is twice that of the conjugate axis, then the distance between its directrices is ____ units.

[AP EAMCET 23-08-2021_Shift-2]

1. $\frac{8b}{\sqrt{5}}$
2. $\frac{8a}{\sqrt{5}}$
3. $\frac{2a}{\sqrt{5}}$
4. $\frac{2b}{\sqrt{5}}$

14. The equation of hyperbola whose eccentricity is $\frac{5}{3}$ and distance between the foci is 10 units is:

[AP EAMCET 24-08-2021_Shift-2]

1. $16x^2-9y^2=16$
2. $16x^2-9y^2=9$
3. $16x^2-9y^2=-144$
4. $16x^2-9y^2=144$

15. If in a hyperbola the distance between the foci is 10 and transverse axis has length 8, then the length of its latus rectum is ______

[AP EAMCET 25-08-2021_Shift-1]

1. 9
2. $\frac{9}{2}$
3. $\frac{32}{3}$
4. $\frac{64}{3}$

16. If the latus rectum subtends a right angle at centre of the hyperbola, then its eccentricity is: **[AP EAMCET 25-08-2021_Shift-2]**

1. $\frac{\sqrt{13}}{2}$
2. $\frac{\sqrt{5}-1}{2}$
3. $\frac{\sqrt{5}+1}{2}$
4. $\frac{\sqrt{3}+1}{2}$

17. If $(\alpha,-1)$ is an interior point of the curve $4x^2-3y^2=1$, then α lies in

[TS EAMCET 04-08-2021_Shift-2]

1. $(-\infty,-1)\cup(1,\infty)$
2. $(-1,1)$
3. $(-\infty,\infty)$
4. $[0,\infty)$

18. The lines $x\cos\alpha+y\sin\alpha=p, \alpha\in R$ are chords of the hyperbola $\frac{x^2}{9}-\frac{y^2}{36}=1$ and they subtend a right angle at the centre of the hyperbola. The locus of the poles of these lines with respect to the given hyperbola is

[TS EAMCET 04-08-2021_Shift-1]

1. $x^2-16y^2=108$
2. $16x^2-y^2=108$
3. $16x^2+y^2=108$
4. $x^2+16y^2=108$

19. If $\frac{x^2}{\alpha+3}+\frac{y^2}{2-\alpha}=1$ represents a hyperbola, then α lies in **[TS EAMCET 05-08-2021_Shift-1]**

1. $(-3,2)$
2. $(-3,\infty)$
3. $(-\infty,-2)$
4. $(-\infty,-3)\cup(2,\infty)$

20. If the latus rectum of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ subtends an angle of 60^0 at the other focus, then the eccentricity of the hyperbola is

[TS EAMCET 05-08-2021_Shift-2]

1. 2
2. $\frac{\sqrt{3}+1}{2}$
3. $2\sqrt{3}$
4. $\sqrt{3}$

21. If the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1(a>b)$ has eccentricity and the length of the latus rectum respectively equal to $\frac{5}{4}$ and 9, then $ab=$

[TS EAMCET 06-08-2021_Shift-2]

1. $12\sqrt{2}$
2. $18\sqrt{3}$
3. 48
4. 20

22. The foci of the hyperbola $5x^2-6y^2-10x-24y-34=0$ are

[TS EAMCET 06-08-2021_Shift-1]

1. $\left(-2\pm\frac{\sqrt{33}}{2},2\right)$
2. $\left(2\pm\frac{\sqrt{33}}{\sqrt{2}},-2\right)$
3. $\left(2\pm\frac{\sqrt{11}}{\sqrt{2}},2\right)$
4. $\left(1\pm\frac{\sqrt{11}}{\sqrt{2}},-2\right)$

23. Let origin be the centre, $(\pm3,0)$ be the foci and $\frac{3}{2}$ be the eccentricity of a hyperbola. Then the line $2x-y-1=0$

[AP EAMCET 04-07-2022_Shift-1]

1. intersects the hyperbola at two points
2. does not intersect the hyperbola
3. touches the hyperbola
4. passes through the vertex of the hyperbola

24. The locus of a variable point whose chord of contact w.r.t. the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ subtends a right angle at the origin is

[AP EAMCET 04-07-2022_Shift-1]

