41. INDEFINITE INTEGRATION

1. If $\int \frac{\cos(13x)-\cos(14x)}{1+2\cos(9x)}dx = \frac{\sin(4x)}{a} - \frac{\sin(5x)}{b} + c$, then $a^b =$ **[AP EAMCET 17-09-20_Shift-2]**
   1. 4^5 2. 5^4
   3. 4^4 4. 5^5

2. Solve $I_n + nI_{n-1}$ if $I_n = \int (\ln x)^n \, dx$ **[AP EAMCET 17-09-20_Shift-2]**
   1. $x(\ln x)^{n-1}+k$ 2. $x(\ln x)^n + k$
   3. $\frac{(\ln x)^n}{x}+k$ 4. $\frac{(\ln x)^{n-1}}{x}+k$

3. $\int \frac{\cos^3(x)}{\sin^2(x)+\sin(x)}dx =$ **[AP EAMCET 17-09-20_Shift-2]**
   1. $\log|\sin(x)|+\sin(x)+c$
   2. $\log|\sin(x)|+\cos(x)+c$
   3. $\log|\cos(x)|-\sin(x)+c$
   4. $\log|\sin(x)|-\sin(x)+c$

4. $\int \left(\log(\log x) + \frac{1}{(\log x)^2} \right) dx =$ **[AP EAMCET 18-09-20_Shift-1]**
   1. $x[\log(\log x)+\log x]+c$
   2. $\frac{x}{\log(\log x)}+c$
   3. $x\log(\log x)+c$
   4. $x\left[\log(\log x) - \frac{1}{\log x}\right]+c$

5. $\int \sqrt{x-1}\left(x\sqrt{x+1}\right)^{-1} dx =$ **[AP EAMCET 18-09-20_Shift-1]**
   1. $\ln\left|x+\sqrt{x^2-1}\right| - Sec^{-1}x + c$
   2. $\ln\left|x-\sqrt{x^2-1}\right| - Tan^{-1}x + c$
   3. $\ln\left|x+\sqrt{x^2-1}\right| + \sec^{-1}x + c$
   4. $\ln\left|x+\sqrt{x^2-1}\right| - Tan^{-1}x + c$

6. $\int \frac{\cos 7x - \cos 8x}{1+2\cos 5x}dx =$ **[AP EAMCET 18-09-20_Shift-1]**
   1. $\sin 2x - \frac{1}{3}\sin 3x + c$
   2. $\frac{1}{2}\sin 2x - \frac{1}{3}\sin 3x + c$
   3. $\frac{1}{2}\sin 2x - \sin 3x + c$
   4. $\frac{1}{3}\sin 2x - \frac{1}{2}\sin 3x + c$

7. Evaluate $\int \sin\left(\sqrt{k}\right) dk$ on $(0,\infty)$ **[AP EAMCET 18-09-20_Shift-1]**
   1. $2\left[\cos\left(\sqrt{k}\right) - \sqrt{k}\sin\left(\sqrt{k}\right)\right]+c$
   2. $2\left[\cos\left(\sqrt{k}\right) + \sqrt{k}\sin\left(\sqrt{k}\right)\right]+c$
   3. $2\left[\sqrt{k}\cos\left(\sqrt{k}\right) - \sqrt{k}\sin\left(\sqrt{k}\right)\right]+c$
   4. $2\left[\sin\left(\sqrt{k}\right) - \sqrt{k}\cos\left(\sqrt{k}\right)\right]+c$

8. $\int e^{x/2}\left(\frac{2+\sin x}{1+\cos x}\right)dx =$ **[AP EAMCET 18-09-20_Shift-2]**
   1. $2e^{x/2}\operatorname{cosec}(x/2)+c$ 2. $2e^{x/2}\tan(x/2)+c$
   3. $2e^{x/2}\cos(x/2)+c$ 4. $2e^{x/2}\sin(x/2)+c$

9. $\int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right]dx =$ **[AP EAMCET 18-09-20_Shift-2]**
   1. $x\log x + c$ 2. $-x\log x + c$
   3. $\frac{\log x}{x}+c$ 4. $\frac{x}{\log x}+c$

10. Let $f(x) = Tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$; $g(x) = Tan^{-1}\left(\frac{\sin x}{1-\cos x}\right)$, then $\int (f(x)+g(x))dx =$ **[AP EAMCET 18-09-20_Shift-2]**
    1. $\frac{\pi x}{2} - \frac{x^2}{4} + C$ 2. $\pi x - \frac{x^2}{2} + C$
    3. $\pi x + \frac{x^2}{4} + C$ 4. $\pi x + \frac{x^2}{2} + C$

11. Let the equation of the curve passing through the point (0,1) be given by $y=\int x^3 e^{x^4} dx$. If the equation of the curve is written in the form x=f(y), then f(y)=

[AP EAMCET 21-09-20_Shift-1]

1. $\log|4y-3|$ 2. $(\log|4y-3|)^{1/4}$
3. $(\log|\frac{3-4y}{4}|)^{1/4}$ 4. $\log|\frac{4y-3}{4}|$

12. $\int \frac{\sin^3(x)+\cos^3(x)}{\sin^2(x).\cos^2(x)} dx =$

[AP EAMCET 21-09-20_Shift-1]

1. sec(x)-cosec(x)+c 2. tan(x)+cot(x)+c
3. cosec(x)-cot(x)+c 4. tan(x)-cot(x)+c

13. $\int \frac{dx}{\cos^2(x)+\sin(2x)} =$

[AP EAMCET 21-09-20_Shift-1]

1. $\frac{1}{2}\log|1+2\cos(x)|+c$
2. $\frac{1}{2}\log|1-2\tan(x)|+c$
3. $\frac{1}{2}\log|1+2\tan(x)|+c$
4. $\frac{1}{2}\log|1+2\cot(x)|+c$

14. $\int \frac{(x+1)^2}{x(x^2+1)} dx =$ **[AP EAMCET 21-09-20_Shift-1]**

1. $\log[x(x^2+1)]+c$
2. $\log|x|+c$
3. $\log|x|+2\text{Tan}^{-1}(x)+c$
4. $2\log|x|+\text{Tan}^{-1}(x)+c$

15. If $\int \frac{1+\cos(4x)}{\cot x-\tan x} dx = k\cos(4x)+c, then$

[AP EAMCET 21-09-20_Shift-2]

1. $k=\frac{1}{8}$ 2. $k=\frac{1}{4}$
3. $k=\frac{-1}{8}$ 4. $k=\frac{-1}{4}$

16. $\int x(\tan^2 x)dx =$

[AP EAMCET 21-09-20 Shift-2]

1. $x\tan(x)-\log(\sec x)-\frac{x^2}{2}+c$
2. $x\tan(x)+\log(\sec x)-\frac{x^2}{2}+c$
3. $x\tan(x)-\log(\sec x)+\frac{x^2}{2}+c$
4. $x\tan(x)+\log(\sec x)+\frac{x^2}{2}+c$

17. $\int x^{2020}(\tan^{-1}x+\cot^{-1}x)dx$

[AP EAMCET 21-09-20_Shift-2]

1. $\frac{x^{2021}}{2020}(\tan^{-1}x+\cot^{-1}x)+c$
2. $\frac{x^{2021}}{2021}(\tan^{-1}x+\cot^{-1}x)+c$
3. $\frac{\pi x^{2021}}{2021}+\frac{\pi}{2}+c$
4. $\frac{x^{52}}{52}+\frac{\pi}{2}+c$

18. Suppose that f and g are integrable on [a,b].then f+g is integrable on........

[AP EAMCET 21-09-20_Shift-2]

1. (a,b) 2. Cannot comment
3. [a,b] 4. Range of f+g

19. Integral $\int\left(\frac{2x^3-3x+5}{2x^2}\right)dx$ is valid for

[AP EAMCET 21-09-20_Shift-2]

1. $x\in R-\{0\}$ 2. $x>0$
3. $x<0$ 4. $x\in R$

20. $\int(1+e^{-x})^{-1}dx =$

[AP EAMCET 22-09-20_Shift-1]

1. $\log(1+e^{-x})+c$ 2. $\log(1+e^{x})+c$
3. $\log(1-e^{x})+c$ 4. $\log(e^{x}-1)+c$

21. $\int_0^1 (1+x)\log(1+x)\,dx =$

[AP EAMCET 22-09-20_Shift-1]

1. $\frac{-3}{4}+\log 2$ 2. $\frac{3}{4}+2\log 2$
3. $2\log 2$ 4. $\frac{-3}{4}+2\log 2$

22. $\int\left(\sqrt{1+\sin(2x)}\right)dx =$

[AP EAMCET 22-09-20_Shift-1]

1. $\cos(x)+\sin(x)+c$
2. $\cos(x)-\sin(x)+c$
3. $\sin(x)-\cos(x)+c$
4. $\sin(x)-\cos ec(x)+c$

23. $\int\frac{\cos x-\sin x}{5+\sin(2x)}dx =$

[AP EAMCET 22-09-20_Shift-1]

1. $\frac{1}{2}\mathrm{Cot}^{-1}\left[\frac{1}{2}(\sin x+\cos x)\right]+c$
2. $\frac{1}{2}\mathrm{Tan}^{-1}\left[\frac{1}{2}(\sin x+\cos x)\right]+c$
3. $\frac{1}{2}\mathrm{Sin}^{-1}\left[\frac{1}{2}(\sin x+\cos x)\right]+c$
4. $\frac{1}{2}\mathrm{Cos}^{-1}\left[\frac{1}{2}(\sin x+\cos x)\right]+c$

24. $\int e^{3\log x}\left(x^4+1\right)^{-1}\,dx =$

[AP EAMCET 22-09-20_Shift-1]

1. $e^{3\log x}+c$ 2. $\frac{1}{4}\log(x^4+1)+c$
3. $\frac{1}{3}\log(x^4+1)+c$ 4. $\frac{x^4}{x^4+1}$

25. $\int\frac{\sin(2x)}{\sin^2(x)+2\cos^2(x)}dx =$

[AP EAMCET 22-09-20 Shift-2]

1. $\log|1+\cos^2(x)|+c$
2. $-\log|1+\sin^2(x)|+c$
3. $\log|1+\tan^2(x)|+c$
4. $-\log|1+\cos^2(x)|+c$

26. If $\int\frac{5\tan x}{(\tan x)-2}dx =$
$\alpha x+\beta\log|\sin x-2\cos x|+\gamma$, then $\alpha-\beta =$

[AP EAMCET 22-09-20_Shift-2]

1. -1 2. 2
3. 0 4. 1

27. $\int e^{x\cos ecx}.\cos ecx.(1-x\cot x)dx =$

[AP EAMCET 22-09-20_Shift-2]

1. $e^{x\cot x}+c$ 2. $e^{x\cos ecx}+c$
3. $e^{-x\cos ecx}+c$ 4. $e^{-x\cot x}+c$

28. $\int\frac{(1+x)e^x}{\cot(xe^x)}dx =$

[AP EAMCET 22-09-20_Shift-2]

1. $\log(\cos(xe^x))+c$ 2. $\log(\cot(xe^x))+c$
3. $\log(\sec(xe^x))+c$ 4. $\log(\mathrm{cosec}(xe^x))+c$

29. $\int\frac{3^x}{\sqrt{1-9^x}}dx =$

[AP EAMCET 23-09-20_Shift-1]

1. $\mathrm{Sin}^{-1}(3^x).(\log 3)^{-1}+c$
2. $-\mathrm{Sin}^{-1}(3^x).\log 3+c$
3. $\frac{1}{3}\mathrm{Sin}^{-1}(3^x)+c$ 4. $\frac{1}{9}\mathrm{Sin}^{-1}(3^x)+c$

30. If the value of $\int_0^{\frac{\pi}{2}}\sin^4(x).\cos^2(x)dx=\frac{\pi}{32}$ then the value of $\int_0^{\frac{\pi}{2}}\cos^4(x).\sin^2(x)dx =$

[AP EAMCET 23-09-20_Shift-1]

1. $\frac{\pi}{32}$ 2. $\frac{\pi}{64}$
3. $\frac{\pi}{4}$ 4. $\frac{\pi}{8}$

31. If $f'(x)$=asinx + bcosx, $f'(0)$ =4, f(0)=3 and f($\frac{\pi}{2}$)=5,then f(x)=

[AP EAMCET 23-09-20_Shift-1]

1. –2cosx-4sinx+1 2. –2cosx+4sinx+1
3. 2sinx-4cosx+1 4. 2sinx+4cosx+1

32. $\int \frac{x^{n-1}}{x^{2n}+4}dx =$ **[AP EAMCET 23-09-20_Shift-1]**

1. $\frac{1}{2n}\mathrm{Tan}^{-1}(\frac{x^n}{2})+c$ 2. $\frac{n}{2}\mathrm{Tan}^{-1}(\frac{x^n}{2})+c$

3. $\frac{n}{2}Sin^{-1}(\frac{x^n}{2})+c$ 4. $\frac{1}{n}Tan^{-1}(\frac{x^n}{2})+c$

33. $\int \frac{1+\tan^2 x}{1-\tan^2 x}dx =$ **[AP EAMCET 23-09-20_Shift-1]**

1. $\log\left(\frac{1-\tan x}{1+\tan x}\right)+c$ 2. $\log\left(\frac{1+\tan x}{1-\tan x}\right)+c$

3. $\frac{1}{2}\log\left(\frac{1-\tan x}{1+\tan x}\right)+c$ 4. $\frac{1}{2}\log\left(\frac{1+\tan x}{1-\tan x}\right)+c$

34. $\int \frac{dx}{\sqrt{2e^x-1}} =$

1. $2\tan^{-1}(\sqrt{2e^x-1})+c$ 2. $2\sin^{-1}(\sqrt{2e^x-1})+c$

3. $2\cos^{-1}(\sqrt{2e^x-1})+c$ 4. $2\operatorname{cosec}^{-1}(\sqrt{2e^x-1})+c$

35. $\int \frac{dx}{e^x+4e^{-x}} = f(x)+c, then\ f(x)\ is$

1. $2Tan^{-1}(2e^x)$ 2. $\frac{1}{2}Tan^{-1}\left(\frac{e^x}{2}\right)$

3. $2Tan^{-1}\left(\frac{e^x}{2}\right)$ 4. $\frac{1}{2}Tan^{-1}(2e^x)$

36. $If\ I_n = \int(\log x)^n dx, then\ I_n + nI_{n-1} =$

1. $(x\log x)^n$ 2. $x(\log x)^n$

3. $n(\log x)^n$ 4. $(\log x)^{n-1}$

37. $\int \frac{2t+1}{t^2+t+1}dt =$

1. $log\left|t^2+t+1\right|+c$ 2. $log\left|t^2-t+1\right|+c$

3. $log\left|t^2-2t-1\right|+c$ 4. $log\left|t^2-3t+1\right|+c$

38. $\int \frac{\tan^{-1}x}{x^3}dx =$

[TS EAMCET 09-09-20_Shift-1]

1. $\frac{-(x^2+1)}{2x}\tan^{-1}x-\frac{1}{2x}+C$

2. $\frac{-(x^2+1)}{2x^2+1}\tan^{-1}x-\frac{1}{2x^2}+C$

3. $\frac{-1}{2x}-\left(\frac{1}{2}+\frac{1}{2x^2}\right)\tan^{-1}x+C$

4. $\frac{1}{2x}+\frac{1}{2x^2}\tan^{-1}x+C$

39. If $\int \frac{1-(\cot x)^{2019}}{\tan x+(\cot x)^{2020}}dx = \frac{1}{n}\ln\left|(f(x))^n+(g(x))^n\right|+c,$ the value of $n\left[(f(x))^4+(g(x))^4\right]_{x=\frac{\pi}{3}} =$

[TS EAMCET 09-09-20_Shift-1]

1. $\frac{10105}{16}$ 2. $\frac{10012}{15}$

3. $\frac{20210}{9}$ 4. $\frac{10105}{8}$

40. If $x \neq -1\ and$

$$\int \frac{(x^3+x^2-x-1)}{(x^5+x^4+3x^3+3x^2+x+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx = A\log(f(x))+C$$

Then $A-\tan(f(2)) =$

[TS EAMCET 09-09-20_Shift-1]

1. $\frac{-3}{2}$ 2. $\frac{-1}{2}$

3. $\frac{7}{2}$ 4. -2

41. If $I_n = \int x^n \sin x dx\ and\ I_6 - 360I_2 = f(x)\cos x + g(x)\sin x,\ then\ f(1)+g(1) =$

[TS EAMCET 09-09-20_Shift-1]

1. -85 2. 0

3. -53 4. 75

42. If $\int \frac{(x-1)dx}{(x+1)\sqrt{x^3+x^2+x}} = f(x)+C, then\ f(1) =$

[TS EAMCET 09-09-20_Shift-2]

1. $\frac{\pi}{4}$ 2. $\frac{2\pi}{5}$

3. $\frac{2\pi}{3}$ 4. $\frac{5\pi}{6}$

43. If $\int \frac{\cos x}{\sqrt{4\sin^2 x+4\sin x+5}}dx = \frac{1}{2}\sinh^{-1}(f(x))+C\ then\ 2f(x) =$

[TS EAMCET 09-09-20_Shift-2]

1. $1+\sin x$ 2. $2\sin x+1$

3. $4\sin x+1$ 4. $2\sin x-\sin 4x+2$

44. $\int \frac{a\cos x - 2\sin x}{b\sin x + 5\cos x}dx = \frac{7}{41}x + \frac{22}{41}\log|b\sin x + 5\cos x| + C,$

$(a>0, b>0)$ then $\int \frac{dx}{b + a\cos x} =$

[TS EAMCET 09-09-20_Shift-2]

1. $\frac{2}{3}\log\left(\frac{3\tan\frac{x}{2} + 4 - \sqrt{3}}{3\tan\frac{x}{2} + 4 + \sqrt{3}}\right) + C$

2. $\frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{7}}\right) + C$

3. $\frac{2}{\sqrt{7}}\log\left(\frac{\sqrt{7} - \tan\frac{x}{2}}{\sqrt{7} + \tan\frac{x}{2}}\right) + C$

4. $2\sinh^{-1}\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + C$

45. If $\int (3x+2)\sqrt{2x^2+3x+4}\,dx$

$= f(x).\sqrt{2x^2+3x+4} + A\sin^{-1}\left(\frac{4x+3}{\sqrt{23}}\right) + C,$

then the ordered pair $(f(1), A) =$

[TS EAMCET 09-09-20_Shift-2]

1. $\left(\frac{73}{8}, \frac{23}{64\sqrt{2}}\right)$
2. $\left(\frac{137}{32}, \frac{-23}{64\sqrt{2}}\right)$
3. $\left(\frac{15}{8}, \frac{-23}{16\sqrt{2}}\right)$
4. $\left(\frac{49}{32}, \frac{23}{16\sqrt{2}}\right)$

46. $\int \frac{y^2 + \sqrt[3]{y^4} + \sqrt[6]{y^2}}{y\left(1 + \sqrt[3]{y^2}\right)} dy =$

[TS EAMCET 10-09-20_Shift-1]

1. $\frac{3}{4}\sqrt[3]{y^4} + \tan^{-1}\left(\sqrt[3]{y}\right) + c$
2. $\frac{3}{2}y^{\frac{2}{3}} + 6\tan^{-1}\left(\sqrt[6]{y^2}\right) + c$
3. $\frac{2}{3\sqrt[3]{y^2}} + 6\log\left(1 + y^2\right) + c$
4. $\frac{3}{1+y} + \tan^{-1}\left(\sqrt[3]{y^2}\right) + c$

47. For $k \in (1, \infty), \int \frac{1}{1 + k\cos x}dx =$

[TS EAMCET 10-09-20_Shift-1]

1. $\frac{2}{\sqrt{1+k^2}}\tan^{-1}\left(\sqrt{\frac{1-k}{1+k}}\tan\frac{x}{2}\right) + c$

2. $\frac{1}{\sqrt{k^2-1}}\log\left(\frac{\sqrt{k+1} + \sqrt{k-1}\tan\frac{x}{2}}{\sqrt{k+1} - \sqrt{k-1}}\right) + c$

3. $\frac{1}{\sqrt{k^2+1}}\log^{-1}\left(\frac{\sqrt{k+1} + \sqrt{k-1}\tan\frac{x}{2}}{\sqrt{k+1} - \sqrt{k-1}\tan\frac{x}{2}}\right) + c$

4. $\frac{1}{\sqrt{k^2-1}}\tan^{-1}\left(\frac{\sqrt{k+1}\cos\frac{x}{2} + \sqrt{k-1}\sin\frac{x}{2}}{\sqrt{k+1}\cos\frac{x}{2} - \sqrt{k-1}\sin\frac{x}{2}}\right) + c$

48. $\int e^{-3x}\left(x^2 + \sin 4x\right)dx$

[TS EAMCET 10-09-20_Shift-1]

1. $-e^{-3x}\left(\frac{x^2}{3} + \frac{2x}{9} + \frac{2}{27} + \frac{3}{25}\sin 4x + \frac{4}{25}\cos 4x\right) + c$
2. $-e^{-3x}\left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27} + \frac{3}{25}\sin 4x + \frac{4}{25}\cos 4x\right) + c$
3. $-e^{-3x}\left(\frac{x^2}{3} + \frac{2x}{9} + \frac{2}{27} + \frac{3}{25}\sin 4x - \frac{4}{25}\cos 4x\right) + c$
4. $-e^{-3x}\left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27} + \frac{3}{25}\sin 4x - \frac{4}{25}\cos 4x\right) + c$