1. $\frac{x^2}{4a^2}-\frac{y^2}{4b^2}=1$
2. $\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)=\frac{x^2}{a^4}+\frac{y^2}{b^4}$
3. $\frac{x}{a}-\frac{y}{b}=\frac{1}{a^2}+\frac{1}{b^2}$
4. $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}-\frac{1}{b^2}$

25. If the normal to the rectangular hyperbola $x^2-y^2=1$ at the point $P\left(\frac{\pi}{4}\right)$ meets the curve again at Q(θ), then $\sec^2\theta+\tan\theta=$

[AP EAMCET 04-07-2022_Shift-2]

1. 43
2. 57
3. 3
4. 1

26. If the vertices and foci of a hyperbola are respectively $(\pm3,0)$ and $(\pm4,0)$ then the parametric equations of that hyperbola are

[AP EAMCET 04-07-2022_Shift-2]

1. $x=3\sec\theta, y=7\tan\theta$
2. $x=\sqrt{3}\sec\theta, y=\sqrt{7}\tan\theta$
3. $x=\sqrt{3}\sec\theta, y=7\tan\theta$
4. $x=3\sec\theta, y=\sqrt{7}\tan\theta$

27. The locus of point of intersection of tangents at the ends of normal chord of the hyperbola $x^2-y^2=a^2$ is

[AP EAMCET 05-07-2022_Shift-1]

1. $y^4-x^4-4a^2x^2y^2$
2. $y^2-x^2=4a^2x^2y^2$
3. $a^2(y^2-x^2)=4x^2y^2$
4. $y^2+x^2=4a^2x^2y^2$

28. If e_1 and e_2 are the eccentricities of the hyperbola $16x^2-9y^2=1$ and its conjugate respectively. Then $3e_1=$

[AP EAMCET 05-07-2022_Shift-1]

1. $5e_2$
2. $4e_2$
3. $2e_2$
4. e_2

29. If $\sqrt{5}y-\sqrt{8}=0$ is the equation of the directrix of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}+1=0$ and $\frac{\sqrt{5}}{2}$ is its eccentricity then $a=$

[AP EAMCET 05-07-2022_Shift-2]

1. $\sqrt{2}$
2. $\sqrt{3}$
3. $\sqrt{5}$
4. $\sqrt{6}$

30. The locus of the point of intersection of the tangents at the end–points of normal chords of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

[AP EAMCET 05-07-2022_Shift-2]

1. $\frac{a^6}{x^2}+\frac{b^6}{y^2}=(a^2+b^2)^2$
2. $\frac{a^6}{x^2}-\frac{b^6}{y^2}=(a^2+b^2)^2$
3. $\frac{a^6}{x^2}-\frac{b^6}{y^2}=(a^2-b^2)^2$
4. $\frac{a^6}{x^2}+\frac{b^6}{y^2}=(a^2-b^2)^2$

31. Let $L(x_1,4)$ be the end of the Latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ lying in the first quadrant and let $S(8,y_1)$ be the focus of the given hyperbola. Then the length of its transverse axis is

[AP EAMCET 06-07-2022_Shift-1]

1. $2(\sqrt{17}-1)$
2. $4(\sqrt{17}-1)$
3. $2(\sqrt{17}+1)$
4. $4(\sqrt{17}+1)$

32. If the line $3x-my+5=0$ is a tangent to the hyperbola $3x^2-4y^2=300$ then the square of the Y−intercept made by this tangent line is

[AP EAMCET 06-07-2022_Shift-1]

1. $\frac{25}{3}$
2. $\frac{35}{3}$
3. $\frac{45}{7}$
4. $\frac{15}{7}$

33. Let $x^2+y^2=16$ be the equation of the auxiliary circle of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and let $(4\sqrt{2},3)$ be a point on the hyperbola. Then the eccentricity of the hyperbola is

[AP EAMCET 06-07-2022_Shift-2]

1. 5/4
2. 5/3
3. 4/3
4. 2

34. The distance between the tangent lines to the hyperbola $x^2-2y^2=18$ which are perpendicular to the line y = x is

[AP EAMCET 06-07-2022_Shift-2]

1. 6
2. $3\sqrt{2}$
3. $2\sqrt{3}$
4. 0

35. For a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, if the length of the transverse axis is 8 and the distance between the foci is $2\sqrt{41}$, then the length of its latus rectum is