49. If $\int \frac{2x^{12} + 5x^9}{\left(1 + x^3 + x^5\right)^3}dx = \frac{x^m}{l\left(1 + x^3 + x^5\right)^r} + c$ then

$\frac{m-l}{r} =$ **[TS EAMCET 10-09-20_Shift-1]**

1. 3
2. 4
3. 5
4. 6

50. For $x \in \left(\frac{3\pi}{4}, \pi\right), \int\left(\sqrt{1+\sin 2x} + \sqrt{1-\sin 2x}\right)dx =$

[TS EAMCET 10-09-20_Shift-2]

1. $-2\cos x + C$
2. $2\sin x + C$
3. $-2\sin x + C$
4. $2\cos x + C$

51. $If \int \frac{x^2(x\sec^2 x+\tan x)}{(x\tan x+1)^2}dx =$

$A\log(|x\sin x+\cos x|)+B\frac{f(x)}{(x\tan x+1)}+C$

then f(A+B)=

[TS EAMCET 10-09-20_Shift-2]

1. 1
2. 0
3. -1
4. 2

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52. $If \int x^3(\log x)^2 dx = x^4\left[A(\log x)^2+B(\log x)+C\log e\right]+K,$

then $A+B+C=$

[TS EAMCET 10-09-20_Shift-2]

1. $\frac{7}{24}$
2. $\frac{4}{25}$
3. $\frac{3}{14}$
4. $\frac{5}{32}$

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53. $If \int \frac{9x+15}{x^3-6x-9}dx = A\log|g(x)|+B\log|f(x)|+C,$

Then $\frac{(A-B)g(4)}{f(-1)}=$

[TS EAMCET 10-09-20_Shift-2]

1. 3
2. $\frac{1}{7}$
3. 1
4. $\frac{3}{7}$

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54. If $\int e^{\sin^2 x}(\sin x\cos x+\cos^3 x\sin x)dx$

$=e^{\sin^2 x}(1+f(x))+c,$ then $f'(x)=$

[TS EAMCET 11-09-20_Shift-1]

1. $\frac{1}{2}\sin^2 x$
2. $\frac{1}{2}\cos^2 x$
3. $-\frac{1}{2}\cos 2x$
4. $-\frac{1}{2}\sin 2x$

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55. $\int \frac{25x^2+8}{\sqrt{25x^2+9}}dx =$

[TS EAMCET 11-09-20_Shift-1]

1. $\frac{x}{2}\sqrt{25x^2+9}+\frac{11}{10}\sinh^{-1}\left(\frac{5x}{3}\right)+C$
2. $\frac{x}{2}\sqrt{25x^2+9}-\frac{7}{10}\log\left(\frac{5x+\sqrt{25x^2+9}}{3}\right)+C$
3. $\frac{x}{2}\sqrt{25x^2+9}+\frac{7}{10}\sinh^{-1}\left(\frac{5x}{3}\right)+C$
4. $\frac{x}{2}\sqrt{25x^2+9}+\frac{11}{10}\log\left(\frac{5x-\sqrt{25x^2+9}}{3}\right)+C$

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56. $I_{m,n}=\int x^m(\log x)^n dx =$

[TS EAMCET 11-09-20_Shift-1]

1. $\frac{x^{m+1}}{m+1}(\log x)^n-\frac{n}{m+1}I_{m,n-1}$
2. $\frac{x^m}{m}(\log x)^n-\frac{n-1}{m+1}I_{m,n-1}$
3. $\frac{x^{m+1}}{m}\frac{(\log x)^{n+1}}{n+1}-\frac{n}{m+1}I_{m,n-1}$
4. $x^m\frac{(\log)^{n+1}}{n+1}-\frac{n}{m+1}I_{m,n-1}$

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57. $\int \frac{x^2}{\left(\sqrt{4-x^2}\right)^3}dx =$

[TS EAMCET 11-09-20_Shift-2]

1. $\frac{x^2}{\sqrt{4-x^2}}-\mathrm{Sin}^{-1}\left(\frac{x}{2}\right)+C$
2. $\frac{x}{\sqrt{4-x^2}}-\mathrm{Tan}^{-1}\left(\frac{x}{\sqrt{4-x^2}}\right)+C$
3. $\frac{x}{\sqrt{4-x^2}}+\mathrm{Sin}^{-1}\left(\frac{2}{\sqrt{4-x^2}}\right)+C$
4. $\sqrt{4-x^2}-\mathrm{Tan}^{-1}\left(\frac{x}{2}\right)+C$

58. $\int \frac{dx}{x\ln(x)\ln^2(x)\ln^3(x).....\ln^m(x)} = \frac{(\ln(x))^K}{K} + C \Rightarrow 2K =$

[TS EAMCET 11-09-20_Shift-2]

1. $(m+1)(m+2)$ 2. $(2-m)(1-m)$
3. $(m+1)(2-m)$ 4. $(m+2)(1-m)$

59. If $I_m = \int x^m \cos nx\, dx = g(x) - \frac{m(m-1)}{n^2} I_{m-2}$, then $g(x) =$

[TS EAMCET 11-09-20_Shift-2]

1. $\frac{x^m \sin nx}{n} + \frac{m(m-1)x^{m-1}\cos nx}{n^2}$
2. $\frac{x^m \cos nx}{n} + \frac{m(m-1)x^{m-1}}{n^2}\sin nx$
3. $\frac{m}{n}\sin nx + \frac{m}{n^2}x^{m-1}\cos nx$
4. $\frac{x^m \sin nx}{n} + \frac{m}{n^2}x^{m-1}\cos nx$

60. Let $I_n = \int \sec^n x\, dx$. If $5I_6 - 4I_4 = f(x)$, then $f\left(\frac{\pi}{4}\right)$ is equal to

[TS EAMCET 11-09-20_Shift-2]

1. 2 2. 4
3. 1 4. $\frac{4}{5}$

61. If $\int \frac{dx}{x\left(\sqrt{x^4 - 1}\right)} = \frac{1}{k}\sec^{-1}\left(x^k\right)$, then the value of k=___

[AP EAMCET 19-08-2021_Shift-1]

1. 1 2. 2
3. 3 4. 4

62. $\int \frac{e^x(x+3)}{(x+5)^3}dx =$

[AP EAMCET 19-08-2021_Shift-1]

1. $\frac{e^x}{(x+5)^2} + c$ 2. $e^x(x+5)^2 + c$
3. $e^x(x+3)^2 + c$ 4. $\frac{e^x}{(x+3)^2} + c$

63. If $\int \frac{(x-1)^2}{(x^2+1)^2}dx = \tan^{-1}(x) + g(x) + k$, then $g(x)$ is equal to____

[AP EAMCET 19-08-2021_Shift-1]

1. $\tan^{-1}\left(\frac{x}{2}\right)$ 2. $\frac{1}{x^2+1}$
3. $\frac{1}{2(x^2+1)}$ 4. $\frac{2}{x^2+1}$

64. If $\int \frac{1-(\cot x)^{2021}}{\tan x + (\cot x)^{2022}}dx = \frac{1}{A}\log\left|(\sin x)^{2023} + (\cos x)^{2023}\right| + c$, Then A=___

[AP EAMCET 19-08-2021_Shift-1]

1. 2020 2. 2021
3. 2022 4. 2023

65. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}dx =$

[AP EAMCET 19-08-2021_Shift-2]

1. $\frac{1}{2}\left(\sqrt{1+x}\right)+C$ 2. $\sqrt{1+x}+C$
3. $2(1+x)^{3/2}+c$ 4. $\frac{2}{3}(1+x)^{3/2}+c$

66. $\int (\cos x)\log\cot\left(\frac{x}{2}\right)dx =$

[AP EAMCET 19-08-2021_Shift-2]

1. $(\sin x)\log\cot\left(\frac{x}{2}\right)+c$
2. $(\cos x)\log\cot\left(\frac{x}{2}\right)+c$
3. $(\sin x)\log\cot\left(\frac{x}{2}\right)+x+c$
4. $(\sin x)\log\cot\left(\frac{x}{2}\right)-x+c$

67. $\int \sqrt{e^{4x}+e^{2x}}\,dx =$

[AP EAMCET 19-08-2021_Shift-2]

1. $\frac{1}{2}e^{x}\left(\sqrt{e^{2x}+1}\right)+\frac{1}{2}\text{Sinh}^{-1}\left(e^{x}\right)+c$
2. $\frac{1}{2}e^{x}\left(\sqrt{e^{2x}+1}\right)+\text{Sinh}^{-1}\left(e^{x}\right)+c$
3. $\frac{1}{2}\left(\sqrt{e^{2x}+1}\right)+\frac{1}{2}\text{Sinh}^{-1}\left(e^{x}\right)+c$
4. $\sqrt{e^{4x}+e^{2x}}+\sqrt{e^{2x}+1}+c$

68. If $\int \frac{1}{1+\sin x}dx = \tan\left(f\left(x\right)\right)+c$, then $f'(0)=$ ___

[AP EAMCET 19-08-2021_Shift-2]

1. 0
2. 1
3. $\frac{1}{2}$
4. $-\frac{1}{2}$

69. $\int \frac{\sin\alpha}{\sqrt{1+\cos\alpha}}d\alpha =$

[AP EAMCET 20-08-2021_Shift-1]

1. $-2\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+c$
2. $2\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+c$
3. $\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+c$
4. $-\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+c$

70. If $\int \frac{\cos 4x+1}{\cot x-\tan x} = k\cos 4x + c$, then "$k$" is equal to ___

[AP EAMCET 20-08-2021_Shift-1]

1. $\frac{-1}{2}$
2. $\frac{-1}{8}$
3. $\frac{-1}{3}$
4. $\frac{-1}{5}$

71. If $\int\left[\cos(x).\frac{d}{dx}(\text{cosec}(x))\right]dx = f(x)+g(x)+c$, then $f(x).g(x)=$

[AP EAMCET 20-08-2021_Shift-1]

1. $x\cot(x)$
2. $x\tan(x)$
3. $x\cos(x)$
4. 1

72. If $\int\frac{(2x+1)^{6}}{(3x+2)^{8}}dx = P\left(\frac{2x+1}{3x+2}\right)^{Q}+R$, then $\frac{P}{Q}=$ ___ **[AP EAMCET 20-08-2021_Shift-1]**

1. $\frac{1}{7^2}$
2. $\frac{1}{7}$
3. 7^2
4. 7

73. If $f'(x)=x+\frac{1}{x}$, then $f(x)=$

[AP EAMCET 20-08-2021_Shift-2]

1. $\frac{x^2}{2a}+\frac{b}{x^2}+\frac{x^{a+1}}{a+1}+\frac{b^x}{\log b}+c$
2. $\frac{x^2}{2a}+b\log|x|+\frac{x^{a+1}}{a+1}+\frac{b^x}{\log b}+abx+c$
3. $\frac{1}{a}+b\log|x|+ax^{a-1}+b^x\log b+ab+c$
4. $\frac{x^2}{2a}+b\log|x|+\frac{x^{a+1}}{a+1}+\frac{b^x}{\log a}+abx+c$

74. $\int(x+1)(x+2)^4(x+3)\,dx=$

[AP EAMCET 23-08-2021_Shift-1]

1. $\frac{(x+1)^2}{2}+\frac{(x+2)^2}{5}+\frac{(x+3)^2}{2}+c$
2. $\frac{(x+2)^7}{7}-\frac{(x+2)^5}{5}+c$
3. $\frac{(x+2)^7}{7}+\frac{(x+2)^5}{5}+c$
4. $\frac{(x+3)^7}{7}-\frac{(x+3)^5}{5}+c$

75. If $\int\frac{\sqrt{1-x^4}}{x^7}dx = f(x)\left\{\sqrt{1-x^4}\right\}^n+c$, then $(f(x))^n=$ ___

[AP EAMCET 23-08-2021_Shift-1]

1. $\frac{-1}{6x^6}$
2. $\frac{-1}{216x^{18}}$
3. $\frac{1}{36x^{12}}$
4. $\frac{1}{216x^{18}}$

76. If $\int\frac{dx}{\cos^4 x+\sin^4 x}=\frac{1}{\sqrt{2}}\tan^{-1}\left[g(x)\right]+c$, then $g(x)$ equals

[AP EAMCET 23-08-2021_Shift-1]

1. $\frac{\tan x-\cot x}{\sqrt{2}}$
2. $\frac{\tan x+\cot x}{\sqrt{2}}$
3. $\frac{\sin x-\cos x}{\sqrt{2}}$
4. $\frac{\sin x+\cos x}{\sqrt{2}}$

77. $\int(1-\cos x)\operatorname{cosec}^2 x\,dx =$

[AP EAMCET 23-08-2021_Shift-2]

1. $\tan\left(\frac{x}{2}\right)+c$
2. $-\tan\left(\frac{x}{2}\right)+c$
3. $2\tan\left(\frac{x}{2}\right)+c$
4. $-2\tan\left(\frac{x}{2}\right)+c$

78. Find the value of k if

$\int\cos^k(x)\sin(x)dx = \frac{-1}{4}\cos^4(x)+c$

[AP EAMCET 23-08-2021_Shift-2]

1. 4
2. 3
3. 2
4. 1

79. If $f'(x)=\tan^2(x)+\cot^2(x)$ and $f\left(\frac{\pi}{4}\right)=0$, then $f(x)=$_____

[AP EAMCET 23-08-2021_Shift-2]

1. $\tan(x)-\cot(x)-x+\frac{\pi}{2}$
2. $\tan(x)-\cot(x)-2x+\frac{\pi}{2}$
3. $\tan(x)+\cot(x)-2x+\frac{\pi}{2}$
4. $\sec(x)-\operatorname{cosec}(x)-2x+\frac{\pi}{2}$

80. $\int\frac{\sin x-\cos x}{\sqrt{\sin 2x}}dx =$

[AP EAMCET 23-08-2021_Shift-2]

1. $-\log\left|\sin x-\cos x+\sqrt{\sin 2x}\right|+c$
2. $-\log\left|\sin x+\cos x-\sqrt{\sin 2x}\right|+c$
3. $-\log\left|\sin x+\cos x+\sqrt{\sin 2x}\right|+c$
4. $-\log\left|\sin x-\cos x-\sqrt{\sin 2x}\right|+c$

81. $\int\frac{x^3-x^2+x-1}{x-1}dx =$

[AP EAMCET 24-08-2021_Shift-1]

1. $\frac{x^3}{3}-x+c$
2. $\frac{x^2}{3}+x+c$
3. $\frac{x^3}{3}+x+c$
4. $2x+c$

82. The value of

$\int\frac{e^{\tan^{-1}(x)}}{1+x^2}\left[\left(\sec^{-1}\sqrt{1+x^2}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]dx$,

for $x>0$ is _______

[AP EAMCET 24-08-2021_Shift-1]

1. $e^{\tan^{-1}(x)}\left(\tan^{-1}x\right)^2+c$
2. $e^{\tan^{-1}(x)}\left(\tan^{-1}x\right)+c$
3. $e^{\tan^{-1}(x)}\left(\tan^{-1}x\right)^3+c$
4. $-e^{\tan^{-1}(x)}\left(\tan^{-1}x\right)^2+c$

83. If $\int\sqrt[3]{x}\left\{1+\sqrt[3]{x^4}\right\}^{1/7}dx = A\left(1+\sqrt[3]{x^4}\right)^B+C$, then value of AB = _____

[AP EAMCET 24-08-2021_Shift-1]

1. $\frac{3}{2}$
2. $\frac{3}{4}$
3. $\frac{3}{32}$
4. $\frac{4}{3}$

84. $\int\frac{\sqrt{2}\sin x}{\sin\left(x+\frac{\pi}{4}\right)}dx$

[AP EAMCET 24-08-2021_Shift-1]

1. $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$
2. $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$
3. $x+\log\left|\sin\left(x+\frac{\pi}{4}\right)\right|+c$
4. $x-\log\left|\sin\left(x+\frac{\pi}{4}\right)\right|+c$

85. $\int 3^x\left(f'(x)+f(x)\log 3\right)dx =$

[AP EAMCET 24-08-2021_Shift-2]

1. $3^x f'(x)+c$
2. $3^x\log 3+c$
3. $3^x f(x)+c$
4. 3^x+c

86. $\int \frac{x+\sin x}{1+\cos x}dx =$

[AP EAMCET 24-08-2021_Shift-2]

1. $x\cot\left(\frac{x}{2}\right)+c$ 2. $\cot\left(\frac{x}{2}\right)+c$
3. $\tan\left(\frac{x}{2}\right)+c$ 4. $x\tan\left(\frac{x}{2}\right)+c$

87. If

$\int \frac{3\cos x-2\sin x}{4\sin x+5\cos x}dx = A\log|5\cos x+4\sin x|+Bx+c,$

then A and B are ____

[AP EAMCET 24-08-2021_Shift-2]

1. $A=\frac{22}{41}$ & $B=\frac{-7}{41}$ 2. $A=\frac{-22}{41}$ & $B=\frac{7}{41}$
3. $A=\frac{-22}{41}$ & $B=\frac{-7}{41}$ 4. $A=\frac{22}{41}$ & $B=\frac{7}{41}$

88. $\int \frac{x-1}{(x-2)(x-3)}dx =$

[AP EAMCET 24-08-2021_Shift-2]

1. $2\log|x-3|+\log|x-2|+c$
2. $\log|x-3|-\log|x-2|+c$
3. $\log|x-3|^2-\log|x+2|+c$
4. $\log\left|\frac{(x-3)^2}{x-2}\right|+c$

89. $\int \frac{dx}{(\sin x)(\cos x)} =$

[AP EAMCET 25-08-2021_Shift-1]

1. $\log|\sin x|+c$ 2. $\log|\cos x|+c$
3. $\log|\tan x|+c$ 4. $\log|\operatorname{cosec} x|+c$

90. If $\int \frac{5\tan(x)}{\tan(x)-2}dx = x+a\log|\sin(x)-2\cos(x)|+k$,

then $a=$ ____

[AP EAMCET 25-08-2021_Shift-1]

1. −1 2. −2
3. 1 4. 2

91. If

$\int \frac{2x+3}{x(x+1)(x+2)(x+3)+1}dx = \frac{-1}{ax^2+bx+c}+\alpha,$

then value of $a+b+c=$

[AP EAMCET 25-08-2021_Shift-1]

1. 3 2. 4
3. 5 4. 6

92. If $\int \frac{2\cos x+3\sin x}{4\cos x+5\sin x}dx =$

$\left(\frac{23}{41}\right)x+k\log|4\cos x+5\sin x|+c$, then $k=$ ____

[AP EAMCET 25-08-2021_Shift-1]

1. $\frac{2}{41}$ 2. $\frac{-2}{41}$
3. $\frac{3}{41}$ 4. $\frac{-3}{41}$

93. $\int \cos\sqrt{x}\,dx =$

[AP EAMCET 25-08-2021_Shift-2]

1. $2\sqrt{x}\sin\sqrt{x}+2\cos\sqrt{x}+c$
2. $2\sqrt{x}\sin\sqrt{x}+2\sin\sqrt{x}+c$
3. $2\sqrt{x}\sin\sqrt{x}-2\cos\sqrt{x}+c$
4. $\sqrt{x}\cos\sqrt{x}-2\sin\sqrt{x}+c$

94. If $\int \frac{x^2+1}{x^4+1}dx = f(x)+c$, then $f(x)=$ ____

[AP EAMCET 25-08-2021_Shift-2]

1. $\frac{1}{\sqrt{2}}\text{Tan}^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right)$ 2. $\frac{1}{\sqrt{2}}\text{Tan}^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)$
3. $\frac{1}{\sqrt{2}}\text{Tan}^{-1}\left(\frac{1-x^2}{\sqrt{2}x}\right)$ 4. $\frac{1}{\sqrt{2}}\text{Tan}^{-1}\left(\frac{1+x^4}{\sqrt{2}x}\right)$

95. If $\int \frac{1+\cos(4x)}{\cot(x)-\tan(x)}dx = A\cos(4x)+B, then A =$

[AP EAMCET 25-08-2021_Shift-2]

1. $\frac{-1}{2}$ 2. $\frac{-1}{4}$
3. $\frac{-1}{3}$ 4. $\frac{-1}{8}$

96. $\int \frac{dx}{\sqrt{7-6x-x^2}} =$

[AP EAMCET 25-08-2021_Shift-2]

1. $\sinh^{-1}\left(\frac{x+3}{4}\right)+c$ 2. $\log\left|\frac{x+3}{4}\right|+c$
3. $\sin^{-1}\left(\frac{x+3}{4}\right)+c$ 4. $\frac{1}{2}\sin^{-1}\left(\frac{x+3}{4}\right)+c$

97. If $f(x)=\frac{3-8x}{3x-1}$ and $\int f(y)dy = Ay + B\log(3y-1)+C$, then $\frac{A-3B}{2}=$

[TS EAMCET 04-08-2021_Shift-2]

1. 0
2. $\frac{-5}{2}$
3. $\frac{1}{2}$
4. $\frac{-3}{2}$

98. $\int\sqrt{\sin x}\cos x dx=\frac{2}{3}(\sin x)^{3/2}+C$ is valid when x lies in the interval

[TS EAMCET 04-08-2021_Shift-2]