[AP EAMCET 07-07-2022_Shift-1]

1. $\frac{25}{2}$
2. $\frac{32}{5}$
3. $\frac{25}{4}$
4. $\frac{16}{5}$

36. If the product of the perpendicular distances from any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to its asymptotes is 6 and eccentricity of the hyperbola is $\sqrt{3}$, then the length of the conjugate axis of the hyperbola is

[AP EAMCET 07-07-2022_Shift-1]

1. 3
2. 6
3. 8
4. 12

37. If (1,2) is the focus, $x+2y=0$ is the directrix and $\sqrt{2}$ is the eccentricity of a hyperbola then the equation of the hyperbola is

[AP EAMCET 07-07-2022_Shift-2]

1. $x^2-y^2=a^2$
2. $3x^2-8xy-3y^2-10x-20y+25=0$
3. $xy=c^2$
4. $3x^2-8xy-3y^2+10x-20y-25=0$

38. The locus of the point of intersection on the line $\sqrt{3}x-y-4\sqrt{3}k=0$ and $\sqrt{3}kx+ky-4\sqrt{3}=0$ for different real values of k is a hyperbola H. If e is the eccentricity of H then $4e^2=$

[AP EAMCET 07-07-2022_Shift-2]

1. 48
2. 39
3. 13
4. 16

39. If one of the roots of the equation $x^2-5x-14=0$ is the length of the semi conjugate axis of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and the square of the other root is the semi-transverse axis then the focus of the hyperbola that lies on the positive x-axis is

[AP EAMCET 08-07-2022_Shift-1]

1. (5,0)
2. $(\sqrt{65},0)$
3. (7,0)
4. $(\sqrt{74},0)$

40. If the chord of contact of a point P w.r.t. the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ passes through a fixed point (α,β), then the locus of P is a straight line. The sum of the squares of the intercepts made by this line on the coordinate axes is

[AP EAMCET 08-07-2022_Shift-1]

1. $\frac{a^4}{\alpha^2}+\frac{b^4}{\beta^2}$
2. $\frac{\alpha^4}{a^2}+\frac{\beta^4}{b^2}$
3. $\frac{\alpha^2}{4a^2}+\frac{\beta^2}{4b^2}$
4. $\frac{a^2}{4\alpha^2}+\frac{b^2}{4\beta^2}$

41. From any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, tangents are drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2$. The area of the figure formed by the chord of contact of that point and the asymptotes is

[AP EAMCET 08-07-2022_Shift-2]

1. $\frac{ab}{2}$
2. ab
3. 2ab
4. 4ab

42. Statement 1: The eccentricity of the hyperbola $9x^2-16y^2-72x+96y-144=0$ is 5/4

Statement 2: The eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\sqrt{1+\frac{b^2}{a^2}}$

[AP EAMCET 08-07-2022_Shift-2]

1. Statement 1 is true , Statement 2 is true; Statement 2 is correct explanation for Statement 1
2. Both statements are true and statement 2 is not the correct explanation of statement 1
3. Statement 1 is false; Statement 2 is true
4. Statement 1 is true; Statement 2 is false

43. If $\frac{x^2}{k-\frac{5}{2}}+\frac{y^2}{\frac{7}{3}-k}=1$ (k is a real number) represents a hyperbola, then the set of all values of k is **[TS EAMCET 18-07-2022_Shift-1]**

1. $\left(-\infty,\frac{7}{3}\right)\cup\left(\frac{5}{2},\infty\right)$
2. $\left(\frac{7}{3},\frac{5}{2}\right)$
3. $\left(-1,\frac{7}{3}\right)\cup\left(\frac{5}{2},1\right)$
4. $R-\left(\frac{7}{3},\frac{5}{2}\right)$

44. Let A (θ_1) and B (θ_2) be two points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and S be the focus of the hyperbola. If A, S, B are the collinear and $a\cos\left(\frac{\theta_1+\theta_2}{2}\right)=k\cos\left(\frac{\theta_1-\theta_2}{2}\right)$ then $k=$

[TS EAMCET 18-07-2022_Shift-1]

1. a^2+b^2
2. $\sqrt{a^2+b^2}$
3. a^2-b^2
4. $a+b$

45. Let S be the focus of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ lying on the positive X- axis and $P(5,y_1)$ be point on the hyperbola. Then SP =