1. $(-\infty,\infty)$
2. $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$
3. $(2n\pi,(2n+1)\pi), n\in N$
4. $((2n+1)\pi,(2n+2)\pi), n\in N$

99. For $x>0\int\left(\frac{\sqrt{1+x+x^2}}{1+x}+\frac{1}{2\sqrt{1+x+x^2}}-\frac{1}{(1+x)\sqrt{1+x+x^2}}\right)dx=$

[TS EAMCET 04-08-2021_Shift-2]

1. $\frac{1}{\sqrt{1+x+x^2}}+C$
2. $\sqrt{1+x}+C$
3. $\frac{1}{\sqrt{1+x}}+C$
4. $\sqrt{x^2+x+1}+C$

100. $\int\frac{dx}{\sin x+\cos x}=$

[TS EAMCET 04-08-2021_Shift-1]

1. $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)\right|+C$
2. $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+C$
3. $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{4}+\frac{\pi}{2}\right)\right|+C$
4. $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{8}+\frac{\pi}{2}\right)\right|+C$

101. $\int(x+2)\sqrt{x+3}\,dx$

[TS EAMCET 04-08-2021_Shift-1]

1. $\frac{2}{15}\sqrt{x+3}\left(3x^2-13x+12\right)+C$
2. $\frac{2}{15}\sqrt{x+3}\left(3x^2+13x+12\right)+C$
3. $\frac{2}{5}\sqrt{x+3}\left(3x^2-12x+13\right)+C$
4. $\frac{2}{5}\sqrt{x+3}\left(3x^2+12x+13\right)+C$

102. $\int\frac{(1-\cos x)^{2/7}}{(1+\cos x)^{9/7}}dx=$

[TS EAMCET 04-08-2021_Shift-1]

1. $\frac{7}{11}\left(\tan\frac{x}{2}\right)^{\frac{11}{7}}+C$
2. $\frac{7}{11}\left(\tan\frac{x}{2}\right)^{\frac{7}{11}}+C$
3. $\frac{7}{11}\left(\cot\frac{x}{2}\right)^{\frac{11}{7}}+C$
4. $\frac{11}{7}\left(\cot\frac{x}{2}\right)^{\frac{7}{11}}+C$

103. $\int\frac{xe^{\frac{x^2}{x^2-2}}}{x^4-4x^2+4}dx=$

[TS EAMCET 05-08-2021_Shift-1]

1. $\frac{-1}{4}e^{\frac{x^2}{x^2-2}}+C$
2. $\frac{1}{4}e^{\frac{x^2}{x^2-2}}+C$
3. $\frac{1}{x^2-2}e^{\frac{x^2}{x^2-2}}+C$
4. $\frac{-1}{(x^2-2)^4}e^{\frac{x^2}{x^2-2}}+C$

104. If $\int\frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx=\frac{1}{2}f(x)+C$, then $f(1)-f(0)=$

[TS EAMCET 05-08-2021_Shift-1]

1. $\frac{1}{2}$
2. $\frac{1}{18}$
3. $\frac{1}{27}$
4. $\frac{1}{54}$

105. $\int \frac{\sqrt{\cot x}}{\sin 2x} dx =$

[TS EAMCET 05-08-2021_Shift-1]

1. $\sqrt{\cot x} + C$ 2. $-\sqrt{\cot x} + C$
3. $\sqrt{\tan x} + C$ 4. $-\sqrt{\tan x} + C$

---

106. If $\int \frac{1}{(x-2)^5 (x-1)^4} dx = \sum_{r=-4}^{-1} A_r \left(\frac{x-2}{x-1}\right)^r + \sum_{r=1}^{3} A_r \left(\frac{x-2}{x-1}\right)^r + Bf(x)$, then $f(x) =$

[TS EAMCET 05-08-2021_Shift-2]

1. $\log(x-2) - \log(x-1)$ 2. $\left(\frac{x-2}{x-1}\right) + \log x$
3. $x + \log\left(\frac{x-2}{x-1}\right)$ 4. $\log x$

---

107. If $f(x) + K$ is obtained by evaluating $\int \frac{x^3}{(1+x^2)^3} dx$ using the substitution $x = \tan\theta$ and $g(x) + C$ is obtained by evaluating $\int \frac{x^3}{(1+x^2)^3} dx$, using the substitution $x^2 + 1 = Z$, then $f(x) - g(x) + K - C =$

[TS EAMCET 05-08-2021_Shift-2]

1. $\frac{1}{4}$ 2. any constant
3. any function of x 4. $\frac{x}{1+x^2}$

---

108. For $-1 < x < 1$ and $y > 1$, if

$$\int \frac{x}{\sqrt{1+x} + \sqrt{1-x}} dx + \int \frac{y}{\sqrt{y+1} + \sqrt{y-1}} dy =$$

$A(1+x)^{3/2} + B(1-x)^{3/2} + f(y)(y+1)^{3/2} + g(y)(y-1)^{3/2} + C,$

then $Af(y) + Bg(y) =$

[TS EAMCET 05-08-2021_Shift-2]

1. $\frac{2y}{15}$ 2. $\frac{-4}{45}$
3. $\frac{-4}{15}$ 4. $\frac{3y+2}{45}$

---

109. $\int \sqrt{x^2 + x + 1}\, dx \times \int \frac{1}{\sqrt{x^2 + x + 1}} dx =$

[TS EAMCET 06-08-2021_Shift-2]

1. $x + C$
2. $\left(\frac{2x+1}{4}\sqrt{x^2+x+1} + \frac{3}{8}\text{Sinh}^{-1}\frac{2x+1}{\sqrt{3}}\right)\text{Sinh}^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$
3. $\frac{2x+1}{2}\text{Sinh}^{-1}\left(\sqrt{x^2+x+1}\right) + \left(\frac{3}{8}\text{Sinh}^{-1}\frac{2x+1}{\sqrt{3}}\right)^2 + C$
4. $\frac{2x+1}{2}\left(\text{Sinh}^{-1}\frac{2x+1}{\sqrt{3}}\right)^2 + C$

---

110. $\int \frac{dx}{\sqrt{(5+2x+x^2)^3}} =$

[TS EAMCET 06-08-2021_Shift-2]

1. $\frac{1}{4}\frac{1}{\sqrt{5+2x+x^2}} + C$ 2. $\frac{1}{\sqrt{5+2x+x^2}} + C$
3. $\frac{x+1}{\sqrt{5+2x+x^2}} + C$ 4. $\frac{1}{4}\frac{x+1}{\sqrt{5+2x+x^2}} + C$

---

101. $\int \frac{x^2}{1+x^6} dx =$

[TS EAMCET 06-08-2021_Shift-1]

1. $x^3 + C$ 2. $\frac{1}{3}Tan^{-1}(x^3) + C$
3. $\log(1+x^3)$ 4. $\frac{1}{1+x^3} + C$

---

102. $\int \frac{dx}{(x^2 - a^2)^{3/2}} =$

[TS EAMCET 06-08-2021_Shift-1]

1. $\frac{a^2 x^2}{\sqrt{(x^2 - a^2)}} + C$
2. $\frac{-1}{a^2}(x^2 - a^2)^{5/2}$
3. $-\frac{x}{a^2\sqrt{(x^2 - a^2)}} + C$
4. $\frac{1}{a^2\sqrt{(x^2 - a^2)}} + C$

103. For $x \geq 0, \int \sqrt{x^2+2x}\,dx =$

[TS EAMCET 06-08-2021_Shift-1]

1. $\frac{x+1}{2}\sqrt{x^2+2x}+\frac{1}{2}Sinh^{-1}\frac{(x+1)}{2}+C$
2. $\frac{x+1}{2}\sqrt{x^2+2x}+\frac{1}{2}Sinh^{-1}(x+1)+C$
3. $\frac{x+1}{2}\sqrt{x^2+2x}-\frac{1}{2}Cosh^{-1}\frac{(x+1)}{2}+C$
4. $\frac{x+1}{2}\sqrt{x^2+2x}-\frac{1}{2}Cosh^{-1}(x+1)+C$

104. Assertion (A) : if

$I_n = \int \cot^n x\,dx, then\ I_6 + I_4 = \frac{-\cot^5 x}{5}$

Reason (R) :

$\int \cot^n x\ dx = \frac{-\cot^{n-1} x}{n} - \int \cot^{n-2} x\,dx$

[AP EAMCET 04-07-2022_Shift-1]

1. A is false, R is false
2. A is true, R is true
3. A is true, R is false
4. A is false, R is true

105. If $I_n = \int \tan^n x\,dx, and\ I_0 + I_1 + 2I_2 + 2I_3 + 2I_4 + I_5 + I_6$

$= \sum_{K=1}^{n} \frac{\tan^K x}{K}, then\ n =$

[AP EAMCET 04-07-2022_Shift-1]

1. 6
2. 5
3. 4
4. 3

106. $\int \frac{e^{\cot x}}{\sin^2 x}(2\log \text{cosec x} + \sin 2\text{ x})\,dx =$

1. $-2e^{\cot x}\log(\cos ec^2 x)+C$
2. $-2e^{\cot x}\log(\cos ec\,x)+C$
3. $-2e^{\cot x}\log(\cos ec\,x+\sin x)+C$
4. $-2e^{\cot x}\log(\cos ec\,x-\cot x)+C$

[AP EAMCET 04-07-2022_Shift-1]

107. The parametric form of a curve is

$x = \frac{t^3}{t^2-1}, y = \frac{t}{t^2-1}, then \int \frac{dx}{x-3y} =$

[AP EAMCET 04-07-2022_Shift-1]

1. $\frac{1}{2}\log(t^2-1)+C$
2. $2\log(t(t^2-1))+C$
3. $\frac{1}{4}\log\left(\frac{t}{t^2-3}\right)+C$
4. $\frac{5}{2}\log\left(t+\frac{1}{t^2}\right)+C$

108. $If\ f(x) = \int x^2\cos^2 x(2x\tan^2 x - 2x - 6\tan x)dx$ and $f(0) = \pi$ then $f(x) =$

[AP EAMCET 04-07-2022_Shift-2]

1. $x^2\sin x+\pi$
2. $\cos x+\pi-1$
3. $-x^3\sin 2x+\pi$
4. $x^3\cos 2x+\pi\cos x$

109. If $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}(x+\sqrt{x})dx = e^{\sqrt{x}}\left[Ax+B\sqrt{x}+C\right]+K$, then $A+B+C=$

[AP EAMCET 04-07-2022_Shift-2]

1. -2
2. 2
3. 4
4. -4

120. $If\ \int \frac{1+\sqrt{\tan x}}{\sin 2x}dx =$

$A\log\tan x + B\tan x + C\ then\ 4A-2B =$

[AP EAMCET 04-07-2022_Shift-2]

1. -1
2. 2
3. 1
4. -2

121 $\int \frac{1+\tan x\tan(x+a)}{\tan x\tan(x+a)}dx =$

[AP EAMCET 04-07-2022_Shift-2]

1. $\tan\alpha(\log(\sec(x+a))+\log\sec x+C)$
2. $\cot\alpha(\log|\sin x|-\log|\sin(x+\alpha|)+C$
3. $\tan\alpha\left(\log\left(\frac{\cos x}{\sin(x+\alpha)}\right)\right)+C$
4. $\cot\alpha\left(\log\frac{\sin(x+a)}{\cos(x+\alpha)}\right)+C$

122. $\int \frac{3x+4}{x^3-2x+4}dx = \log f(x)+C \Rightarrow f(3)=$

[AP EAMCET 05-07-2022_Shift-1]

1. $\frac{1}{\sqrt{17}}$
2. $\frac{1}{17}$
3. $\frac{2}{15}$
4. $\frac{2}{17}$

123. $\int \frac{e^{\tan^{-1}x}}{1+x^2}\left[\left(\sec^{-1}\sqrt{1+x^2}\right)^2+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]dx =$

[AP EAMCET 05-07-2022_Shift-1]

1. $e^{\tan^{-1}x}\left(\tan^{-1}x\right)^2+C$
2. $e^{\tan^{-1}x}\left(\sec^{-1}x\right)^2+C$
3. $e^{\tan^{-1}x}\left(\sec^{-1}\left(\sqrt{1+x^2}\right)\right)+C$
4. $e^{\tan^{-1}x}\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)+C$

124. $\int \frac{dx}{(x-3)^{4/5}(x+1)^{6/5}}=$

[AP EAMCET 05-07-2022_Shift-1]

1. $\frac{5}{4}\sqrt{\frac{x-3}{x+1}}+C$
2. $\frac{1}{5}\left(\frac{x-3}{x+1}\right)^{1/5}+C$
3. $\frac{1}{5}\left(\frac{x-3}{x+1}\right)^{1/5}+C$
4. $\frac{5}{4}\left(\frac{x-3}{x+4}\right)^{4/3}+C$

125. If $I_n=\int\left(\cos^n x+\sin^n x\right)dx$, and $I_n-\frac{n-1}{n}I_{n-2}=\frac{\sin x\cos x}{n}f(x)$, then $f(x)=$

[AP EAMCET 05-07-2022_Shift-1]

1. $\cos^{n-2}x+\sin^{n-2}x$
2. $\cos^{n-2}x-\sin^{n-2}x$
3. $\frac{\cos^{n-2}x-\sin^{n-2}x}{n}$
4. $\frac{\cos^{n-2}x+\sin^{n-2}x}{n}$

126. There exist θ such that $a>|\sec\theta|$, then $\int\frac{dx}{1+a\cos x}=$

[AP EAMCET 05-07-2022_Shift-2]

1. $\frac{1}{\sqrt{a^2-1}}\tan^{-1}\left(\frac{\sqrt{a-1}}{\sqrt{a+1}}\tan\frac{x}{2}\right)+C$
2. $\frac{1}{\sqrt{a^2-1}}\tan^{-1}\left(\sqrt{\frac{1-a}{1+a}}\tan\frac{x}{2}\right)+C$
3. $\frac{1}{\sqrt{a^2-1}}\log\left(\frac{\sqrt{a+1}\cos\frac{x}{2}-\sqrt{a-1}\sin\frac{x}{2}}{\sqrt{a-1}\cos\frac{x}{2}+\sqrt{a-1}\sin\frac{x}{2}}\right)+C$
4. $\frac{1}{\sqrt{a^2-1}}\log\left(\frac{\sqrt{a+1}\cos\frac{x}{2}+\sqrt{a-1}\sin\frac{x}{2}}{\sqrt{a+1}\cos\frac{x}{2}-\sqrt{a-1}\sin\frac{x}{2}}\right)+C$

127 $\int\frac{x^2-2}{x^3\sqrt{x^2-1}}dx=$

[AP EAMCET 05-07-2022_Shift-2]

1. $\frac{-x^2}{\sqrt{x^2-1}}$
2. $\frac{-\sqrt{x^2-1}}{x}$
3. $\frac{-x}{\sqrt{x^2-1}}$
4. $\frac{-\sqrt{x^2-1}}{x^2}$

128. Let $f(x)=\int\frac{e^{3x}}{4+8e^{2x}+e^{4x}}dx$ and $g(x)=\int\frac{2dx}{e^{3x}+8e^{x}+4e^{-x}}dx$, then $f(x)-g(x)=$

[AP EAMCET 05-07-2022_Shift-2]

1. $\frac{1}{2}\tan^{-1}\left(\frac{e^x+2e^{-x}}{2}\right)+C$
2. $\frac{1}{2}\tan^{-1}\left(\frac{e^x+e^{-x}}{2}\right)+C$
3. $\frac{1}{2}\tan^{-1}\left(\frac{2e^{-x}+e^{-2x}}{2}\right)+C$
4. $\frac{1}{2}\tan^{-1}\left(\frac{e^{2x}+2e^{x}}{2e^{-x}}\right)+C$

129. If *a, b, c, d*≠0 and $f\left(\frac{at+b}{ct+d}\right)=t$, then $\int f(x)dx=$

[AP EAMCET 05-07-2022_Shift-2]

1. $\frac{-d}{c}x+\frac{bc-ad}{c^2}\log(cx-a)+K$
2. $\frac{-d}{c}x+\frac{bc-ad}{c^2}\log(cx+d)+K$
3. $\frac{a}{c}x+\frac{ad-bc}{c^2}\log(cx+a)+K$
4. $\frac{a}{c}x-\frac{ad-bc}{c}\log(cx+a)+K$

130. If $\int e^{x}\left(f(x)-f'(x)\right)=g(x)+C$, then $\int e^{x}f'(x)dx=$

[AP EAMCET 06-07-2022_Shift-1]

1. $\frac{1}{2}\left[e^{x}f(x)-g(x)\right]+C$
2. $\frac{1}{2}\left[e^{x}f(x)+g(x)\right]+C$
3. $\frac{e^{x}f'(x)+g(x)}{2}+C$
4. $\frac{1}{2}\left[e^{x}f(x)+e^{x}g(x)\right]+C$

---

131. If $\int\frac{\sin^{3}x\left(\tan^{-1}(\sec x+\cos x)\right)^{-1}}{\left(\cos^{4}x+3\cos^{2}x+1\right)}dx=f(x)+C$, then $e^{f(x)}=$

[AP EAMCET 06-07-2022_Shift-1]

1. $\tan^{-1}(\sec x+\cos x)$
2. $\tan(\sec x+\cos x)$
3. $\frac{1}{\cos^{4}x+3\cos^{2}x+1}$
4. $\frac{\sin x}{\sin^{3}+\cos^{4}x+1}$

---

132. $\int\frac{1}{\left(\sin x+\cos x+\sqrt{2}\sqrt{\sin 2x}\right)^{2}}dx=$

[AP EAMCET 06-07-2022_Shift-1]

1. $\frac{-\left(1+3\sqrt{\tan x}\right)}{\left(3+\tan^{2}x\right)^{3}}+C$ 2. $\frac{-\left(1+3\sqrt{\tan x}\right)}{3\left(1+\sqrt{\tan x}\right)^{3}}+C$
3. $\frac{-\left(1+\sqrt{\tan x}\right)}{3\left(1+3\sqrt{\tan x}\right)^{2}}+C$ 4. $\frac{1}{\left(1+3\sqrt{\tan x}\right)^{3}}+C$

---

133. If $\int\frac{dx}{x^{2022}\left(1+x^{2022}\right)^{1/2022}}=\frac{-\left(1+x^{m}\right)^{n/m}}{nx^{n}}+C$, then $m-n=$

[AP EAMCET 06-07-2022_Shift-1]

1. 1 2. 2
3. 3 4. 0

---

134. $\int\sqrt{1+2\cot x(\cot x+\operatorname{cosec}x)}dx=$

[AP EAMCET 06-07-2022_Shift-2]

1. $2log\left(\sin\frac{x}{2}\right)+C$
2. $2\log(\sin x)-\log(\text{cosec}x+\cot x)+C$
3. $\frac{1}{2}\log\left(\operatorname{cosec}\frac{x}{2}+\cot\frac{x}{2}\right)+C$
4. $4\log\cos\frac{x}{2}+C$

---

135. $\int\frac{\cos^{4}x}{(\sin^{2}x+\sin^{-3}x\cos^{5}x)^{3}}dx=$

[AP EAMCET 06-07-2022_Shift-2]

1. $\frac{1}{5}(1+\cot^{5}x)^{-2}+C$ 2. $\frac{1}{10}(1+\cot^{2}x)^{-5}+C$
3. $\frac{1}{10}(1+\cot^{5}x)^{-2}+C$ 4. $\frac{1}{5}(1+\cot^{5}x)^{-5}+C$

---

136. $\left(\int\frac{2\cos x+1}{(2+\cos x)^{2}}dx\right)-\frac{\sin x}{2+\cos x}=$

[AP EAMCET 06-07-2022_Shift-2]

1. $\frac{1}{2+\cos x}+C$ 2. $\sin x+C$
3. $\frac{2}{2+\cos x}+C$ 4. C

---

137. $\int\frac{1}{\cos 4x\cos 2x}dx=\frac{1}{4\sqrt{2}}\log\left(\frac{1+f(x)}{1-f(x)}\right)-\frac{1}{2}$ log g (x)+ C, then $g\left(\pi/6\right)-\sqrt{2}f\left(\pi/6\right)$

[AP EAMCET 06-07-2022_Shift-2]

1. $\frac{\pi}{2\sqrt{2}}$ 2. $\pi+3$
3. 2 4. 1

---

138. If f(x) is anti derivative of g(x) and $\int f(x)g(x)\left(1+f^{2}(x)\right)dx=F(x)$ then F(x)=

[AP EAMCET 07-07-2022_Shift-1]

1. $\frac{\left(1+f^{2}(x)\right)^{2}}{4}+C$ 2. $\frac{\left(1+f^{2}(x)\right)^{2}}{2}+C$

3. $\frac{f^2(x)g(x)}{4}+C$ 4. $\frac{g^2(x)f(x)}{4}+C$

139. $k\in N,\int\frac{1-k\cos^2 x}{\sin^k x.xos^2 x}dx=$

[AP EAMCET 07-07-2022_Shift-1]

1. $\frac{\tan x}{\sin^{k+1} x}+C$ 2. $\frac{\tan x}{\sin^k x}+C$

3. $\sin^k x\sec^2 x+C$ 4. $k\sin^{k-1} x\cos x+C$

140. $\int\frac{\cos ec^2 x-2022}{\cos^{2022} x}dx=f(x)+C\Rightarrow f\left(\frac{\pi}{4}\right)=$

[AP EAMCET 07-07-2022_Shift-1]

1. $\left(\frac{1}{2}\right)^{1011}$ 2. -2^{1011}

3. 2^{2011} 4. -2^{2022}

141. $\int\cos^{-1}\left(\sqrt{\frac{x}{a+x}}\right)dx=f(x)+C\Rightarrow f'(a)=$

[AP EAMCET 07-07-2022_Shift-1]

1. $\frac{\pi}{6}$ 2. $\frac{\pi}{2}$

3. $\frac{\pi}{3}$ 4. $\frac{\pi}{4}$

142. $\int\frac{dx}{\left(1+\sqrt{x}\right)^{2022}}=$

[AP EAMCET 07-07-2022_Shift-2]

1. $\frac{2}{\left(1+\sqrt{x}\right)^{2021}}\left[\frac{1+\sqrt{x}}{2020}-\frac{1}{2021}\right]+C$

2. $\frac{2}{\left(1+\sqrt{x}\right)^{2022}}\left[\frac{1+\sqrt{x}}{2020}-\frac{\sqrt{x}}{2021}\right]+C$

3. $\frac{2}{\left(1+\sqrt{x}\right)}\left[\frac{\left(1+\sqrt{x}\right)^{2022}}{2022}-\frac{\left(1+\sqrt{x}\right)^{2021}}{2021}\right]+C$

4. $\frac{1}{\left(1+\sqrt{x}\right)^2}\left[\frac{1}{\left(1+\sqrt{x}\right)^{1010}}-\frac{1}{\left(1+\sqrt{x}\right)^{1011}}\right]+C$

143. $\int\frac{x^8+4}{x^4-2x^2+2}dx=Ax^5+Bx^3+Cx+K$, then 5A+3B+C=

[AP EAMCET 07-07-2022_Shift-2]

1. 7 2. 5

3. 3 4. 1

144. $\int\sqrt{x+\sqrt{x^2+2}}\,dx=$

[AP EAMCET 07-07-2022_Shift-2]

1. $\frac{3}{2}\left(x+\sqrt{x+2}\right)^{\frac{3}{2}}-2\left(x+\sqrt{x^2+2}\right)^{\frac{1}{4}}+C$

2. $\frac{1}{3}\left(x+\sqrt{x^2+2}\right)^{\frac{3}{2}}-2\left(x+\sqrt{x^2+2}\right)^{\frac{1}{4}}+C$

3. $\left(x+\sqrt{x^2+2}\right)^{\frac{-3}{2}}-2\left(x+\sqrt{x^2+2}\right)^{\frac{-1}{2}}+C$

4. $\frac{\left(x+\sqrt{x^2+2}\right)^2-6}{3\sqrt{x+\sqrt{x^2+2}}}+C$

145. If $\int\frac{\sin\left(x-\frac{\pi}{4}\right)}{2+\sin 2x}dx=-\frac{1}{\sqrt{2}}\tan^{-1}\left(f(x)\right)+C$, then $f(x)=$ **[AP EAMCET 07-07-2022_Shift-2]**

1. $\sin x-\cos x$ 2. $\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$

3. $\sin\left(x-\frac{\pi}{4}\right)$ 4. $\sqrt{2}\tan\left(x-\frac{\pi}{4}\right)$

146. $\int\tan^{-1}\left(1-x+x^2\right)dx+\int\tan^{-1}(x)dx+\int\tan^{-1}(1-x)dx=$

[AP EAMCET 08-07-2022_Shift-1]

1. $\frac{\pi}{2}x+C$ 2. $\frac{\pi}{4}x+C$

3. $x+C$ 4. $\pi x+C$

147. $0<x<1$, $\int\frac{dx}{\sqrt{x^2-x^5}}=\frac{1}{3}\log|f(x)|+C$, then $f\left(\frac{1}{2}\right)$ **[AP EAMCET 08-07-2022_Shift-1]**

1. $\frac{\left(\sqrt{8}-\sqrt{7}\right)}{\left(\sqrt{8}+\sqrt{7}\right)}$ 2. $\frac{\left(\sqrt{8}+\sqrt{7}\right)}{\left(\sqrt{8}-\sqrt{7}\right)}$

3. $2\left(\sqrt{8}-\sqrt{7}\right)$ 4. $2\left(\sqrt{8}-\sqrt{7}\right)^2$

---

148. $\int \frac{dx}{\sin\left(x-\frac{\pi}{3}\right)\cos x}=$

[AP EAMCET 08-07-2022_Shift-1]

1. $2\log\left(\frac{\tan x-\sqrt{3}}{2}\right)+C$
2. $2\log\left(\sin\left(x-\frac{\pi}{3}\right)\cos x\right)+C$
3. $2\log\left(\tan x+\sqrt{3}\right)+C$
4. $2\log\left(\sin x+\sqrt{3}\cos x\right)+C$

---

149. $\int(1+x)\log\left(1+x^2\right)dx=\left(x+\frac{x^2}{2}+\frac{1}{2}\right)\log\left(1+x^2\right)+g(x)+C$ then g(x)

[AP EAMCET 08-07-2022_Shift-1]

1. $-2x-\frac{x^2}{2}+2\tan^{-1}x$
2. $2\tan^{-1}x+\frac{x^2}{2}+\frac{x^3}{3}$
3. $2\tan^{-1}x-\frac{x^2}{2}+3x$
4. $2\tan^{-1}x+3x+\frac{x^3}{2}$

---

150. $\int\frac{3e^x-7e^{-x}}{7e^x+3e^{-x}}dx=Kx+L\log\left(e^{-2x}+\frac{7}{3}\right)+C$, then K+L=

[AP EAMCET 08-07-2022_Shift-2]

1. $\frac{-3}{38}$ 2. $\frac{21}{38}$
3. $\frac{38}{21}$ 4. $\frac{-38}{3}$

---

151. $\left(\sqrt{x+\sqrt{12x-36}}+\sqrt{x-\sqrt{12x-36}}\right)dx=$

[AP EAMCET 08-07-2022_Shift-2]

1. $2\sqrt{3}x+C,\ \forall x$ 2. $\frac{4(x-3)^{\frac{3}{2}}}{3}+C,\ \forall x$

3. $\begin{cases}\frac{4}{3}(x-3)^{\frac{1}{2}}+C, & if\ x>6\\ 2\sqrt{3}x+C & if\ 3\le x\le 6\end{cases}$ 4. $\begin{cases}\frac{4}{3}(x-3)^{\frac{1}{2}}+C, & if\ 3\le x\le 6\\ 2\sqrt{3}x+C & if\ x>6\end{cases}$

---

152. $\int\frac{xdx}{\sqrt[15]{\left(1+x^2\right)^{12}\left(2+x^2\right)^{18}}}=\alpha\left(\frac{1+x^2}{2+x^2}\right)^{\frac{1}{n}}+C\Rightarrow\frac{n}{\alpha}=$

[AP EAMCET 08-07-2022_Shift-2]

1. 6 2. 4
3. 2 4. 8

---

153. $\int(\log x)^2x^3dx=\frac{x^4}{32}f(x)+C\Rightarrow f(x)=$

[AP EAMCET 08-07-2022_Shift-2]

1. $8(\log x)^2-4\log x+1$ 2. $8\log x-4x^4+x^3$
3. $8(\log x)^2+4x-x^2$ 4. $4(\log x)^2-4x^2+x+1$

---

154. Let $f(x)=\int\frac{2x^3-3x^2+4x-5}{x^2}dx$ and $f(1)=1$. Then $f(5)=$

[TS EAMCET 18-07-2022_Shift-1]

1. 10 + 4 log 5 2. 10 – 4 log 5
3. 9 + 4 log 5 4. 9 – 4 log 5

---

155. If $x>0$ and $x\neq(2n+1)\frac{\pi}{2}$ then

$$\int\left(x\sqrt{x}-e^{\log(\sec x\tan x)}+\frac{3x^2-2x+1}{x^2}\right)dx=$$

[TS EAMCET 18-07-2022_Shift-1]

1. $x\sqrt{x}-\sec x+3x-2\log x-\frac{1}{x}+c$
2. $\frac{2}{5}x^2\sqrt{x}-\sec x+3x+\frac{2}{x^2}-\frac{1}{x}+c$
3. $x\sqrt{x}-\sec x+3x+\frac{2}{x^2}-\frac{1}{x}+c$
4. $\frac{2}{5}x^2\sqrt{x}-\sec x+3x-2\log x-\frac{1}{x}+c$

---

156. $\int(2x-3)\sqrt{3x+2}\,dx=$

[TS EAMCET 18-07-2022_Shift-1]

1. $\frac{2}{135}\left(54x^2-123x+106\right)\sqrt{3x+2}+c$
2. $\frac{2}{135}\left(54x^2+123x-106\right)\sqrt{3x+2}+c$

3. $\frac{2}{135}\left(54x^2-123x-106\right)\sqrt{3x+2}+c$

4. $\frac{2}{135}\left(54x^2-195x-106\right)\sqrt{3x+2}+c$

157. If $\int \frac{1+\cos 8x}{\tan 2x-\cot 2x}dx = f(x).\cos(g(x))+c$, then $f\left(\frac{1}{4}\right)+g\left(\frac{1}{4}\right)=$

[TS EAMCET 18-07-2022_Shift-2]

1. 2
2. $\frac{17}{8}$
3. $\frac{15}{8}$
4. $\frac{33}{16}$

158. Let $x\neq \frac{-3}{5},\frac{2}{5}$, if $f\left(\frac{2x+1}{5x+3}\right)=x+2$, then $\int f(x)\,dx=$

[TS EAMCET 18-07-2022_Shift-2]

1. $\frac{7}{5}x-\frac{1}{5}\log|5x+3|+c$
2. $\frac{7}{5}x-\frac{1}{25}\log|5x+3|+c$
3. $\frac{7}{5}x-\frac{1}{25}\log|5x-2|+c$
4. $\frac{7}{5}x-\frac{1}{5}\log|5x-2|+c$

159. If $\int e^x \cos x dx = \frac{e^x}{2}(\cos x+\sin x)$ and

$$\int \frac{\cos\left(\log\left(\frac{2x+3}{3-2x}\right)\right)}{(3-2x)^2}dx =$$

$\frac{f(x)}{24}\left[\cos(g(x))+\sin(g(x))\right]+c$ then g(1)

[TS EAMCET 18-07-2022_Shift-2]

1. 5
2. log f(2)
3. log f(1)
4. 0

160. Given that $\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$ and $\frac{d}{dx}(\sinh^{-1}x)=\frac{1}{\sqrt{1+x^2}}$. Then

$$\int\frac{3x^6-2x^4+x^2-2}{x^2+1}dx=$$

[TS EAMCET 19-07-2022_Shift-I]

1. $\frac{3}{7}x^7-\frac{2}{5}x^5+\frac{1}{3}x^3-2x+c$
2. $\frac{\frac{3}{7}x^7-\frac{2}{5}x^5+\frac{1}{3}x^3-2x}{\frac{x^3}{3}+x}+c$
3. $\frac{3}{5}x^5-\frac{5}{3}x^3+6x-8\tan^{-1}x+c$
4. $\frac{3}{5}x^5-\frac{5}{3}x^3+6x-8\sinh^{-1}x+c$

161. $\int\frac{\sin x.\sec^2 x-\tan x.\sin x+\cos x}{(1-\cos 2x)}dx=$

[TS EAMCET 19-07-2022_Shift-I]

1. $\frac{1}{2}\left[\sec x-\cos ecx-\log\left|\tan\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|\right]+c$
2. $\sec x-\cos ecx+\log\left|\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right|+c$
3. $\frac{1}{2}\left[\sec x-\cos ecx-\log\left|\frac{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}\right|\right]+c$
4. $\sec x+\cos ecx-\log\left|\tan\left(\frac{x}{2}\right)\right|+c$

162. If $f(x)=\int\frac{16x^7+5x^{10}}{\left(x^3+2+3x^8\right)^2}dx\ (x\geq 0)$ and $f(0)=1$, then the value of $f(-1)$ is

[TS EAMCET 19-07-2022_Shift-I]

1. $\frac{7}{6}$
2. $\frac{5}{4}$
3. $\frac{-3}{4}$
4. $\frac{-5}{6}$

163. Let g(x) be anti–derivative of f(x).Then the function for which $\log_e(1+(g(x))^2)+c$ is an antiderivative is

[TS EAMCET 19-07-2022_Shift-2]

1. $\frac{(1+(g(x))^2)}{g'(x)f(x)}$
2. $\frac{-2f(x)g(x)}{1+g(x)}$
3. $\frac{2f(x)g(x)}{1+(g(x))^2}$
4. $\frac{2g(x)}{1+(g(x))^2}$

164. If $f(x)=\int\left[\tan^2 x+\cot^2 x+\frac{4(\sin^3 x+\cos^3 x)}{\sin^2 2x}\right]$ dx and $f\left(\frac{\pi}{4}\right)=0$, then $3\left[f\left(\frac{\pi}{6}\right)+2\right]=$

[TS EAMCET 19-07-2022_Shift-2]

1. $\frac{\pi}{2}$
2. $\frac{\pi}{4}$
3. 0
4. $\frac{-\pi}{2}$

165. $\int\sqrt{4\cos^2 x-5\sin^2 x}\cos x dx=$

[TS EAMCET 19-07-2022_Shift-2]

1. $\frac{1}{2}\sin x\sqrt{4-9\sin^2 x}+\frac{2}{3}\sin^{-1}\left(\frac{3\sin x}{2}\right)+c$
2. $\frac{1}{2}\cos x\sqrt{4-9\cos^2 x}+\frac{2}{3}\sin^{-1}\left(\frac{3\cos x}{2}\right)+c$
3. $\frac{1}{2}\sin x\sqrt{4-9\sin^2 x}+\frac{2}{3}\cos^{-1}\left(\frac{3\cos x}{2}\right)+c$
4. $\frac{1}{2}\cos x\sqrt{4-9\sin^2 x}+\frac{2}{3}\sin^{-1}\left(\frac{3\sin x}{2}\right)+c$

166. If $\frac{3\pi}{4}<x<\frac{7\pi}{4}$ then

$\int\left(2^x-\sqrt{1+\sin 2x}+\frac{1}{x^2}-\frac{1}{x}\right)dx=$

[TS EAMCET 20-07-2022_Shift-1]

1. $\frac{2^x}{\log 2}-\sin x+\cos x-\frac{1}{x}-\log x+c$
2. $2^x\log 2+\sin x-\cos x-\frac{1}{x}+\frac{1}{x^2}+c$
3. $\frac{2^x}{\log 2}+\sin x-\cos x-\frac{1}{x}-\log x+c$
4. $2^x\log 2-\sin x+\cos x-\frac{1}{x}+\frac{1}{x^2}+c$

167 If $\tan\alpha=\frac{4}{3}$, then $\int\frac{1}{3\cos x-4\sin x}dx=$

[TS EAMCET 20-07-2022_Shift-1]

1. $\frac{1}{5}\log\left|\tan\left(\frac{x}{2}+\frac{\alpha}{2}\right)\right|+c$
2. $\frac{1}{5}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}+\frac{\alpha}{2}\right)\right|+c$
3. $\frac{1}{5}\log\left|\tan\left(\frac{\pi}{4}-\frac{x}{2}-\frac{\alpha}{2}\right)\right|+c$
4. $\frac{1}{5}\log\left|\tan\left|\sec x+\tan x\right|\right|+c$

168. If $x\neq(2n+1)\frac{\pi}{2}$, then $\int\frac{\cos^3 x}{(1+\sin x)^4}dx=$

[TS EAMCET 20-07-2022_Shift-1]

1. $\frac{\cos^4 x}{(1+\sin x)^3}+c$
2. $\frac{\cos^3 x}{(1+\sin x)^3}+c$
3. $\frac{\cos^4 x}{(1+\sin x)^4}+c$
4. $\frac{\cos^4 x}{4(1+\sin x)^4}+c$

169. If $f(x)=\int\frac{2-3\sin^2 x}{1+\cos 2x}dx$ and $f\left(\frac{\pi}{4}\right)=1$ then $f(0)=$ **[TS EAMCET 20-07-2022_Shift-2]**

1. $\frac{3}{8}(4-\pi)$
2. $3-\frac{\pi}{4}$
3. 0
4. 1

170. If $x\neq(2n+1)\frac{\pi}{2}, n\in\mathbb{Z}$ and $\cos x\neq\frac{-1}{2}$ then

$\int\left(\frac{\sin x+\sin 2x}{1+\cos x+\cos 2x}\right)^2 dx=$

[TS EAMCET 20-07-2022_Shift-2]

1. $\frac{\tan^3 x}{3}-x+c$
2. $\frac{\sec^3 x}{3}-x+c$
3. $\cot x-x+c$
4. $\tan x-x+c$

171. Given that $\int\frac{1}{x^2+a^2}dx=\frac{1}{a}Tan^{-1}\left(\frac{x}{a}\right)+c$. If

$\int\frac{1}{x^4+3x^2+1}dx=a.$

$Tan^{-1}\left(\frac{b(x^2-1)}{x}\right)+cTan^{-1}\left(\frac{d(x^2+1)}{x}\right)+k$

where k is a constant of integration, then $5(c+d+ab)=$

[TS EAMCET 20-07-2022_Shift-2]

1. 3
2. 5
3. 8
4. 10

172.If $\int\left(\frac{4e^{x}+6e^{-x}}{9e^{x}-4e^{-x}}\right)dx = Ax + B\log\left|\left(9e^{2x}-4\right)\right|+C$, then $(A, B)=$ **[15th May 2023 Shift 1]**

1. $\left(\frac{3}{2},\frac{35}{36}\right)$
2. $\left(\frac{-3}{2},\frac{-35}{36}\right)$
3. $\left(\frac{-3}{2},\frac{35}{36}\right)$
4. $\left(\frac{3}{2},\frac{-35}{36}\right)$

173 If $f(x)=\int\frac{dx}{\left(x^{2}+2\right)}$ and $f\left(\sqrt{2}\right)=0$, then $f(0)=$ **[15th May 2023 Shift 1]**

1. $\frac{\pi}{2\sqrt{2}}$
2. $\frac{-\pi}{2\sqrt{2}}$
3. $\frac{-\pi}{4\sqrt{2}}$
4. $\frac{\pi}{4\sqrt{2}}$

174 $\int e^{x}\left(\frac{2+\sin 2x}{1+\cos 2x}\right)dx=$ **[15th May 2023 Shift 1]**

1. $e^{x}\sec x+C$
2. $e^{x}\tan x+C$
3. $e^{x}\cot x+C$
4. $e^{x}\operatorname{cosec}x+C$

175 $\int\frac{dx}{4+5\cos x}=$ **[15th May 2023 Shift 2]**

1. $-\frac{1}{3}\log\left|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right|+C$
2. $\frac{1}{3}\log\left|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right|+C$
3. $-\frac{1}{9}\log\left|\frac{3-\tan\frac{x}{2}}{3+\tan\frac{x}{2}}\right|+C$
4. $\frac{1}{9}\log\left|\frac{3-\tan\frac{x}{2}}{3+\tan\frac{x}{2}}\right|+C$

176 If $f(x)=\int\frac{5x^{8}+7x^{6}}{\left(x^{2}+2x^{7}+1\right)^{2}}\mathrm{dx}\,(x\geq 0)$ and f(0)=0, then the value of f(1)=

[15th May 2023 Shift 2]

1. $\frac{-1}{2}$
2. $\frac{-1}{4}$
3. $\frac{1}{4}$
4. $\frac{1}{2}$

177 $\int x^{3}(\log x)^{2}\,\mathrm{dx}=$ **[15th May 2023 Shift 2]**

1. $(\log x)^{2}\frac{x^{4}}{4}+\frac{1}{2}\left[(\log x)\frac{x^{4}}{4}+\frac{x^{4}}{16}\right]+C$
2. $(\log x)^{2}\frac{x^{4}}{4}-\frac{1}{2}\left[(\log x)\frac{x^{4}}{4}+\frac{x^{4}}{16}\right]+C$
3. $(\log x)^{2}\frac{x^{4}}{4}-\frac{1}{2}\left[(\log x)\frac{x^{4}}{4}-\frac{x^{4}}{16}\right]+C$
4. $(\log x)^{2}\frac{x^{4}}{4}+\frac{1}{2}\left[(\log x)\frac{x^{4}}{4}-\frac{x^{4}}{16}\right]+C$

178 If $\int\frac{1}{x\left[(\log x)^{2}+4\log x-1\right]}dx = A\log\left[\frac{\log x+B}{\log x+C}\right]+K$

where K is the constant of integration then

[15th May 2023 Shift 2]

1. A=$\frac{1}{2\sqrt{5}}, B=\left(2-\sqrt{5}\right), C=\left(2+\sqrt{5}\right)$
2. A=$-\frac{1}{2\sqrt{5}}, B=\left(2-\sqrt{5}\right), C=\left(2+\sqrt{5}\right)$
3. A=$\frac{1}{2\sqrt{5}}, B=\left(2+\sqrt{5}\right), C=\left(2-\sqrt{5}\right)$
4. A=$-\frac{1}{2\sqrt{5}}, B=\left(2+\sqrt{5}\right), C=\left(2-\sqrt{5}\right)$

179 $\int\frac{x^{2}-1}{x^{3}\sqrt{2x^{4}-2x^{2}+1}}dx=$

[16th May 2023 Shift 1]