[TS EAMCET 18-07-2022_Shift-2]

1. $\frac{1}{4}$
2. $\frac{3}{4}$
3. $\frac{9}{4}$
4. $\frac{5}{4}$

46. If $P(\theta)=\left(x_1,\frac{3\sqrt{5}}{2}\right), 0<\theta<\frac{\pi}{2}$ is a point on the hyperbola $\frac{x^2}{25}-\frac{y^2}{9}=1$, where θ is the parameter in its parametric form, then $2x_1+9\sin^2\theta$

[TS EAMCET 18-07-2022_Shift-2]

1. 8
2. 10
3. 20
4. 34

47. Assertion(A): The image of $\frac{x^2}{25}+\frac{y^2}{16}=1$ in the line $x+y=10$ is $\frac{(x-10)^2}{16}+\frac{(y-10)^2}{25}=1$.

Reason(R): The image of a curve 'C' in a line L is the locus of the image of every point of C with respect to the line L.

The correct option among the following is:

[TS EAMCET 19-07-2022_Shift-I]

1. A is true, R is true and R is the correct explanation for A
2. A is true, R is true but R is not the correct explanation for A.
3. A is true but R is false
4. A is false but R is true

48. If the latus rectum of a hyperbola subtends an angle of 120° at its centre, then its eccentricity is

[TS EAMCET 19-07-2022_Shift-I]

1. $\frac{\sqrt{3}+2}{\sqrt{2}}$
2. $\frac{\sqrt{3}+\sqrt{5}}{2}$
3. $\frac{\sqrt{3}-\sqrt{2}}{3}$
4. $\frac{\sqrt{3}+\sqrt{7}}{2}$

49. Let $P\left(\frac{\pi}{4}\right), Q\left(\frac{5\pi}{4}\right), R\left(\frac{3\pi}{4}\right), T\left(\frac{7\pi}{4}\right)$ be the points on the hyperbola $x^2-4y^2-4=0$ in the parametric form. Then the area of the quadrilateral PQRT is (in square units)

[TS EAMCET 19-07-2022_Shift-1]

1. $4\sqrt{2}$
2. $16\sqrt{2}$
3. $32\sqrt{2}$
4. $8\sqrt{2}$

50. A hyperbola having its centre at the origin is passing through the point (5, 2) and has transverse axis of length 8 along the X axis. Then the eccentricity of its conjugate hyperbola is

[TS EAMCET 19-07-2022_Shift-2]

1. $\frac{\sqrt{13}}{3}$
2. $\sqrt{\frac{13}{3}}$
3. $\frac{\sqrt{13}}{2}$
4. $\sqrt{\frac{13}{2}}$

51. If e_1 is the eccentricity of the hyperbola $x=\sec\theta, y=\sqrt{2}\tan\theta$ and e_2 is the eccentricity of the hyperbola $x=\sqrt{2}\sec\theta, y=\tan\theta, then \frac{e_2^2}{e_1^2}=$

[TS EAMCET 19-07-2022_Shift-2]

1. 1
2. 2
3. $\frac{1}{2}$
4. $\frac{1}{4}$

52. Let e_1 be the eccentricity of a hyperbola for which distance between its focii is 2 times the distance between its directrices and e_2 be the eccentricity of another hyperbola for which the length of its transverse axis is twice the length of it's the conjugate axis. Then e_1e_2=

[TS EAMCET 20-07-2022_Shift-1]

1. 1
2. $\frac{\sqrt{10}}{2}$
3. $\sqrt{5}$
4. $\frac{\sqrt{5}}{2}$

53. Assertion(A): The distance between the points $p\left(\frac{\pi}{4}\right) and p\left(\frac{\pi}{3}\right)$ on the hyperbola $9x^2+16y^2=9$ is $\frac{1}{2\sqrt{2}}\sqrt{66-33\sqrt{2}-9\sqrt{3}}$

Reason(R): $x=a\cosh t, y=b\sinh t$ are the parametric equations of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

[TS EAMCET 20-07-2022_Shift-1]

1. (A) is true,(R) is true and (R) is the correct explanation for (A)
2. (A) is true,(R) is true and (R) is not the correct explanation for (A)
3. (A) is true but (R) is false
4. (A) is false but (R) is true