1. $\frac{1}{2x^{2}}\sqrt{2x^{4}+2x^{2}+1}+C$
2. $\frac{1}{2x^{2}}\sqrt{2x^{4}-2x^{2}+1}+C$

3. $\frac{1}{2x^2}\sqrt{4x^4-2x^2+1}+C$

4. $\frac{1}{2x^2}\sqrt{4x^4+2x^2+1}+C$

180 If the graph of the anti derivative g(x) of f(x) = log(log x) + $(\log x)^2$ passes through (e,2023-e) and the term independent of x in g(x) is k, then the sum of all the digits of k is [16th May 2023 Shift 1]

1. 5
2. 6
3. 7
4. 8

181 $\int 3^{-\log_a x^2} dx =$ **[16th May 2023 Shift 1]**

1. $2\log|x|+C$
2. $\log|x|+C$
3. $-\log|x|+C$
4. $-2\log|x|+C$

182. $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sqrt{x}(\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x})}dx =$

[16th May 2023 Shift 2]

1. $\frac{2}{\pi}\left[\sin^{-1}\sqrt{x}(2x-1)+\sqrt{x(1-x)}\right]+x+C$
2. $\frac{8}{\pi}\left[\sqrt{x}\sin^{-1}\sqrt{x}+\sqrt{1-x)}\right]-2\sqrt{x}+C$
3. $\frac{2}{\pi}\left[(2x-1)\sin^{-1}\sqrt{x}-\sqrt{x(1-x)}\right]-x+C$
4. $\frac{2}{\pi}\left[(2x-1)\sin^{-1}\sqrt{x}-\sqrt{x(1-x)}\right]+x+C$

183. $If\int\frac{3x+2}{4x^2+4x+5}dx = A\log(4x^2+4x+5)+BTan^{-1}\left(x+\frac{1}{2}\right)+C,$ $then(A,B)=$

[16th May 2023 Shift 2]

1. $\left(\frac{3}{8},\frac{1}{8}\right)$
2. $\left(\frac{5}{8},\frac{1}{8}\right)$
3. $\left(\frac{-3}{8},\frac{1}{8}\right)$
4. $\left(\frac{-5}{8},\frac{1}{8}\right)$

184. $\int e^x\left(\log x+\frac{1}{x^2}\right)dx =$

[16th May 2023 Shift 2]

1. $e^x\left(\log x+\frac{1}{x}\right)+C$
2. $e^x\left(\log x-\frac{1}{x}\right)+C$
3. $e^x\left(\log x+\frac{2}{x}\right)+C$
4. $e^x\left(\log x-\frac{2}{x}\right)+C$

185 $\int\frac{\sqrt{x^4+x^{-4}+2}}{x^3}dx =$

[16th May 2023 Shift 2]

1. $\log|x|-\frac{1}{4x^4}+C$
2. $\log|x|+\frac{1}{4x^4}+C$
3. $\log|x|-\frac{1}{x^4}+C$
4. $\log|x|+\frac{1}{x^4}+C$

186 $\int e^{\sin x}\frac{\left(x\cos^3 x-\sin x\right)}{\cos^2 x}dx =$

[17th May 2023 Shift 1]

1. $e^{\sin x}(x-\sec x)+C$
2. $e^{\sin x}(x-\operatorname{cosec} x)+C$
3. $e^{\sin x}(x+\sec x)+C$
4. $e^{\sin x}(x+\operatorname{cosec} x)+C$

187 If $f'(x)=a\cos x+b\sin$ x and

$f'(0)=4, f(0)=3, f\left(\frac{\pi}{2}\right)=5,$ then $f(x)=$

[17th May 2023 Shift 1]

1. $2\cos x+4\sin x+1$
2. $4\cos x+2\sin x+1$
3. $2\cos x+3\sin x+1$
4. $4\cos x+\sin x+1$

188. If $\int f(x)dx=\Psi(x)$, then $\int x^5 f(x^3)dx =$

[17th May 2023 Shift 1]

1. $\frac{1}{3}\left[x^3\Psi(x^3)\right]-\int x^2\Psi(x^3)dx$
2. $\frac{1}{3}\left[x^3\Psi(x^3)\right]+\int x^2\Psi(x^3)dx$
3. $-\frac{1}{3}\left[x^3\Psi(x^3)\right]-\int x^3\Psi(x^3)dx$
4. $-\frac{1}{3}\left[x^3\Psi(x^3)\right]+\int x^3\Psi(x^3)dx$

189 $If\int\frac{\sin^2\alpha-\sin^2 x}{\cos x-\cos\alpha}dx = f(x)+Ax+B\ and\ B\in R,$

then **[17th May 2023 Shift 1]**

1. $f(x)=2\sin x,\ A=\cos\alpha$

2. $f(x)=2\sin x,\ A=2\cos\alpha$

3. $f(x)=\sin x,\ A=\cos\alpha$

4. $f(x)=\sin x,\ A=2\cos\alpha$

190 $\int\left(x^{3m}+x^{2m}+x^{m}\right)\left(2x^{2m}+3x^{m}+6\right)^{\frac{1}{m}}dx=$

[17th May 2023 Shift 1]

1. $\frac{1}{6(m+1)}\left(2x^{3m}+3x^{2m}+6x^{m}\right)^{\frac{m+1}{m}}+C$

2. $\frac{1}{6(m+1)}\left(2x^{3m}+3x^{2m}+6x^{m}\right)^{\frac{m-1}{m}}+C$

3. $\frac{1}{6(m+1)}\left(2x^{3m}+3x^{2m}+6\right)^{\frac{m+1}{m}}+C$

4. $\frac{1}{6(m-1)}\left(2x^{3m}+mx^{2m}+6x^{m}\right)^{\frac{m-1}{m}}+C$

191. $\int\frac{dx}{(2ax+x^2)^{\frac{3}{2}}}=$ **[17th May 2023 Shift 2]**

1. $\frac{1}{a^2}\left(\frac{(x+a)}{\sqrt{2ax+x^2}}\right)+c$ 2. $\frac{1}{a^2}\left(\frac{(x-a)}{\sqrt{2ax+x^2}}\right)+c$

3. $\frac{-1}{a^2}\left(\frac{(x-a)}{\sqrt{2ax+x^2}}\right)+c$ 4. $\frac{-1}{a^2}\left(\frac{(x+a)}{\sqrt{2ax+x^2}}\right)+c$

192. If f(x)= $\lim_{n\to\infty} n^2\left(x^{\frac{1}{n}}-x^{\frac{1}{n+1}}\right)$, x>0, then

$\int xf(x)dx=$ **[17th May 2023 Shift 2]**

1. $\frac{x^2}{2}\log x+c$ 2. $\frac{x^2}{2}\log x+\frac{x^2}{4}+c$

3. $\frac{x^2}{2}\log x-\frac{x^2}{4}+c$ 4. $-\frac{x^2}{2}\log x+\frac{x^2}{4}+c$

193. $\int\frac{1-\cos x}{\cos x(1+\cos x)}dx=$

[17th May 2023 Shift 2]

1. $\log|\sec x+\tan x|-2(\operatorname{cosec}x-\cot x)+C$

2. $\log|\sec x+\tan x|-2(\operatorname{cosec}x+\cot x)+C$

3. $\log|\sec x+\tan x|+2(\operatorname{cosec}x-\cot x)+C$

4. $\log|\sec x+\tan x|+2(\operatorname{cosec}x+\cot x)+C$

194. g(x) is an anti derivative of f(x) = $1+2^x\log 2$ and the graph of y=g(x) passes through $\left(-1,\frac{1}{2}\right)$ Then the curve meets the y–axis at

[17th May 2023 Shift 2]

1. (0, 1) 2. (0, 2)

3. (0, –2) 4. (1, 1)

195. $\int\frac{x^3}{\sqrt{1+x^2}}dx=A(1+x^2)^{\frac{3}{2}}+B(1+x^2)^{\frac{1}{2}}+C$

then A+B= **[17th May 2023 Shift 2]**

1. $\frac{2}{3}$ 2. $-\frac{2}{3}$

3. $\frac{1}{3}$ 4. $-\frac{1}{3}$

196. $\int\frac{dx}{\sin(x-a)\cos(x-b)}=$

[18th May2023 shift -1]

1. $\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+C$ 2. $\frac{1}{\cos(b-a)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+C$

3. $\frac{1}{\cos(b-a)}\left[\log|\sin(x-a)\cos(x-b)|\right]+C$ 4. $\frac{1}{\sin(a-b)}\left[\log|\sin(x-a)\cos(x-b)|\right]+C$

197. $If\ I_n=\int\tan^n x\,dx(n>1), then\ I_4+I_6=$

[18th May2023 shift -1]

1. $\frac{1}{5}\tan^5 x+C$ 2. $-\frac{1}{5}\tan^5 x+C$

3. $\frac{1}{10}\tan^5 x+C$ 4. $-\frac{1}{10}\tan^5 x+C$

198. $If\int\sqrt{x}(1-x^3)^{\frac{-1}{2}}dx=\frac{2}{3}g(f(x))+c,\ then$

[18th May2023 shift -1]

1. $f(x)=\sqrt{x},g(x)=\sin^{-1}x$ 2. $f(x)=x^{\frac{3}{2}},g(x)=\sin^{-1}x$

3. $f(x)=x^{\frac{3}{2}},g(x)=\cos^{-1}x$ 4. $f(x)=\sqrt{x},g(x)=\cos^{-1}x$

199 $\int\frac{x^2}{(x^2-1)(x^2+1)}dx=$

1. $\frac{1}{4}\log\left|\frac{x+1}{x-1}\right|-\frac{1}{2}Tan^{-1}x+c$

2. $\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|+\frac{1}{2}Tan^{-1}x+c$

3. $\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}Tan^{-1}x+c$

4. $\frac{1}{4}\log\left|\frac{x+1}{x-1}\right|+\frac{1}{2}Tan^{-1}x+c$

[18th May2023 shift -1]

200 If $\int\frac{dx}{1+\sin x}=\tan\left(\frac{x}{2}-\theta\right)+C, then\,\theta=$

[18th May 2023 Shift 2]

1. $\frac{\pi}{2}$ 2. $\frac{\pi}{4}$

3. π 4. $\frac{\pi}{6}$

201 $\int\left(\frac{8^{1+x}+4^{1+x}}{2^{2x}}\right)dx=$ **[18th May 2023 Shift 2]**

1. $\frac{2^x}{\log 2}+4x+C$ 2. $8.\frac{2^x}{\log 2}-4x+C$

3. $8.\frac{2^x}{\log 2}+4x+C$ 4. $\frac{2^x}{\log 2}-4x+C$

202.If

$f(x)=\frac{x}{\left(1+nx^n\right)^{\frac{1}{n}}}$ for $n\geq 2$, then $\int x^{n-2}f(x)\,dx=$

[19th May 2023 Shift 1]

1. $\frac{1}{n(n-1)}\left(1+nx^n\right)^{1-\frac{1}{n}}+C$ 2. $\frac{1}{n-1}\left(1+nx^n\right)^{1-\frac{1}{n}}+C$

3. $\frac{1}{n(n-1)}\left(1+nx^n\right)^{1+\frac{1}{n}}+C$ 4. $\frac{1}{n+1}\left(1+nx^n\right)^{1+\frac{1}{n}}+C$

203. $\int\frac{1+x\cos x}{x\left[1-x^2\left(e^{\sin x}\right)^2\right]}dx=$

[19th May 2023 Shift 1]

1. $\frac{1}{2}\log\left|\frac{\left(xe^{\sin x}\right)^2}{\left(xe^{\sin x}\right)^2+1}\right|+c$ 2. $-\frac{1}{2}\log\left|\frac{\left(xe^{\sin x}\right)^2}{\left(xe^{\sin x}\right)^2+1}\right|+c$

3. $\frac{1}{2}\log\left|\frac{\left(xe^{\sin x}\right)^2}{1-\left(xe^{\sin x}\right)^2}\right|+c$ 4. $-\frac{1}{2}\log\left|\frac{\left(xe^{\sin x}\right)^2}{\left(xe^{\sin x}\right)^2-1}\right|+c$

204. $\int\frac{dx}{(x-1)\sqrt{x+2}}=$ **[19th May 2023 Shift 1]**

1. $\frac{2}{\sqrt{3}}\log\left|\frac{\sqrt{(x+2)}+\sqrt{3}}{\sqrt{(x+2)}-\sqrt{3}}\right|+c$ 2. $\frac{-1}{\sqrt{3}}\log\left|\frac{\sqrt{(x+2)}-\sqrt{3}}{\sqrt{(x+2)}+\sqrt{3}}\right|+c$

3. $\frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{(x+2)}+\sqrt{3}}{\sqrt{(x+2)}-\sqrt{3}}\right|+c$ 4. $\frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{(x+2)}-\sqrt{3}}{\sqrt{(x+2)}+\sqrt{3}}\right|+c$

205. $\int e^{-2x}\left(\frac{1-\sin 2x}{1+\cos 2x}\right)dx=$

[19th May 2023 Shift 1]

1. $\frac{1}{2}e^{-2x}\tan x+c$ 2. $-\frac{1}{2}e^{-2x}\tan x+c$

3. $\frac{1}{2}e^{-2x}\cot x+c$ 4. $-\frac{1}{2}e^{-2x}\cot x+c$

206. $\int\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx=$

[12TH MAY 2023 SHIFT-1]

1. $\frac{1}{2}\cos 2x+c$ 2. $\frac{-1}{2}\cos 2x+c$

3. $\frac{-1}{(1+\tan x)^2}+c$ 4. $\frac{-1}{2}\sin 2x+c$

207 If $\int\frac{x^2\left(x\sec^2 x+\tan x\right)}{(x\tan x+1)^2}dx=\frac{-x^2}{x\tan x+1}+$ $f(x)+c$, then $f(x)=$

[12TH MAY 2023 SHIFT-1]

1. $\log|x\sin x+\cos x|+c$

2. $\log|x\cos x+\sin x|+c$

3. $2\log|x\sin x+\cos x|+c$

4. $2\log|x\cos x+\sin x|+c$

208 If $\int\sin(101x)(\sin x)^{99}dx=\frac{\sin(100x)(\sin x)^{\lambda}}{\mu}$ $+c$ then $\frac{\lambda}{\mu}=$

[12TH MAY 2023 SHIFT-1]

1. 1 2. 2

3. 4 4. 8

209 $\int e^x\left(\sin^2 2x-8\cos 4x\right)dx=$
If $e^x f(x)+c, then\ f\left(\frac{\pi}{4}\right)=$

[12TH MAY 2023 SHIFT-1]

1. 0
2. 1
3. -1
4. e

210 If n is a positive integer greater than 1 and $I_n=\int\frac{\sin nx}{\sin x}dx$, then $I_{n+1}-I_{n-1}=$

[12TH MAY 2023 SHIFT-1]

1. $\frac{2}{n-1}\cos(n-1)x$
2. $\frac{2}{n-1}\sin(n-1)x$
3. $\frac{2}{n}\cos nx$
4. $\frac{2}{n}\sin nx$

211. Match the following items from List I into List II

List –I	List–II
1. $\int\frac{\sin^2 x}{\cos^4 x}dx$	A. $\frac{\tan^2 x}{2}+\ln\lvert\cos x\rvert+c$
2. $\int\frac{\sin^4 x}{\cos^2 x}dx$	B. $\cos x+\sec x+c$
3. $\int\frac{\sin^3 x}{\cos^2 x}dx$	C. $\frac{\tan^3 x}{3}+c$
4. $\int\frac{\sin^3 x}{\cos^3 x}dx$	D. $\tan x+\frac{\sin 2x}{4}-\frac{3x}{2}+c$
	E. $\cos x-\sec x+c$

[12TH MAY 2023 SHIFT -2]

1. 1–C, 2–E, 3–B, 4–A
2. 1–C, 2–D, 3–B, 4–A
3. 1–D, 2–C, 3–A, 4–B
4. 1–C, 2–E, 3–A, 4–D

212 If $\int\frac{x}{(a+x)^5}dx=\frac{1}{k(a+x)^4}(f(x))+c$ then $\frac{f(-a)}{ak}=$

[12TH MAY 2023 SHIFT -2]

1. $\frac{1}{3}$
2. $\frac{1}{2}$
3. $\frac{5}{6}$
4. $\frac{1}{4}$

213. If $\int x^4(\log x)^3dx=x^5\left[A(\log x)^3+B(\log x)^2+C\log x+D\right]+k$, then $A+B+C+5D=$

[12TH MAY 2023 SHIFT -2]

1. $\frac{2}{25}$
2. $\frac{8}{25}$
3. $\frac{12}{125}$
4. $\frac{16}{125}$

214. $\int\frac{1}{16-7\sin^2 x}dx=$

[13TH MAY 2023 SHIFT-1]

1. $\frac{1}{12}\text{Tan}^{-1}\left(\frac{3\tan x}{4}\right)+c$
2. $\frac{1}{3}\text{Sin}^{-1}\left(\frac{3\sin x}{4}\right)+c$
3. $\frac{1}{12}\log\left(\frac{4-\sqrt{7}\sin x}{4+\sqrt{7}\sin x}\right)+c$
4. $\frac{1}{12}\log\left(\frac{4+\sqrt{7}\sin x}{4-\sqrt{7}\sin x}\right)+c$

215 $\int\frac{\sec^2 x}{(\sec x+\tan x)^2}dx=$

[13TH MAY 2023 SHIFT-1]

1. $\frac{3+(\sec x+\tan x)^2}{2(\sec x+\tan x)^3}+c$
2. $-\frac{1+3(\sec x+\tan x)^2}{6(\sec x+\tan x)^3}+c$
3. $-\frac{3+(\sec x+\tan x)^2}{2(\sec x+\tan x)^3}+c$
4. $-\frac{1+(\sec x+\tan x)}{3(\sec x+\tan x)^2}+c$

215. $\int\frac{1}{3\cos x-4\sin x+5}dx=$

[13TH MAY 2023 SHIFT-1]

1. $\frac{2}{\sqrt{5}}\text{Tan}^{-1}\left(\frac{3\tan\frac{x}{2}+4}{\sqrt{5}}\right)+c$

2. $\frac{3}{4}\mathrm{Tan}^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$

3. $\frac{1}{2-\tan\frac{x}{2}}+c$

4. $\frac{1}{1+\tan\frac{x}{2}}+c$

217 $\int\frac{1}{(x-2)(x^2+1)}dx=$

[13TH MAY 2023 SHIFT-1]

1. $\log\frac{\sqrt{x^2+1}}{|x-2|}+2\mathrm{Tan}^{-1}x+c$ 2. $\log\frac{|x-2|}{x^2+1}+2\mathrm{Tan}^{-1}x+c$

3. $\frac{1}{5}\left[\log\frac{|x-2|}{\sqrt{7+x^2}}+2\mathrm{Tan}^{-1}x\right]+c$ 4. $\frac{1}{5}\left[\log\frac{|x-2|}{\sqrt{1+x^2}}-2\mathrm{Tan}^{-1}x\right]+c$

218 If $\int\frac{x^{49}Tan^{-1}\left(x^{50}\right)}{\left(1+x^{100}\right)}$ dx =k $\left(Tan^{-1}\left(x^{50}\right)\right)^2+c,$

then k = **[EAPCET 14-05-23 SHIFT-1]**

1. $\frac{-1}{100}$
2. $\frac{1}{50}$
3. $\frac{-1}{50}$
4. $\frac{1}{100}$

219.If $\int(\log x)^3x^5\,dx=\frac{x^6}{A}\left[B(\log x)^3+C(\log x)^2+D(\log x)-1\right]+k$

and A,B,C,D are integers, then

$A-(B+C+D)=$ **[EAPCET 14-05-23 SHIFT-1]**

1. 172
2. 184
3. 192
4. 216

220. $\int\frac{dx}{\left(x^2+1\right)\left(x^2+4\right)}=$ **[EAPCET 14-05-23 SHIFT-1]**

1. $\frac{1}{3}Tan^{-1}x+\frac{1}{6}Tan^{-1}\left(\frac{x}{2}\right)+c$

2. $\frac{1}{3}Tan^{-1}x-\frac{1}{3}Tan^{-1}\left(\frac{x}{2}\right)+c$

3. $\frac{1}{3}Tan^{-1}x+\frac{1}{3}Tan^{-1}\left(\frac{x}{2}\right)+c$

4. $\frac{1}{3}Tan^{-1}x-\frac{1}{6}Tan^{-1}\left(\frac{x}{2}\right)+c$

221. $\int\frac{dx}{(x-1)^{\frac{3}{4}}(x+2)^{\frac{5}{4}}}=$

[EAPCET 14-05-23 SHIFT-1]

1. $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+c$
2. $\frac{3}{4}\left(\frac{x-1}{x-2}\right)^{\frac{1}{4}}+c$
3. $\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}}+c$
4. $\frac{3}{4}\left(\frac{x-2}{x-1}\right)^{\frac{1}{4}}+c$

222. $\int\frac{1}{\left(x+\frac{2}{x}\right)\sqrt{x^4+4x^2+3}}dx=$

[EAPCET 13-05-23 SHIFT-2]

1. $\frac{1}{2}\sec^{-1}\left(x^2+2\right)+c$ 2. $-Co\sec h^{-1}\left(x^2+2\right)+c$

3. $\frac{1}{2}Tan^{-1}\left(x+\frac{2}{x}\right)+c$ 4. $\frac{-1}{2}Cot^{-1}\left(x+\frac{2}{x}\right)+c$

223 If $\frac{3\pi}{2}<x<\frac{5\pi}{2}$ and $\int\left(\sqrt{1-\sin x}+\sqrt{1+\sin x}\right)dx=f(x)+c$

Where c is the constant of integration, then $f\left(\frac{\pi}{3}\right)-f(0)=$ **[EAPCET 13-05-23 SHIFT-2]**

1. 2
2. -2
3. $2\sqrt{2}$
4. $-2\sqrt{2}$

224 If $\int\frac{2\sin 2x-3\cos x}{2\sin^2 x-3\sin x+4}dx=f(x)+c$ where c is the constant of integration, then $f\left(\frac{\pi}{2}\right)-f(0)=$

[EAPCET 13-05-23 SHIFT-2]

1. $2\log_2$
2. 0
3. $\log\left(\frac{3}{4}\right)$
4. 1

225. $\int\frac{2x+3}{\sqrt{3x^2-2x+1}}dx=$

[EAPCET 13-05-23 SHIFT-2]

1. $\frac{2}{3}\sqrt{3x^2-2x+1}+\frac{11}{3}\sinh^{-1}\left(\frac{3x-1}{\sqrt{2}}\right)+c$

[EAPCET 13-05-23 SHIFT-2]

2. $\frac{1}{3}\sqrt{3x^2-2x+1}+\frac{11}{3}\sinh^{-1}\left(\frac{\sqrt{3}x-1}{\sqrt{2}}\right)+c$

3. $\frac{1}{3}\sqrt{3x^2-2x+1}+\frac{11}{3}\sinh^{-1}\left(\frac{3x-1}{\sqrt{3}}\right)+c$

4. $\frac{2}{3}\sqrt{3x^2-2x+1}+\frac{11}{3\sqrt{3}}\sinh^{-1}\left(\frac{3x-1}{\sqrt{2}}\right)+c$

KEY

1) 1	2) 2	3) 4	4) 4	5) 1
6) 2	7) 4	8) 2	9) 4	10) 2
11) 2	12) 1	13) 3	14) 3	15) 3
16) 1	17) 2	18) 3	19) 2	20) 2
21) 4	22) 3	23) 2	24) 2	25) 4
26) 1	27) 2	28) 3	29) 1	30) 1
31) 2	32) 1	33) 4	34) 1	35) 2
36) 2	37) 1	38) 3	39) 4	40) 1
41) 1	42) 3	43) 2	44) 2	45) 2
46) 1	47) 3	48) 1	49) 2	50) 4
51) 1	52) 4	53) 1	54) 4	55) 3
56) 1	57) 2	58) 4	59) 4	60) 2
61) 2	62) 1	63) 2	64) 4	65) 4
66) 3	67) 1	68) 3	69) 1	70) 2
71) 1	72) 1	73) 2	74) 2	75) 2
76) 1	77) 1	78) 2	79) 2	80) 3
81) 3	82) 1	83) 2	84) 4	85) 3
86) 4	87) 4	88) 4	89) 3	90) 4
91) 3	92) 2	93) 1	94) 2	95) 4
96) 3	97) 4	98) 3	99) 4	100) 1
101) 2	102) 1	103) 1	104) 2	105) 2
106) 1	107) 2	108) 2	109) 2	110) 4
111) 2	112) 3	113) 4	114) 3	115) 2
116) 2	117) 1	118) 3	119) 2	120) 3
121) 2	122) 1	123) 1	124) 1	125) 2
126) 4	127) 4	128) 1	129) 1	130) 1
131) 1	132) 2	133) 1	134) 1	135) 3
136) 4	137) 3	138) 1	139) 2	140) 2
141) 4	142) 1	143) 2	144) 4	145) 2
146) 1	147) 1	148) 2	149) 1	150) 3
151) 3	152) 3	153) 1	154) 3	155) 4
156) 3	157) 4	158) 3	159) 3	160) 3
161) 3	162) 2	163) 3	164) 1	165) 1
166) 3	167) 2	168) 4	169) 1	170) 4
171) 1	172) 3	173) 3	174) 2	175) 2
176) 3	177) 3	178) 1	179) 2	180) 3
181) 2	182) 2	183) 1	184) 2	185) 1
186) 1	187) 1	188) 1	189) 3	190) 1
191) 4	192) 3	193) 1	194) 2	195) 2
196) 2	197) 1	198) 2	199) 2	200) 2
201) 3	202) 1	203) 3	204) 4	205) 1
206) 4	207) 3	208) 1	209) 2	210) 4
211) 2	212) 4	213) 1	214) 1	215) 2
216) 3	217) 4	218) 4	219) 3	220) 4
221) 1	222) 1	223) 2	224) 3	225) 4

SOLUTIONS

1.

$$\frac{\cos 13x-\cos 14x}{1+2\cos 9x}=\frac{2\sin\frac{27x}{2}\sin\frac{x}{2}}{1+2\left[1-2\sin^2\frac{9x}{2}\right]}$$

$$=\frac{2\sin\frac{27x}{2}\sin\frac{x}{2}}{\sin\frac{9x}{2}\left[3-4\sin^2\frac{9x}{2}\right]\cdot\frac{1}{\sin\frac{9x}{2}}}$$

$$=2\sin\frac{x}{2}\sin\frac{9x}{2}=\cos(-4x)-\cos 5x=$$

$$\cos 4x-\cos 5x$$

$$\int\frac{\cos 13x-\cos 14x}{1+2\cos 9x}dx=\int(\cos 4x-\cos 5x)dx$$

$$=\frac{\sin 4x}{4}-\frac{\sin 5x}{5}+c$$

here

$a=4, b=5$

$a^b=4^5$

2. Given

$$I_n=\int(\ln x)^n.1\ dx$$

$$=(\log x)^n.x-\int n\frac{(\log x)^{n-1}}{x}xdx$$

$$\therefore I_n=I_n.x-n.I_{n-1}+k$$

$$\therefore I_n+nI_{n-1}=I_n.x-n.I_{n-1}+k+nI_{n-1}$$

$$=I_n.x+K$$

$$=(\log x)^n.x+k$$

3. $\int \frac{\cos^3(x)}{\sin^2(x)+\sin(x)}dx$

$= \int \frac{(1-\sin^2 x)\cos x}{\sin^2 x+\sin x}dx$

put $\sin x = t$

$\int \frac{(1-t^2)}{t^2+t}dt$

$\int \left(\frac{1-t}{t}\right)dt = \log|t| - t + c$

$= \log|\sin x| - \sin x + c$

4. Let logx =t

$\Rightarrow \frac{1}{x}dx = dt$

$\Rightarrow dx = e^t dt$

$\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right]dx = \int \left(\log t + \frac{1}{t^2}\right)e^t dt$

$= \int e^t \left[\log t + \frac{1}{t} - \frac{1}{t} + \frac{1}{t^2}\right]dt$

$= \int e^t \left[\left(\log t - \frac{1}{t}\right) + \left(\frac{1}{t} + \frac{1}{t^2}\right)\right]dt$

$= e^t \left(\log t - \frac{1}{t}\right) + c$

$= x\left(\log(\log x) - \frac{1}{\log x}\right) + c$

5. $\int \frac{\sqrt{x-1}}{x\sqrt{x+1}}dx = \int \frac{x-1}{x\sqrt{x^2-1}}dx$

$= \int \frac{1}{\sqrt{x^2-1}}dx - \int \frac{1}{x\sqrt{x^2-1}}dx$

$= \log\left(x+\sqrt{x^2-1}\right) - \sec^{-1}x + c$

6. $\int \frac{2\sin\frac{15x}{2}\sin\frac{x}{2}}{1+2\cos 5x}dx$

$= \int \frac{2\left(3\sin\frac{5x}{2} - 4\sin^3\frac{5x}{2}\right)\sin\frac{x}{2}}{3-4\sin^2\frac{5x}{2}}dx$

$= \int 2\sin\frac{x}{2}\sin\frac{5x}{2}dx = \int \cos 2x - \cos 3x\, dx$

$= \frac{\sin 2x}{2} - \frac{\sin 3x}{3} + c$

7. Put $\sqrt{k} = t$

8. $I = \int e^{x/2}\left(\sec^2\frac{x}{2} + \tan\frac{x}{2}\right)dx$

put $\frac{x}{2} = t$

$= 2\int e^t\left(\sec^2 t + \tan t\right)dt$

$= 2e^t \tan t + c$

$= 2.e^{x/2}\tan\frac{x}{2} + c$

9. Put logx=t

$\Rightarrow dx = e^t dt$

$\therefore I = \int\left[\frac{1}{t} - \frac{1}{t^2}\right]e^t dt$

$= e^t\left(\frac{1}{t}\right) + c$

$= \frac{x}{\log x} + c$

10. $f(x) = \mathrm{Tan}^{-1}\left(\cot\frac{x}{2}\right);\ g(x) = \mathrm{Tan}^{-1}\left(\cot\frac{x}{2}\right)$

$f(x) + g(x) = 2\tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right] = \pi - x$

$\therefore I = \pi x - \frac{x^2}{2}$

11. $y = \int x^3 e^{x^4}dx.$

$Let\ t = x^4$

$dt = 4x^3 dx$

$x^3 dx = \frac{1}{4}dt$

$y = \int e^t \frac{1}{4}dt$

$= \frac{1}{4}e^t + c$

$= \frac{1}{4}e^{x^4} + c$

Passes through (0,1)

$$c=\frac{3}{4}$$

$$y=\frac{1}{4}e^{x^4}+\frac{3}{4}$$

$$4y-3=e^{x^4}$$

$$x^4=\log|4y-3|$$

$$f(y)=x=(\log|4y-3|)^{1/4}$$

12. $\int \frac{\sin^3(x)+\cos^3(x)}{\sin^2(x).\cos^2(x)}dx$

$$=\int(\sec x+\tan x+\operatorname{cosec}x.\cot x)dx$$

$$=\sec x-\operatorname{cosec}x+c$$

13. $\int \frac{dx}{\cos^2(x)+2\sin x\cos x}$

$$\int\frac{\sec^2 x}{1+2\tan x}dx$$

$$t=\tan x\Rightarrow dt=\sec^2 x\,dx$$

$$=\int\frac{1}{1+2t}dt$$

$$=\frac{1}{2}\log|1+2t|+c$$

$$=\frac{1}{2}\log|1+2\tan x|+c$$

14. $\int \frac{x^2+1+2x}{x(x^2+1)}dx$

$$=\int\left[\frac{x^2+1}{x(x^2+1)}+\frac{2x}{x(x^2+1)}\right]dx$$

$$=\int\left[\frac{1}{x}+\frac{2}{(x^2+1)}\right]dx$$

$$=\log|x|+2\tan^{-1}x+c$$

15. Given $\int\frac{1+\cos(4x)}{\cot x-\tan x}dx$

$$=\int\frac{2\cos^2 2x}{\left(\frac{\cos^2 x-\sin^2 x}{\sin x.\cos x}\right)}dx$$

$$=\int\cos 2x.\sin 2x.dx$$

$$=\frac{1}{2}\int\sin 4x.dx$$

$$=\frac{-1}{8}\cos 4x+c$$

$$k=\frac{-1}{8}$$

16. Given $\int x(\tan^2 x)dx$

$$=\int\frac{x.(1-\cos^2 x)}{\cos^2 x}dx$$

$$=\int x.\sec^2 x.dx-\int x.dx$$

$$=x.\operatorname{Tan}x-\int \operatorname{Tan}x.dx-\frac{x^2}{2}+c$$

$$=x.\operatorname{Tan}x-\log|\sec x|-\frac{x^2}{2}+c$$

17. W.k.t $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$

Given $I=\int x^{2020}(\tan^{-1}x+\cot^{-1}x).dx$

$$=\frac{\pi}{2}\int x^{2020}dx$$

$$=\frac{\pi}{2}\left(\frac{x^{2021}}{2021}\right)+c$$

$$=\frac{x^{2021}}{2021}(\tan^{-1}x+\cot^{-1}x)+c$$

18. Suppose that f & g are integrable on [a,b] and also f+g is also integrable on [a,b]

19. $\int\left(\frac{2x^3-3x+5}{2x^2}\right)dx$

$$=\int\left(x-\frac{3}{2}.\frac{1}{x}+\frac{5}{2}.\frac{1}{x^2}\right)dx$$

$$=\frac{x^2}{2}-\frac{3}{2}\log|x|-\frac{5}{2x}$$

Now $x\neq 0, x>0$

20. $\int(1+e^{-x})^{-1}dx$

$$=\int\frac{1}{1+e^{-x}}dx$$

$$=\int \frac{e^x}{e^x+1}dx$$

$$=\log\left(e^x+1\right)+c\left(\because \int \frac{f'(x)}{f(x)}dx=\log\left|f(x)\right|+c\right)$$

21. Conceptual

22. $\int\left(\sqrt{1+\sin(2x)}\right)dx$

$$=\int(\sin x+\cos x)dx$$

$$=\sin x-\cos x+c$$

23. Conceptual

24. $\int e^{3\log x}\left(x^4+1\right)^{-1}\ dx$

$$=\frac{1}{4}\int\frac{4x^3dx}{x^4+1}$$

$$=\frac{1}{4}\log\left(1+x^4\right)+c$$

25. $\int\frac{\sin(2x)}{\sin^2(x)+2\cos^2(x)}\ dx=$

$$\int\frac{\sin 2x}{1+\cos^2 x}dx$$

Put $1+\cos^2 x=t$

$$\Rightarrow\int\frac{f'(x)}{f(x)}dx=\log f(x)+c$$

$$=-\log\left|1+\cos^2 x\right|+c$$

26. $\int\frac{5\tan x}{(\tan x)-2}dx$

$$=\int\ \frac{5\sin x}{\sin x-2\cos x}dx$$

w.k.t

$$\int\ \frac{a\cos x+b\sin x}{c\cos x+d\sin x}dx$$

$$=\frac{ac+bd}{c^2+d^2}x+\frac{ad-bc}{c^2+d^2}\log\left|c\cos x+d\sin x\right|+c$$

$$=\frac{5}{5}x+\frac{10}{2}\log\left|\sin x-2\cos x\right|+c$$

$$=x+5\log\left|\sin x-2\cos x\right|+c$$

$$\Rightarrow\alpha=1,\beta=2\Rightarrow\alpha-\beta=-1$$

27. $\int e^{\frac{x}{\sin x}}.\left(\frac{1}{\sin x}-\frac{x\cos x}{\sin x}\right)dx$

$$\frac{x}{\sin x}=t$$

$$\left(\frac{1}{\sin x}-\frac{x\cos x}{\sin x}\right)dx=dt$$

$$\int\ e^t dt=e^{\frac{x}{\sin x}}+c=e^{\cos ecx}+c$$

28. $\int\frac{(1+x)e^x}{\cot(xe^x)}\ dx$

put $xe^x=t$

$$(1+x)e^x dx=dt$$

$$\int\ \tan t dt=-\log\left|\cos t\right|+c$$

$$=\log\left|\sec\left(xe^x\right)\right|+c$$

29. $3^x=t\Rightarrow 3^x dx=(\log 3)^{-1}$

$$\int\frac{dt}{\sqrt{1-t^2}}(\log 3)^{-1}=\sin^{-1}(3^x)(\log 3)^{-1}+C$$

30. $\int_0^{\pi/2}\sin^4 x\cos^2 x\ dx=\int_0^{\pi/2}\cos^4 x\sin^2 x\ dx=\frac{\pi}{32}$

31. $f'(x)=a\sin x+b\cos x$

$f'(0)=4\Rightarrow b=4$

$f(x)=-a\cos x+b\sin x+C$

$f(0)=3\Rightarrow C-a=3$

$$f\left(\frac{\pi}{2}\right)=5\Rightarrow b+c=5\Rightarrow C=1,a=-2$$

$f(x)=-2\cos x+4\sin x+1$

32. $x^n=t\Rightarrow x^{n-1}dx=\frac{1}{n}dt$

$$\frac{1}{n}\int\frac{dt}{2^2+t^2}=\frac{1}{2n}\mathrm{Tan}^{-1}\left(\frac{x^n}{2}\right)+C$$

33. $\int\sec 2x\ dx=\frac{1}{2}\log|\sec 2x+\tan 2x|+C$

$$=\frac{1}{2}\log\left|\frac{(1+\tan x)^2}{(1+\tan x)(1-\tan x)}\right|+C$$

$$=\frac{1}{2}\log\left|\frac{1+\tan x}{1-\tan x}\right|+C$$

34. $2e^x-1=t^2$

$$dx=\frac{2t}{t^2+1}$$

$$\therefore\int\frac{dx}{\sqrt{2e^x-1}}=\int\frac{2t}{(t^2+1)t}dt$$

$$=2\tan^{-1}t+C$$

$$=2\tan^{-1}\sqrt{2e^x-1}+C$$

35. $\int\frac{dx}{e^x+4e^{-x}}=f(x)+C$

$$=\int\frac{e^x}{\left(e^x\right)^2+4}\,dx$$

Let $e^x=t\ \Rightarrow e^x dx=dt$

$$=\int\frac{dt}{t^2+2^2}=\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)+C$$

$$=\frac{1}{2}\tan^{-1}\left(\frac{e^x}{2}\right)+C$$

$$\therefore f(x)=\frac{1}{2}\tan^{-1}\left(\frac{e^x}{2}\right)+C$$

36. $I_n=\int(\log x)^n\ dx$

Let $\log x=t\Rightarrow x=e^t\Rightarrow dx=e^t\,dt$

$$I_n=\int t^n.e^t\ dt$$

$$I_n+nI_{n-1}=(\log x)^n x.$$

37. $\int\frac{2t+1}{t^2+t+1}\,dt$

We know that,

$$\int\frac{f'(t)}{f(t)}\,dt=\log|f(t)|+C$$

$$\therefore\int\frac{2t+1}{t^2+t+1}\,dt=\log|t^2+t+1|+C$$

38. $\tan^{-1}x\times\frac{-1}{2x^2}+\frac{1}{2}\int\frac{1}{\left(1+x^2\right)x^2}dx$

$$=\frac{-1}{2x^2}\tan^{-1}x+\frac{1}{2}\left[\frac{-1}{x}-\tan^{-1}x\right]+C$$

$$=\frac{-1}{2x}-\left(\frac{1}{2}+\frac{1}{2x^2}\right)\tan^{-1}x+C$$

39. $\int\frac{\sin^{2019}x-\cos^{2019}x}{\sin^{2019}x}\times\frac{\cos x\sin^{2020}x}{\sin^{2021}x+\cos^{2021}x}dx$

$$=\int\frac{\sin^{2020}x\cos x-\cos^{2020}x\sin x}{\sin^{2021}x+\cos^{2021}x}dx$$

$$=\frac{1}{2021}\log\left|(\sin x)^{2021}+(\cos x)^{2021}\right|+C$$

$$f(x)=\sin x,g(x)=\cos x,n=2021$$

$$n\left[\left(f(x)\right)^4+\left(g(x)\right)^4\right]_{x=\frac{\pi}{3}}=2021\left(\frac{9}{16}+\frac{1}{16}\right)$$

$$=\frac{10105}{8}$$

40. $\int\frac{(x+1)\left(x^2-1\right)}{(x+1)\left(x^4+3x^2+1\right)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$

$$\tan^{-1}\left(\frac{x^2+1}{x}\right)=t$$

$$\left(\frac{x^2-1}{x^4+3x^2+1}\right)dx=dt$$

$$=\int\frac{1}{t}dt=\log\left|\tan^{-1}\left(\frac{x^2+1}{x}\right)\right|$$

A=1 $\quad f(x)=\tan^{-1}\left(\frac{x^2+1}{x}\right)$

$$f(2)=\tan^{-1}\left(\frac{5}{2}\right)$$

$$A-\mathrm{Tan}\left(f(2)\right)=1-\frac{5}{2}=\frac{-3}{2}$$

41. $I_n=\int x^n\sin x\,dx$

$$I_n=-x^n\cos x+nx^{n-1}\sin x-n(n-1)I_{n-2}$$

$$I_6=-x^6\cos x+6x^5\sin x-30I_4$$

$$=-x^6\cos x+6x^5\sin x-30\left[-x^4\cos x+4x^3\sin x-12I_2\right]$$

$$=\left(30x^4-x^6\right)\cos x+\left(6x^5-120x^3\right)\sin x+360I_2$$

$$f(x)=30x^4-x^6, g(x)=6x^5-120x^3$$

$$f(1)+g(1)=29-114=-85$$

42. $$\int\frac{x^2-1}{(x+1)^2x\sqrt{x+1+\frac{1}{x}}}dx=\int\frac{x^2-1}{(x^2+2x+1)x\sqrt{x+1+\frac{1}{x}}}dx$$

$$x+1+\frac{1}{x}=t^2\Rightarrow\left(1-\frac{1}{x^2}\right)dx=2tdt$$

$$=\int\frac{2tdt}{(t^2+1)t}=2\,\mathrm{Tan}^{-1}t+C$$

$$=2\,\mathrm{Tan}^{-1}\sqrt{x+1+\frac{1}{x}}+C$$

$$f(x)=2\,\mathrm{Tan}^{-1}\sqrt{x+1+\frac{1}{x}}$$

$$f(1)=2\,\mathrm{Tan}^{-1}\sqrt{3}=2\frac{\pi}{3}=\frac{2\pi}{3}$$

43. $\sin x=t\Rightarrow\cos x dx=dt$

$$\int\frac{dt}{\sqrt{4t^2+4t+5}}=\frac{1}{2}\int\frac{dt}{\sqrt{t^2+t+\frac{5}{4}}}$$

$$=\frac{1}{2}\mathrm{Sinh}^{-1}(\sin x+\frac{1}{2})+C$$

$$2f(x)=2\left(\sin x+\frac{1}{2}\right)=2\sin x+1$$

44. $$\int\frac{a\cos x+b\sin x}{c\cos x+d\sin x}dx=\frac{ac+bd}{c^2+d^2}x+\frac{ad-bc}{c^2+d^2}\log|f(x)|+C$$

$$\int\frac{a\cos x-2\sin x}{5\cos x+b\sin x}dx=\frac{7}{41}x+\frac{22}{41}\log|5\cos x+b\sin x|+C$$