54. Let S be the focus of the hyperbola $x^2-2y^2=1$ lying on the positive X-axis. Let $P(-1,1)$ be a given point. Then the area of the triangle formed by the line PS with the coordinate axes is (in sq. units)

[TS EAMCET 20-07-2022_Shift-2]

1. $\frac{\sqrt{2}}{2(\sqrt{2}+3)}$
2. $\frac{\sqrt{6}}{2(2+\sqrt{6})}$
3. $\frac{3}{2(2+\sqrt{6})}$
4. $\frac{\sqrt{3}}{2(\sqrt{2}+\sqrt{3})}$

55. If $P\left(\frac{\pi}{6}\right)$ is a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, S, S^1 are its foci and SP + S'P = 2|SP – S'P|, then e =

[TS EAMCET 20-07-2022_Shift-2]

1. $\sqrt{2}$
2. 2
3. $\sqrt{3}$
4. 3

KEY

1)	3	2)	3	3)	4	4)	4	5)	3
6)	4	7)	3	8)	1	9)	1	10)	2
11)	3	12)	2	13)	1	14)	4	15)	2
16)	3	17)	2	18)	3	19)	4	20)	4
21)	3	22)	4	23)	2	24)	4	25)	2
26)	4	27)	3	28)	2	29)	4	30)	2
31)	2	32)	4	33)	1	34)	2	35)	1
36)	2	37)	2	38)	4	39)	2	40)	1
41)	4	42)	1	43)	1	44)	2	45)	3
46)	3	47)	1	48)	4	49)	4	50)	3
51)	3	52)	2	53)	3	54)	3	55)	3

SOLUTIONS

1. $\dfrac{x^2}{\frac{144}{25}}-\dfrac{y^2}{\frac{81}{25}}=1$ $\quad\quad$ $\dfrac{x^2}{16}+\dfrac{y^2}{b^2}=1$

$e=\dfrac{5}{4}$ $\quad\quad$ $e=\sqrt{\dfrac{16-b^2}{16}}$

$foci=(\pm3,0)$ $\quad\quad$ $foci=\left(\pm\sqrt{16-b^2},0\right)$

$3=\sqrt{16-b^2}$

$b^2=7$

2. $polar\ is\ 9\alpha x-16\beta y-144=0$

$3x-16y+48=0$

$\dfrac{9\alpha}{3}=\dfrac{-16\beta}{-6}=\dfrac{-144}{48}$

$\alpha=-1,\ \beta=-3$

3. $\theta+\phi=\dfrac{\pi}{2}$

Equation of normal at $p(\theta)$ is

$\dfrac{ax}{\sec\theta}+\dfrac{by}{\tan\theta}=a^2+b^2....(1)$

Equation of normal at $p(\phi)$ is

$\dfrac{ax}{\sec\phi}+\dfrac{by}{\tan\phi}=a^2+b^2.....(2)$

Solving (1) and (2) we get (h, k)