$a=3,\ b=4$

$$\int\frac{dx}{b+a\cos x}=\int\frac{dx}{4+3\cos x}=\frac{2}{\sqrt{7}}\mathrm{Tan}^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{7}}\right)+C$$

$$\left(\begin{array}{l}let\quad\tan\frac{x}{2}=t\\ dx=\frac{2dt}{1+t^2},\ \cos x=\frac{1-t^2}{1+t^2}\end{array}\right)$$

45. Conceptual (I.P.E model)

$$px+q=A\frac{d}{dx}(ax^2+bx+c)+B$$

46. Let

$$y^2=t^6$$

$$2ydy=6t^5dt$$

$$dy=3t^2dt$$

$$\int\frac{\left(t^6+t^4+t\right)}{t^3\left(1+t^2\right)}3t^2dt$$

$$=3\int\frac{\left(t^5+t^3+1\right)}{\left(1+t^2\right)}dt$$

$$=3\int\left(t^3+\frac{1}{\left(1+t^2\right)}\right)dt$$

$$=3\left[\frac{t^4}{4}+\tan^{-1}(t)\right]+c$$

$$=\frac{3}{4}y^{\frac{4}{3}}+\tan^{-1}\left(y^{\frac{1}{3}}\right)+c$$

47. Conceptual

48. Conceptual

49. $$\int\frac{2x^{12}+5x^9}{x^{15}\left(1+x^{-2}+x^{-5}\right)^3}dx$$

$put\ 1+x^{-2}+x^{-5}=t$

$$\left(2x^{-3}+5x^{-6}\right)dx=-dt$$

$$\int\frac{2x^{-3}+5x^{-6}}{\left(1+x^{-2}+x^{-5}\right)^3}dx$$

$$=\int\frac{-dt}{t^3}=\frac{1}{2t^2}+c$$

$$=\frac{1}{2\left(1+x^{-2}+x^{-5}\right)^2}+c$$

$$=\frac{x^{10}}{2\left(1+x^3+x^5\right)^2}+c$$

Here ,

$m=10, l=2, r=2$

$$\frac{m-l}{r}=\frac{10-2}{2}=4$$

50. $$\int\left(\frac{\sqrt{\sin^2x+\cos^2x+2\sin x\cos x}+}{\sqrt{\sin^2x+\cos^2x-2\sin x\cos x}}\right)dx$$

$$=2\int\sin xdx$$

$$=2\cos x+c$$

51. $x^2\int\left[\frac{x\sec^2 x+\tan x}{(x\tan x+1)^2}\right]dx-$

$\int\left[2x\int\frac{x\sec^2 x+\tan x}{(x\tan x+1)^2}dx\right]dx$

$=x^2\left(\frac{-1}{x\tan x+1}\right)-2\int\frac{-x}{x\tan x+1}dx$

$=\frac{-x^2}{x\tan x+1}+2\int\frac{x\cos x+\sin x-\sin x}{x\sin x+\cos x}dx$

$=\frac{-x^2}{x\tan x+1}+2\log|x\sin x+\cos x|+c$

$A=2, B=-1, f(x)=x^2$

A+B=1

$f(A+B)=1$ $\left[\begin{array}{l}\because x\tan x+1=t\\ \int\left[\frac{x\sec^2 x+\tan x}{(x\tan x+1)^2}\right]dx=\int\frac{dt}{t^2}=\frac{-1}{t}+c\end{array}\right]$

52. $\frac{x^4}{4}(\log x)^2-\int\left(\frac{2\log x}{x}.\frac{x^4}{4}\right)dx$

$=\frac{x^4}{4}(\log x)^2-\frac{1}{2}\left[\log x.\frac{x^4}{4}-\int\frac{1}{x}.\frac{x^4}{4}dx\right]$

$=x^4\left[\frac{1}{4}(\log x)^2-\frac{1}{8}\log x+\frac{1}{32}\right]+c$

$A=\frac{1}{4}, B=\frac{-1}{8}, C=\frac{1}{32}$

53. $\frac{9x+15}{(x-3)(x^2+3x+3)}=\frac{A}{x-3}+\frac{Bx+C}{x^2+3x+3}$

After simplification we get

$A=2, B=-2, C=-3$

$\int\frac{9x+15}{(x-3)(x^2+3x+3)}dx$

$=\int\frac{2}{x-3}dx+\int\frac{-2x-3}{x^2+3x+3}dx$

$=2\log|x-3|-\log|x^2+3x+3|+C$

Here

$A=2, g(x)=x-3$

$B=-1, f(x)=x^2+3x+3$

$\frac{(A-B)g(4)}{f(-1)}=3$

54. $\int e^{\sin^2 x}\sin x\cos x\left(1+\cos^2 x\right)dx$

Put $\sin^2 x=t$

$2\sin x\cos dx=dt$

$\frac{1}{2}\int e^t(2-t)dt$

$=\frac{1}{2}e^t(2-t-1)+c$

$=\frac{1}{2}e^{\sin^2 x}\left(1-\sin^2 x\right)+c$

$=e^{\sin^2 x}\left(1-\frac{1}{2}-\frac{\sin^2 x}{2}\right)+c$

$\therefore f(x)=\frac{-1}{2}\left(1+\sin^2 x\right)$

$f^1(x)=\frac{-1}{2}(2\sin x\cos x)=\frac{-1}{2}\sin 2x$

55. $\int\frac{(25x^2+8)}{\sqrt{25x^2+9}}dx=\int\frac{(25x^2+9-1)}{\sqrt{25x^2+9}}dx$

$=\int\sqrt{(5x)^2+3^2}dx-\int\frac{1}{\sqrt{(5x)^2+3^2}}dx$

$=\frac{5x}{2(5)}\sqrt{25x^2+9}+\frac{9}{2(5)}\sinh^{-1}\left(\frac{5x}{3}\right)-\frac{1}{5}\sinh^{-1}\left(\frac{5x}{3}\right)+c$

$=\frac{x}{2}\sqrt{25x^2+9}+\frac{7}{10}\sinh^{-1}\left(\frac{5x}{3}\right)+c$

56. $I_{m,n}=\int x^m(\log x)^n dx$

$=(\log x)^n.\frac{x^{m+1}}{m+1}-\int\frac{x^{m+1}}{m+1}.n(\log x)^{n-1}\frac{1}{x}dx$

$=\frac{x^{m+1}}{m+1}(\log x)^n-\frac{n}{m+1}I_{m,n-1}$

57. $\int\frac{x^2}{\left(\sqrt{4-x^2}\right)^3}dx$

$x=2\sin\theta,\ dx=2\cos\theta d\theta$

Put $=\int\frac{4\sin^2\theta}{(4-4\sin^2\theta)^{\frac{3}{2}}}2\cos\theta d\theta=\int\frac{\sin^2\theta}{\cos\theta}d\theta$

$=\int\tan^2\theta d\theta=\tan\theta-\theta$

$=\frac{x}{\sqrt{4-x^2}}-\sin^{-1}\frac{x}{2}=\frac{x}{\sqrt{4-x^2}}-\tan^{-1}\left(\frac{x}{\sqrt{4-x^2}}\right)$

58. Put

$$\log x = t \Rightarrow \frac{1}{x}dx = dt$$

$$\int \frac{dt}{t.t^2.t^3\ldots t^m} = \int t^{\frac{-m(m+1)}{2}}\,dt$$

$$= \frac{t^{\frac{-m(m+1)}{2}+1}}{\frac{-m(m+1)}{2}+1}$$

$$2K = -m(m+1)+2 \Rightarrow 2k = -m^2 - m + 2$$

$$= (m+2)(1-m)$$

59. $I_m = \int x^m.\cos(nx)\,dx$

$$u = x^m \qquad dv = \cos(nx)\,dx$$

$$du = m.x^{m-1}dx \qquad v = \frac{\sin(nx)}{n}$$

$$I_m = \frac{x^m.\sin(nx)}{n} - \frac{m}{n}\int x^{m-1}.\sin(nx)\,dx$$

$$I_m = \frac{x^m.\sin(nx)}{n} - \frac{m}{n}\left\{\frac{-x^{m-1}\cos nx}{n} + \int \frac{\cos nx}{n}(m-1)x^{m-2}dx\right.$$

$$I_m = \frac{x^m.\sin(nx)}{n} + \frac{m}{n^2}x^{m-1}(\cos nx) - \frac{m(m-1)}{n^2}I_{m-2}$$

60. $I_n = \int \sec^n x\,dx$

$$(n-1)I_n - (n-2)I_{n-2} = \sec^{n-2} x.\tan x$$

$$put \;\; n = 6$$

$$5.I_6 - 4.I_4 = \sec^4 x.\tan x = f(x)$$

$$f\left(\frac{\pi}{4}\right) = \sec^4\frac{\pi}{4}.\tan\frac{\pi}{4} = 2^2 = 4$$

61. $\int \frac{dx}{x\sqrt{x^4-1}}$

$$x^2 = t \Rightarrow \frac{dx}{dt} = \frac{dt}{2t}$$

$$= \int \frac{dt}{2t\sqrt{t^2-1}} = \frac{1}{2}\sec^{-1}(t) + C$$

$$= \frac{1}{2}\sec^{-1}(x^2) + C$$

62. $\int e^x\left[\frac{x+5-2}{(x+5)^3}\right]dx$

$$= \int e^x\left[\frac{1}{(x+5)^2} - \frac{2}{(x+5)^3}\right]dx$$

$$= \frac{e^x}{(x+5)^2} + C$$

63. $\int \frac{(x-1)^2}{(x^2+1)^2}dx = Tan^{-1}x + g(x) + k$

$$= \int\left[\frac{1}{x^2+1} - \frac{2x}{(x^2+1)^2}\right]dx$$

$$= Tan^{-1}x + \frac{1}{1+x^2} + k$$

64. $\int \frac{1-\cot^{2021} x}{Tanx + \cot^{2022} x}dx$

$$= \int \frac{(\sin^{2022} x\cos x - \cos^{2022} x\sin x)}{(\sin^{2023} x + \cos^{2023} x)} \times \frac{1}{2023}dx$$

$$= \frac{1}{2023}\log|\sin^{2023} x + \cos^{2023} x| + C$$

65. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}dx$

$$= \int \frac{\sqrt{1+x}\left(\sqrt{1+x}+\sqrt{x}\right)}{\sqrt{x}+\sqrt{1+x}}dx$$

$$= \frac{2}{3}(1+x)^{3/2} + C$$

66. $\int \cos x \log\left(\cot\frac{x}{2}\right)dx$

Using by parts

$$= \sin x\log\left(\cot\frac{x}{2}\right) - \int \frac{\sin x.\left(-\operatorname{cosec}^2\frac{x}{2}\right)}{\cot\frac{x}{2}}\frac{1}{2}dx$$

$$= \sin x\log\left(\cot\frac{x}{2}\right) + x + C$$

67. $\int \sqrt{e^{4x}+e^{2x}}\,dx = \int e^x\sqrt{e^{2x}+1}\,dx$

Put $e^x = t$

$$= \int \sqrt{t^2+1}\,dt$$

$$= \frac{t}{2}\sqrt{t^2+1} + \frac{1}{2}\sinh^{-1}(t) + C$$

$$= \frac{e^x}{2}\sqrt{e^{2x}+1} + \frac{1}{2}\sinh^{-1}(e^x) + C$$

68. $\int \frac{dx}{1+\sin x} = Tan\left(f\left(x\right)\right)+C$

$= \int \left(\sec^2 x - \sec x Tanx\right) dx$

$= Tanx - \sec x + C \Rightarrow Tan\left(\frac{x}{2}-\frac{\pi}{4}\right)+C$

$f\left(x\right) = \frac{x}{2}-\frac{\pi}{4}, f'(0) = \frac{1}{2}$

69.

$\int \frac{\sin\alpha}{\sqrt{1+\cos\alpha}} d\alpha = \int \frac{2\sin\frac{\alpha}{2}}{\sqrt{2}} d\alpha$

$= -2\sqrt{2}\cos\frac{\alpha}{2} + C$

70. $\int \frac{\cos 4x + 1}{\cot x - Tanx} dx = k\cos 4x + C$

$= \int \frac{2\cos^2 2x \cos x \sin x}{\cos 2x} dx$

$= \frac{1}{2}\int \sin 4x dx = \frac{-1}{8}\cos 4x + C$

71. $\int \cos x\left(-\cos ecx \cot x\right) dx = f\left(x\right) + g\left(x\right) + C$

$\Rightarrow \int \frac{-\cos^2 x}{\sin^2 x} dx$

$= \int \left(1 - \cos ec^2 x\right) dx = x + \cot x + C$

$f\left(x\right).g\left(x\right) = x\cot x$

72. $\int \left(\frac{2x+1}{3x+2}\right)^6 \frac{1}{\left(3x+2\right)^2} dx$

Put $\frac{2x+1}{3x+2} = t \Rightarrow \frac{1}{\left(3x+2\right)^2} dx = dt$

$= \int t^6\, dt = \frac{1}{7}\left(\frac{2x+1}{3x+2}\right)^7 + C \Rightarrow \frac{P}{Q} = \frac{1}{7^2}$

73. $f'\left(x\right) = x + \frac{1}{x}$

Integrate on both sides

$f\left(x\right) = \frac{x^2}{2} + \log x + C$

74. $\int \left(x+1\right)\left(x+2\right)^4\left(x+3\right) dx$

$= \int \left(x+2\right)^4\left[\left(x+2\right)^2 - 1\right] dx$

$= \frac{\left(x+2\right)^7}{7} - \frac{\left(x+2\right)^5}{5} + C$

75. $\int \frac{\sqrt{1-x^4}}{x^7} dx = f\left(x\right)\left(\sqrt{1-x^4}\right)^n + C$

$= \int \frac{\sqrt{x^{-4}-1}}{x^5} dx$

Put $x^{-4} - 1 = t^2 \Rightarrow \frac{dx}{x^5} = \frac{-t}{2} dt$

$= \int \frac{-t^2}{2} dt = \frac{-t^3}{6} + C$

$= \frac{-1}{6}\frac{\left(\sqrt{1-x^4}\right)^3}{x^6} + C$

$f\left(x\right) = \frac{-1}{6x^6} \Rightarrow \left(f\left(x\right)\right)^3 = \frac{-1}{216x^{18}}$

76. $\int \frac{dx}{\sin^4 x + \cos^4 x} = \frac{1}{\sqrt{2}} Tan^{-1}\left(g\left(x\right)\right) + C$

$= \int \frac{\sec^2 x \sec^2 x}{1 + Tan^4 x} dx$

Put $Tanx = t$

$= \int \frac{1+t^2}{1+t^4} dt = \int \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2 + \left(\sqrt{2}\right)^2} dt$

$= \frac{1}{\sqrt{2}} Tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right) + C$

$= \frac{1}{\sqrt{2}} Tan^{-1}\left(\frac{Tanx - \cot x}{\sqrt{2}}\right) + C$

77. $\int \left(1 - \cos x\right)\cos ec^2 x\, dx$

$= \int \left(\cos ec^2 x - \cos ecx \cot x\right) dx$

$= -\cot x + \cos ecx + C$

$= \frac{1-\cos x}{\sin x} + C = Tan\frac{x}{2} + C$

78. $\int \cos^k x \sin x\, dx = \frac{-1}{4}\cos^4 x + C$

Put $\cos x = t$

$= \frac{-1}{k+1}\cos^{k+1} x + C \Rightarrow k = 3$

79. $f'\left(x\right) = \sec^2 x + \cos ec^2 x - 2$

Integrate on both sides

$f(x) = Tanx - \cot x - 2x + C$

$f\left(\frac{\pi}{4}\right) = 0, C = \frac{\pi}{2}$

$f(x) = Tanx - \cot x - 2x + \frac{\pi}{2}$

80. $\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}} dx$

$= (-1)\int \frac{\cos x - \sin x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx$

Put $\sin x + \cos x = t$

$= -\int \frac{dt}{\sqrt{t^2 - 1}} = -\log|\sqrt{t^2-1} + t| + C$

$= -\log|\sqrt{\sin 2x} + \sin x + \cos x| + C$

81. $\int \frac{(x-1)(x^2+x+1) - x(x-1)}{x-1} dx$

$= \frac{x^3}{3} + x + C$

82. $\int \frac{e^{Tan^{-1}x}}{1+x^2}\left[\sec^{-1}\sqrt{1+x^2} + \cos^{-1}\frac{1-x^2}{1+x^2}\right] dx$

Put $x = Tan\theta$

$= \int \frac{e^\theta}{\sec^2\theta}(\theta + 2\theta)\sec^2\theta d\theta = 3\int e^\theta \theta d\theta$

$= 3e^\theta(\theta - 1) + C$

$= 3e^{Tan^{-1}x}(Tan^{-1}x - 1) + C$

83. $\int x^{1/3}(1+x^{4/3})^{1/7} dx = A(1+x^{4/3})^B + C$

Put $1 + x^{4/3} = t \Rightarrow \frac{4}{3}x^{1/3}dx = dt$

$= \frac{3}{4}\int t^{1/7} dt = \frac{3}{4}.\frac{7}{8}(1+x^{4/3})^{8/7} + C$

$AB = \left(\frac{3}{4}\frac{7}{8}\right)\left(\frac{8}{7}\right) = \frac{3}{4}$

84. $\int \frac{\sqrt{2}\sin x}{\sin\left(x + \frac{\pi}{4}\right)} dx = \sqrt{2}\int \frac{\sin\left(x + \frac{\pi}{4} - \frac{\pi}{4}\right)}{\sin\left(x + \frac{\pi}{4}\right)} dx$

$= \sqrt{2}\int\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\frac{\cos\left(x + \frac{\pi}{4}\right)}{\sin\left(x + \frac{\pi}{4}\right)}\right] dx$

$= x - \log\left|\sin\left(x + \frac{\pi}{4}\right)\right| + C$

85. $\int 3^x(f'(x) + f(x)\log 3) dx$

$= \int 3^x f'(x) dx + \int 3^x f(x)\log 3 dx$

By parts

$= f(x)3^x - \int 3^x f(x)\log 3 dx + \int 3^x f(x)\log 3 dx + C$

$= f(x)3^x + C$

86. $\int \frac{x + \sin x}{1 + \cos x} dx$

$= \int\left(\frac{x}{2}\right)\sec^2\frac{x}{2} dx + \int Tan\frac{x}{2} dx$

By parts

$= \left(\frac{x}{2}\right)2Tan(x/2) - \int Tan\frac{x}{2}dx + \int Tan\frac{x}{2}dx + C$

$= xTan\left(\frac{x}{2}\right) + C$

87. $\int \frac{3\cos x - 2\sin x}{4\sin x + 5\cos x} dx$

$= A\log|5\cos x + 4\sin x| + Bx + C$

Formula:

$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x} dx$

$= \frac{ac+bd}{c^2+d^2}x + \frac{ad-bc}{c^2+d^2}\log|c\cos x + d\sin x| + C$

$I = \frac{7}{41}x + \frac{22}{41}\log|4\sin x + 5\cos x| + C$

88. $I = \int \frac{x-1}{(x-2)(x-3)} dx$

$\frac{x-1}{(x-2)(x-3)}$ Resolve into partial fractions.