4. $e=\sqrt{2}$

$2ae=16$

$a=4\sqrt{2}$

$a^2=32$

$b^2=a^2\left(e^2-1\right)$

$=32(2-1)=32$

Eq. of hyperbola is

$\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

$\dfrac{x^2}{32}-\dfrac{y^2}{32}=1$

$x^2-y^2=32$

5. $x^2+y^2=a^2\rightarrow(1)\quad\quad xy=b^2\rightarrow(2)$

From (1) and (2) $y^4-a^2y^2+b^4=0$

$y_1y_2y_3y_4=b^4$

6. Let, the asymptotes are $x=a,\ and\ y=b$

w.k.t. P.O.I of perpendicular tangents is

$(1,1)$ i.e. $a=1\ and\ \ b=1\ \therefore x=1\, and\ \ y=1$

Now, the equation of hyperbola is

$(x-1)(y-1)+\lambda=0$

Clearly it passes through $(3,2)$

$\therefore\lambda=-2$

Equation of hyperbola is $xy=x+y+1$

7. $2a=12$

$a=6$

$\dfrac{x^2}{36}-\dfrac{y^2}{b^2}=1$

$(8,2)$ lies on the hyperbola $\Rightarrow\dfrac{64}{36}-\dfrac{4}{b^2}=1$

$b^2=\dfrac{36}{7},a^2=36\ e=\sqrt{\dfrac{a^2+b^2}{a^2}}=\sqrt{\dfrac{8}{7}}$

8. $\dfrac{1}{p^2}+\dfrac{1}{q^2}=1$

$By\ solving\ \dfrac{x^2}{p^2}+\dfrac{y^2}{q^2}=1\ and\ x^2-y^2=0, we get$

$the\ points\ (1,1),(-1,1),(-1,-1),(1,-1)$

$Ar(Square)=\dfrac{1}{2}\begin{vmatrix}2 & -2\\ 2 & 2\end{vmatrix}=4\,sq.units$

9. Area of Δ^{le} formed by asymptotes & tangent $=ab$

$ab=a^2Tan\alpha\Rightarrow b=aTan\alpha$

Now, $e=\sqrt{1+\dfrac{b^2}{a^2}}=\sqrt{1+\dfrac{a^2\tan^2\alpha}{a^2}}=\sec\alpha$

10. C= Centre = foot of perpendicular from S to direction.

A divides CS in $1:e-1$ ratio.

11. Standard

12. Rectangular hyperbola

13. Standard

14. Standard

15. Standard

16. Standard

17. $S=4x^2-3y^2-1$

$S_{11}<0$

$4\alpha^2-4<0$

$(\alpha+1)(\alpha-1)<0$

$\alpha\in(-1,1)$

18. $\frac{x^2}{9}-\frac{y^2}{36}=1\ldots(1)$

$x\cos\alpha+y\sin\alpha=p$

$pole=\left(\frac{-a^2l}{n},\frac{b^2m}{n}\right)$

$=\left(\frac{-9\cos\alpha}{-p},\frac{36\sin\alpha}{-p}\right)$

$=\left(\frac{9\cos\alpha}{p},\frac{-36\sin\alpha}{p}\right)$

$\frac{x^2}{9}-\frac{y^2}{36}=1(1)^2$

$\frac{x^2}{9}-\frac{y^2}{36}=1\left(\frac{x\cos\alpha+y\sin\alpha}{p}\right)^2$

$\frac{1}{9}-\frac{\cos^2\alpha}{p^2}-\frac{1}{36}-\frac{\sin^2\alpha}{p^2}=0$

$\frac{1}{9}-\left(\frac{x_1}{9}\right)^2-\frac{1}{36}-\left(\frac{y_1}{-36}\right)^2=0$

$\Rightarrow 16x_1^2+y_1^2=108$

19. $\alpha+3>0, 2-\alpha<0$

Case –(i) $\alpha>-3$ and $\alpha>2$

$\alpha\in(-3,\infty)$ and $\alpha\in(2,\infty)$

$\alpha\in(-3,\infty)$ on $(-\infty,2)$

$\therefore \alpha\in(2,\infty)$

Case –(ii) $\alpha+3<0$ and $2-\alpha>0$

$\alpha<-3$ and $\alpha<2$ $\alpha\in(-\infty,-3),\alpha\in(-\infty,2)$

$\therefore \alpha\in(-\infty,-3)$

From case –(i) and case-(ii)

$\alpha\in(-\infty,-3)\cup(2,\infty)$

20. Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$SS'=2ae, PS=b^2/a$

In $\Delta S'SP, \tan 30^0=\frac{SP}{SS'}$

$\Rightarrow \frac{1}{\sqrt{3}}=\frac{\frac{b^2}{a}}{2ae}=\frac{b^2}{2a^2e}$

$\Rightarrow \frac{1}{\sqrt{3}}=\frac{a^2(e^2-1)}{2a^2e}=\frac{e^2-1}{2e}$

$\Rightarrow \sqrt{3}e^2-2e-\sqrt{3}=0$

$(e-\sqrt{3})(\sqrt{3}e+1)=0$

$e=\sqrt{3}, e\neq -1/\sqrt{3}$

21. $e=\frac{5}{4},\frac{2b^2}{a}=9$

$e^2=\frac{25}{16}$

$\frac{a^2+b^2}{a^2}=\frac{25}{16}$

$a^2+\frac{9a}{2}=\frac{25}{16}a^2$

$\Rightarrow a=8, b=6, ab=48$

22 Given hyperbola is

$5x^2-6y^2-10x-24y-34=0$

$5(x-1)^2-6(y+2)^2=15$

$\frac{(x-1)^2}{3}-\frac{(y+2)^2}{3/2}=1$

$e=\frac{\sqrt{a^2+b^2}}{a}\Rightarrow ae=\frac{\sqrt{11}}{\sqrt{2}}$

Foci $=S=(h\pm ae,k)=\left(1\pm\frac{\sqrt{11}}{\sqrt{2}},-2\right)$

23. $2ae = 6, e = 3/2$

on solving a = 2, $b^2=5$

24. Equation of chord with respect to

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \to (1)$ $from(x_1, y_1)$ is $S_1 = 0$

$i.e., \frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1 \to (2)$

Homogenise (1) with respect (2) then apply

$x^2 coeff + y^2 coeff = 0.$

25. Any point

$P\left(\pi/4\right) on\ x^2 - y^2 = 1 \ .....(1)$

$is\left(\sqrt{2}, 1\right) p$

At p, tangent slope $=\sqrt{2}$ *Then* normal slope

slope $= \frac{-1}{\sqrt{2}}$

Normal Eq:-

$x + \sqrt{2}\, y = 2\sqrt{2} \to (2)$

Solve (1) & (2)

we get $\left(\sqrt{2}, 1\right)\left(5\sqrt{2}, 7\right) = (\sec\theta, \tan\theta)$

26. Given that 2a = 6; 2ae = 8.

Solving we get, a = 3, b= $\sqrt{7}$

27. Let p(h, k) be the point of intersection of tangents at the ends of a normal chord

Its equation :- $hx - ky - a^2 = 0 \to (1)$

(1) is a normal to hyperbola. So, it must be of the form $x \cos\theta + y \cot\theta = 2a \to (2)$

(1), (2) are same lines

$$\Rightarrow \frac{\cos\theta}{h} = \frac{\cot\theta}{-k} = \frac{2a}{a^2}.$$

$$\Rightarrow \sec\theta = \frac{a}{2h}, \ tan\theta = \frac{-a}{2k}.$$

Eliminate 'θ' *we get*, $a^2(k^2 - h^2) = 4h^2k^2.$

28. Use $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

29. Data is in correct.

30. Locus be P(h, k)

$a\ x\cos\theta + by\ \cot\theta = a^2 + b^2 \to (1)$

$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1 \Rightarrow \frac{hx}{a^2} - \frac{ky}{b^2} = 1 \ \to (2)$

$(1) \& (2) \Rightarrow \frac{a\cos\theta}{\frac{h}{a^2}} = \frac{b\cot\theta}{\frac{-k}{b^2}} = \frac{a^2+b^2}{1}$

$\therefore \cos\theta = \frac{h\left(a^2+b^2\right)}{a^3} \Rightarrow \sec\theta = \frac{a^3}{h\left(a^2+b^2\right)}$

$\cot\theta = \frac{-k\left(a^2+b^2\right)}{b^3} \Rightarrow \tan\theta = \frac{-b^3}{k\left(a^2+b^2\right)}$

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \frac{a^6}{x^2} - \frac{b^6}{y^2} = \left(a^2+b^2\right)^2$

31. ae=8, $\frac{b^2}{a} = 4$

$\Rightarrow a^2\left(e^2-1\right) = 4a$

$\Rightarrow a = 2\left(\sqrt{17}-1\right)$

$\therefore 2a = 4\left(\sqrt{17}-1\right)$

32. Apply tangent condition to get $m^2 = 35/3$.

We get y-intercept :- $\frac{5}{m}$.

Sq. of y- intercept= $\frac{15}{7}$

33. Here a = 4. (radius of auxiliary circle)

$P(4\sqrt{2}, 3) \in$ hyperbola then we get $b^2 = 9$.

34. Tangent $\perp^{er}$ *to* $y = x$ *is* $x + y + k = 0 \to (1)$

If (1) is a tangent to $\frac{x^2}{18} - \frac{y^2}{9} = 1$ *Then* $k = \pm 3$.

35. Conceptual

36. Product of $\perp rs = \frac{a^2b^2}{a^2+b^2} = 6 \to (1)$

$\& e = \sqrt{3} \Rightarrow \frac{a^2+b^2}{a^2} = 3 \to (2)$

By (1)& (2)

$\frac{a^2b^2}{3a^2} = 6 \Rightarrow b^2 = 18 \Rightarrow b = 3\sqrt{2}$

Length of conjugate axis 2b $= 6\sqrt{2}$.

37. Use $\frac{SP}{PM} = e.$

38. By eliminating ' K', we get $\frac{x^2}{16} - \frac{y^2}{64} = 1$

39. $a = (-2)^2 = 4, b = 7$

then $e = \frac{\sqrt{65}}{4}$

40. Chord of contact eq: $s_1 = 0.ie. \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1 = 0.$

(α,β) lies on it then $\frac{x_1}{a^2}\alpha - \frac{y_1}{b^2}\beta - 1 = 0.$

This is a straight line.

Sum of squares of intercepts $= \frac{a^4}{\alpha^2} + \frac{b^4}{\beta^2}$

41. $P(a,o) \in \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Chord of contact of P to

$\frac{x^2}{a_2} - \frac{y^2}{b^2} = 2 \ is \ \frac{ax}{a^2} - \frac{ay}{b^2} - 2 = 0 \rightarrow (1)$

Solve (1) with $ay = bx \Rightarrow Q(2a,2b)$

Solve (1) *with* $ay + bx = 0 \Rightarrow R(2a,-2b).$

$\therefore area \ of \Delta PQR == 4ab$

42. Conceptual.

43. $K - \frac{5}{2} > 0 \ \& \ \frac{7}{3} - k > 0.$

44. Conceptual.

45. Find semi latusrectum ie, $\frac{b^2}{a}$

46. $P(\theta) = (5\sec\theta, 3\tan\theta) = \left(x_1, \frac{3\sqrt{5}}{2}\right)$

$\therefore x_1 = 5\sec\theta$ and $\tan\theta = \frac{\sqrt{5}}{2}$

And

$\sec^2\theta = \frac{9}{4} \Rightarrow \sec\theta = \frac{3}{2}$

$\therefore 2x_1 = 15.$

$: 2x_1 + 9\sin^2\theta = 20$

47. Canceptual

48. Conceptual

49. $P(2\sqrt{2},1)Q(-2\sqrt{2},1)R(-2\sqrt{2},-1)$

$R(2\sqrt{2},-1)$

50. $2a = 8. \ \& \ (5,2) \in \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $b^2 = \frac{64}{9}$

Then $e_H = \frac{\sqrt{13}}{3}$

$WKT, \frac{1}{e_H^2} + \frac{1}{e_e^2} = 1 \ of \ e_e^2 = \frac{\sqrt{13}}{2}$

51. Given curves $x^2 - \frac{y^2}{2} = 1 \ \& \frac{x^2}{2} - y^2 = 1$

52. $2ae_1 = 2\left(\frac{2a}{e_1}\right) \Rightarrow e_1 = \sqrt{2}$

$\& 2a = 2(2b) \Rightarrow a = 2b.$

$e_2 = \sqrt{\frac{1+b^2}{a^2}} = \frac{\sqrt{5}}{2}$

$\therefore e_1 e_2 = \frac{\sqrt{5\times 2}}{\sqrt{2}} = \frac{\sqrt{10}}{2}$

53. $\frac{x^2}{1} + \frac{y^2}{\frac{9}{16}} = 1$

$p\left(\frac{\pi}{4}\right) = \left(\sqrt{2}, \frac{3}{4}\right)$

$p\left(\frac{\pi}{3}\right) = \left(2, \frac{3\sqrt{3}}{4}\right)$

$dis\tan ce = \frac{1}{2\sqrt{2}}$

$\sqrt{66 - 32\sqrt{2} - 9\sqrt{3}}$

54. $\frac{x^2}{1} - \frac{y^2}{\frac{1}{2}} = 1 \Rightarrow e = \frac{\sqrt{3}}{\sqrt{2}}.$

$\therefore s\left(\frac{\sqrt{3}}{\sqrt{2}} \quad 0\right)$

Equation of PS is

$\frac{x}{\frac{\sqrt{3}}{\sqrt{2}}} + \frac{y}{\frac{\sqrt{3}}{\sqrt{2}+\sqrt{3}}} = 1$

$area = \frac{1}{2}ab = \frac{3}{2(2+\sqrt{6})}$

55. $P = \left(\frac{2a}{\sqrt{3}}, \frac{b}{\sqrt{3}}\right)$

$SP + S'P = 4a$

$\Rightarrow e^2 = 3$

$\Rightarrow e = \sqrt{3}$