$I = \int\left(\frac{-1}{x-2} + \frac{2}{x-3}\right) dx$

$= -\log(x-2) + 2\log(x-3) + C$

$= \log\left|\frac{(x-3)^2}{x-2}\right| + C$

89. $\int \frac{dx}{\sin x\cos x}$

$= \int \frac{\sec^2 x}{Tanx} dx = \log|Tanx| + C$

90. $\int \frac{5Tanx}{Tanx-2}dx = \int \frac{5\sin x}{\sin x - 2\cos x}dx$

$a=0, b=5, c=-2, d=1$

$= \frac{ac+bd}{c^2+d^2}x + \frac{ad-bc}{c^2+d^2}\log|c\cos x + d\sin x| + C$

$I = \frac{5}{5} + \frac{10}{5}\log|\sin x - 2\cos x| + C$

$= 1 + 2\log|\sin x - 2\cos x| + C$

91. $\int \frac{2x+3}{x(x+1)(x+2)(x+3)+1}dx = \frac{-1}{ax^2+bx+c} + \alpha$

$= \int \frac{2x+3}{\left(x^2+3x+1\right)^2}dx$

$= \frac{-1}{x^2+3x+1} + \alpha$

$\Rightarrow a=1, b=3, c=1 \Rightarrow a+b+c=5$

92. $\int \frac{2\cos x + 3\sin x}{4\cos x + 5\sin x}dx = \frac{23}{41}x + k\log|4\cos x + 5\sin x| + C$

Formula:

$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}dx$

$= \frac{ac+bd}{c^2+d^2}x + \frac{ad-bc}{c^2+d^2}\log|c\cos x + d\sin x| + C$

$k = \frac{2(5)-3(4)}{4^2+5^2} = \frac{-2}{41}$

93. $\int \cos\sqrt{x}\,dx$, put $\sqrt{x} = t$

$= 2\int t\cos t\,dt = 2(t\sin t + \cos t) + C$

$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$

94. $\int \frac{x^2+1}{x^4+1}dx = f(x) + C$

$\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2 + \left(\sqrt{2}\right)^2}dx$

$= \frac{1}{\sqrt{2}}\text{Tan}^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) + C$

$= \frac{1}{\sqrt{2}}\text{Tan}^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + C$

95. $\int \frac{1+\cos 4x}{\cot x - \tan x}dx = A\cos 4x + B$

96. $\int \frac{dx}{\sqrt{7-6x-x^2}}$

$= \int \frac{dx}{\sqrt{16-(x+3)^2}} = \sin^{-1}\left(\frac{x+3}{4}\right) + C$

97. $f(x) = \frac{3-8x}{3x-1} \Rightarrow f(y) = \frac{3-8y}{3y-1}$

Now $\int \frac{3-8y}{3y-1}dy = 3\log(3y-1) - 8\int \frac{y}{3y-1}dy$

$= 3\log(3y-1) - \frac{8}{3}\left[y + \frac{1}{3}\log(3y-1)\right] + C$

$= \log\left[\frac{(3y-1)^3}{(3y-1)^{8/9}}\right] - \frac{8y}{3} + C$

$= -\frac{8}{3}y + \frac{19}{9}\log(3y-1) + C$

Compare with $Ay + B\log(3y-1) + C$

$A = \frac{-8}{3}, B = \frac{19}{9}$

Now, $\frac{A-3B}{2} = \frac{-9}{2}$

98. Given $\int \sqrt{\sin x}\cos x\,dx = \frac{2}{3}(\sin x)^{3/2} + C$

Valid $\sqrt{\sin x} \geq 0, \sin x \geq 0$

$(2n\pi, (2n+1)\pi)\ \forall n \in N$

99. $\int\left(\frac{\sqrt{1+x+x^2}}{1+x} + \frac{1}{2\sqrt{1+x+x^2}} - \frac{1}{(1+x)\sqrt{1+x+x^2}}\right)dx$

$= \int \frac{2x(1+x)+(1+x)}{2(1+x)\sqrt{1+x+x^2}}dx$

$= \frac{1}{2}\int \frac{2x+1}{\sqrt{1+x+x^2}}dx$

$= \frac{1}{2}\frac{\left(1+x+x^2\right)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} = \sqrt{1+x+x^2} + C$

100. $\int \frac{dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}}\int \frac{1}{\sin\left(\frac{\pi}{4}+x\right)}dx$

$= \frac{1}{\sqrt{2}}\int \text{cosec}\left(\frac{\pi}{4}+x\right)dx$

$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{8}+\frac{x}{2}\right)\right| + C$

101. $\int (x+2)\sqrt{x+3}\,dx$

$x+3=t^2 \Rightarrow x+2=t^2-1$

$dx=2tdt$

$=\int (t^2-1)t\,2tdt$

$=\int 2(t^4-t^2)dt$

$=2\left(\frac{t^5}{5}-\frac{t^3}{3}\right)+C$

$=\frac{2}{15}\sqrt{x+3}\left[3x^2+13x+12\right]+C$

102. $\int \frac{(1-\cos x)^{2/7}}{(1+\cos x)^{9/7}}dx$

$=\int \frac{(1-\cos x)^{2/7}}{(1+\cos x)^{2/7}\cdot(1+\cos x)}dx$

$=\frac{1}{2}\int \left(\tan^2\frac{x}{2}\right)^{2/7}\sec^2\frac{x}{2}dx$

$\tan\frac{x}{2}=t$

$=\int (t^2)^{2/7}dt=\int (t)^{4/7}dt$

$=\frac{7}{11}t^{11/7}+C=\frac{7}{11}\left(\tan\frac{x}{2}\right)^{11/7}+C$

103.

$\int \frac{xe^{\frac{x^2}{x^2-2}}}{x^4-4x^2+4}dx=\int \frac{x.e^{\frac{x^2}{x^2-2}}}{(x^2-2)^2}...(1)$

Let

$\frac{x^2}{x^2-2}=t \Rightarrow \frac{-4x}{(x^2-2)^2}dx=dt$

From (1)

$=\frac{-1}{4}\int e^t dt=-\frac{e^t}{4}+C=\frac{-1}{4}e^{\frac{x^2}{x^2-2}}+C$

104. $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx=\int \frac{2x^{12}+5x^9}{x^{15}(x^{-5}+x^{-2}+1)^3}dx$

$=\int \frac{2x^{-3}+5x^{-6}}{(x^{-5}+x^{-2}+1)^3}dx......(1)$

$x^{-5}+x^{-2}+1=t \Rightarrow -\left[5x^{-6}+2x^{-3}\right]dx=dt$

From (1)

$=-\int \frac{dt}{t^3}=-\left(\frac{t^{-2}}{-2}\right)+C$

$=\frac{1}{2}(x^{-5}+x^{-2}+1)^{-2}+C$

$\therefore f(x)=(x^{-5}+x^{-2}+1)^{-2}$

$=\frac{1}{(x^{-5}+x^{-2}+1)^2}=\frac{x^{10}}{(x^5+x^3+1)^2}$

$f(1)=3^{-2}=\frac{1}{9}, f(0)=0$

$\Rightarrow f(1)-f(0)=\frac{1}{9}$

105. $\int \frac{\sqrt{\cot x}}{\sin 2x}dx=\int \frac{\sqrt{\cot x}}{2\sin x.\cos x}dx$

$=\frac{1}{2}\int \frac{\sqrt{\cot x}}{\sin^2 x.\cot x}dx=-\frac{1}{2}\int \frac{d(\cot x)}{\sqrt{\cot x}}dx$

$=-\frac{1}{2}\left(2\sqrt{\cot x}\right)+C=-\sqrt{\cot x}+C$

106. Let $\frac{x-2}{x-1}=t$

Diff w.r.to t

$\frac{1}{(x-1)^2}dx=dt \quad \left[x-1=\frac{-1}{t-1}\right]$

$=\int \frac{dt}{t^5(x-1)^7}$

$=\int \frac{(1-t)^7}{t^5}dt$

$\int \frac{\left(1-{}^7C_1t+{}^7C_2t^2-{}^7C_3t^3+{}^7C_4t^4-{}^7C_5t^5+{}^7C_6t^6-{}^7C_7t^7\right)}{t^5}dt$

$=\int \frac{1}{t^5}dt-{}^7C_1\int \frac{1}{t^4}dt+{}^7C_2\int \frac{1}{t^3}dt-{}^7C_3\int \frac{1}{t^2}dt+{}^7C_4\int \frac{1}{t}dt$
$-{}^7C_5\int dt+{}^7C_6\int tdt-{}^7C_7\int t^2dt$

$={}^7C_4\log t={}^7C_3\log\left[\frac{x-2}{x-1}\right]$

$=35\log\left[\frac{x-2}{x-1}\right]=Bf(x)$

$f(x)=\log(x-2)-\log(x-1)$

107. $f(x)=\int \frac{x^3}{(1+x^2)^3}$

$x=\tan\theta$

$dx=\sec^2\theta d\theta$

$=\int \frac{\tan^3\theta}{(1+\tan^2\theta)^3}\times\sec^2\theta d\theta$

$=\int \frac{\tan^3\theta}{\sec^6\theta}\times\sec^2\theta d\theta$

$=\int \frac{\tan^3\theta}{\sec^4\theta}d\theta$

$=\int \frac{\sin^3\theta}{\cos^3\theta}\times\cos^4\theta d\theta$

$=\int \sin^3\theta.\cos\theta\,d\theta$

$=\frac{\sin^4\theta}{4}+K$

$$=\frac{x^4}{4(1+x^2)^2}+K$$

$$g(x)+C=\int\frac{x^2.x}{(1+x^2)^3}dx$$

$$x^2+1=z\Rightarrow 2xdx=dz$$

$$=\frac{1}{2}\int\frac{(z-1)}{z^3}dz$$

$$=\frac{1}{2}\int\left(\frac{1}{z^2}-\frac{1}{z^3}\right)dz$$

$$=\frac{1}{2}\left[\frac{-1}{1+x^2}+\frac{1}{2(1+x^2)^2}\right]+C$$

$$=\frac{-1-2x^2}{4(1+x^2)^2}+C$$

Then $f(x)-g(x)+k-C$

$$=\frac{x^4}{4(1+x^2)^2}+\frac{2x^2+1}{4(1+x^2)^2}+K-C$$

$$=\frac{(x^2+1)^2}{4(1+x^2)^2}+K-C=\frac{1}{4}+K-C$$

any constant

108. $\frac{1}{2}\int\left(\sqrt{1+x}-\sqrt{1-x}\right)dx+\frac{1}{2}\int y\left(\sqrt{y+1}-\sqrt{y-1}\right)dy$

$$=\frac{1}{3}(1+x)^{3/2}+\frac{1}{3}(1-x)^{3/2}+\frac{1}{2}\int y\sqrt{y+1}dy-\frac{1}{2}\int y\sqrt{y-1}dy$$

$y+1=t^2$ $\quad$ $y-1=z^2$

$dy=2tdt$ $\quad$ $dy=2zdz$

$$\frac{1}{3}(1+x)^{3/2}+\frac{1}{3}(1-x)^{3/2}+\frac{t^5}{5}-\frac{t^3}{3}-\frac{z^5}{5}+\frac{z^3}{3}+C$$

$$\frac{1}{3}(1+x)^{3/2}+\frac{1}{3}(1-x)^{3/2}$$
$$+(y+1)^{3/2}\left[\frac{3y+3-5}{15}\right]-(y-1)^{3/2}\left[\frac{3y-3+5}{15}\right]+C$$

$$\frac{1}{3}(1+x)^{3/2}+\frac{1}{3}(1-x)^{3/2}$$
$$+(y+1)^{3/2}\left[\frac{3y-2}{15}\right]-(y-1)^{3/2}\left[\frac{3y+2}{15}\right]+C$$

$$A=1/3,B=1/3,f(y)=\frac{3y-2}{15},g(y)=-\left(\frac{3y+2}{15}\right)$$

Then

$$Af(y)+Bg(y)=\frac{1}{3}\left[\frac{3y-2}{15}\right]-\frac{1}{3}\left[\frac{3y+2}{15}\right]=\frac{-4}{45}$$

109. $\int\sqrt{x^2+x+1}\,dx\times\int\frac{1}{\sqrt{x^2+x+1}}dx$

$$\int\sqrt{x^2+2\left(\frac{1}{2}\right)x+\frac{1}{4}-\frac{1}{4}+1}\,dx\times\int\frac{dx}{\sqrt{x^2+2\left(\frac{1}{2}\right)x+\frac{1}{4}-\frac{1}{4}+1}}$$

$$\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx\times\int\frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}}$$

$$\left(\frac{2x+1}{4}\sqrt{x^2+x+1}+\frac{3}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

110. $\int\frac{dx}{\sqrt{(5+2x+x^2)^3}}$,

$$x+1=2\operatorname{Tan}\theta\Rightarrow dx=2\sec^2\theta$$

$$\int\frac{dx}{\sqrt{(x^2+2x+1-1+5)^3}}$$

$$\int\frac{dx}{\sqrt{((x+1)^2+4)^3}}$$

$$\int\frac{2\sec^2\theta d\theta}{\sqrt{((2\tan\theta)^2+4)^3}}=\frac{1}{4}\int\cos\theta\,d\theta$$

$$=\frac{1}{4}\sin\theta=\frac{1}{4}\frac{x+1}{\sqrt{x^2+2x+5}}$$

111. $\int\frac{x^2}{1+x^6}dx=\int\frac{x^2}{1+(x^3)^2}.dx$

Take $x^3=t\Rightarrow x^2dx=\frac{1}{3}dt$

$$=\frac{1}{3}\int\frac{1}{1+t^2}dt=\frac{1}{3}\text{Tan}^{-1}\left(x^3\right)+C$$

112. Put $x=a\sin\theta\Rightarrow dx=a.\cos\theta.d\theta$

$$\int\frac{1}{\left(a^2\sin^2\theta-a^2\right)^{3/2}}.a\cos\theta d\theta$$

$$=\int\frac{a\cos\theta}{-a^3.\cos^3\theta}d\theta=\frac{-1}{a^2}\int\sec^2\theta d\theta$$

$$=\frac{-1}{a^2}\text{Tan}\,\theta+c\quad\left[\because\tan\theta=\frac{x}{\sqrt{x^2-a^2}}\right]$$

$$=\frac{-1}{a^2}.\frac{x}{\sqrt{x^2-a^2}}+c$$

113. $\int\sqrt{x^2+2x}\,dx=\int\sqrt{(x+1)^2-(1)^2}\,dx$

$$=\frac{(x+1)}{2}\sqrt{x^2+2x}-\frac{1}{2}\cosh^{-1}(x+1)+c$$

$$\left[\because\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right)+c\right]$$

114 In= $\int\cot^{n-2}x\left(\cot^2x\right)dx$

$$I_n+I_{n-2}=\frac{-\cot^{n-1}x}{n-1}$$

115. In = $\int\text{Tan}_x^n\,dx$

$$=\int\text{Tan}^{n-2}x\left(\sec^2x-1\right)dx$$

$$I_n+I_{n-2}=\frac{\text{Tan}^{n-1}x}{n-1}$$

Put n=2,3,4,5,6 and adding

$I_0+2\left(I_2+I_3+I_4\right)+I_5+I_6$

$$=\sum_{k=1}^{5}\frac{\tan^kx}{k}$$

k = 5

116. $\int\frac{e^{\cot x}}{\sin^2x}\left[2\log\left(\cos ecx\right)+\sin2x\right]dx$

Put cotx = t

$$\Rightarrow-\int e^t\left[\log\left(1+t^2\right)+\frac{2t}{1+t^2}\right]dt$$

$$=-e^t\log\left(1+t^2\right)$$

$$=-2e^{\cot x}\log(\cos ecx)+c$$

117. Conceptual

118. Conceptual

119. $\int\frac{e^{\sqrt{x}}}{\sqrt{x}}\left(x+\sqrt{x}\right)ctx$

Put $\sqrt{x}=t$

$$\int e^t\left(t^2+t\right)2dt$$

$$=e^t\left(t^2-t+2\right)+k$$

$$=e^{\sqrt{x}}\left(x-\sqrt{x}+2\right)+k$$

A=1 B=–1 C=2

120. $\int\frac{1+\sqrt{\tan x}}{\sin2x}dx$

Put tanx=t

$$=\frac{1}{2}\int\left(\frac{l+\sqrt{t}}{t}\right)dt$$

$$=\frac{1}{2}\log t+t^{1/2}+c$$

$$=\frac{1}{2}\log\left(\tan x\right)+\sqrt{\tan x}+C$$

$A=1/2,\ B=1$

121. $\int\frac{\cos a}{\sin x\ sin(x+a)}dx$

$$=\cot a\int\frac{\sin(a+x-x)}{\sin x\sin\left(x+a\right)}dx$$

$$=\cot a(\log\left|\sin x\right|-\log\left|\sin\left(x+a\right)\right|+c$$

122. Conceptual

123. $$\int \frac{e^{\tan^{-1}x}}{1+x^2}\left[\left(\sec^{-1}\sqrt{1+x^2}\right)^2+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]dx$$

Put x = $\tan\theta$

$$\int e^{\theta}\left(\theta^2+2\theta\right)d\theta$$

$$=e^{\theta}\ \theta^2\ +c$$

$$=e^{\tan^{-1}x}\left(\mathrm{Tan}^{-1}x\right)^2+C$$

124. $$\int \frac{dx}{\left(\frac{x-3}{x+1}\right)^{4/5}(x+1)^2}$$

Put $\frac{x-3}{x+1}=t$

$$\frac{dx}{(x+1)^2}=\frac{1}{4}dt$$

$$\frac{1}{4}\int t^{-4/5}dt$$

$$=\frac{5}{4}\left(\frac{x-3}{x+1}\right)^{1/5}+C$$

125. $\int \cos^n xdx+\int \sin^n xdx$

$$I_n=\int \sin^n x\ dx$$

$$I_n-\frac{n-1}{n}I_{n-2}=\frac{-\sin^{n-1}x\cos x}{n}$$

$$I_n=\int \cos^n x\ dx$$

$$I_n-\frac{n-1}{n}I_{n-2}=\frac{\cos^{n-1}x\ \sin x}{n}$$

Add $\int \sin^n xdx+\int \cos^n x\ dx$

$$I_n-\left(\frac{n-1}{n}\right)I_{n-2}=\frac{\sin x\cos x}{2n}\left(\cos^{n-2}x-\sin^{n-2}x\right)$$

126. $$\int \frac{dx}{1+a\cos x}$$

Put

$$\tan\frac{x}{2}=t$$

$$dx=\frac{2dt}{1+t^2}$$

$$=\frac{1}{a-1}\int \frac{2dt}{t^2-\left(\frac{a+1}{a-1}\right)}$$

$$=\frac{1}{\sqrt{a^2-1}}\log\left|\frac{\sqrt{a-1}\tan\frac{x}{2}+\sqrt{a+1}}{\sqrt{a-1}\,\mathrm{Tan}\frac{x}{2}-\sqrt{a+1}}\right|+C$$

127. $$\int \frac{x^2-2}{x^3\sqrt{x^2-1}}dx$$

put $x^2-1=t^2$

$$=\int \frac{t^2-1}{\left(t^2+1\right)^2}dt$$

$$=\frac{-t}{1+t^2}=\frac{-\sqrt{x^2-1}}{x^2}+C$$

128. Conceptual

129. $\frac{at+b}{ct+d}=x\to t=\frac{dx-b}{a-cx}$

$$\int f(x)dx=\int \frac{dx-b}{-cx+a}dx$$

$$=\int\left[\frac{-d}{c}+\frac{ad-bc}{c}\left(\frac{1}{a-cx}\right)\right]dx$$

$$=-\frac{d}{c}x+\frac{ad-bc}{c^2}\log|cx-a|+k$$

130. $\int e^x f(x)-\int e^x f'(x)dx=g(x)+c$

$$\Rightarrow e^x f(x)-\int e^x f'(x)dx-\int e^x f'(x)dx=g(x)+c$$

$$\Rightarrow \int e^x f'(x)\,dx = \frac{1}{2}\left[e^x f(x) - g(x)\right] + c$$

131. $\left[\mathrm{Tan}^{-1}(\sin x + \cos x)\right] = t$

$$\frac{\sin^3 x}{\cos^4 x + 3\cos^2 x + 1}\,dx = dt$$

$$\int \frac{dt}{t} = \log t + c$$

$$= \log\left[\mathrm{Tan}^{-1}(\sec x + \cos x)\right] + c$$

132. $\int \frac{dx}{\left(\sqrt{\sin x} + \sqrt{\cos x}\right)^4}$ put $\tan x = t^2$

$$= \int 2t(1+t)^{-4}\,dt$$

Apply by parts

$$= 2\left[\frac{-t}{3(1+t)^3} - \frac{1}{6}\,\frac{1}{(1+t)^2} + c\right]$$

$$= \frac{-3t-1}{3(t+1)^3} + c$$

$$= \frac{-3\sqrt{\mathrm{Tan}\,x} + 1}{3(1+\sqrt{\mathrm{Tan}\,x})^3} + c$$

133. $\int \frac{dx}{x^{2022}\left(1 + x^{2022}\right)^{\frac{1}{2022}}}$

$$\int \frac{dx}{x^{2023}\left(1 + x^{-2022}\right)^{\frac{1}{2000}}}$$

put $1 + x^{-2000} = t$

$$\frac{-1}{2022}\int \frac{dt}{(t)^{1/2022}}$$

$m = 2022,\ n = 2021$

134. $\int (\cot x + \operatorname{cosec} x)\,dx$

$$= \log|\sin x| + \log\left|\tan\frac{x}{2}\right| + c$$

$$= 2\log\left|2\sin\frac{x}{2}\right| + c$$

135. $\int \frac{\cos^4 x}{\left(\sin^2 x + \frac{\cos^5 x}{\sin^3 x}\right)}\,dx$

$$\int \frac{\cot^4 x \operatorname{cosec}^2 x}{\left(1 + \cot^5 x\right)^3}\,dx$$

$$= \frac{-1}{5}\,\frac{\left(1 + \cot^5 x\right)^{-2}}{-2} + c$$

$$= \frac{1}{10}\,\frac{1}{\left(1 + \cot^5 x\right)^2} + c$$

136. $\int \frac{2\cos x + 1}{(2 + \cos x)^2}\,dx$

put $\frac{\sin x}{2 + \cos x} = A$

Differentiate w.r to x

$$\frac{1 + 2\cos x}{(2 + \cos x)^2} = \frac{dA}{dx}$$

Integrate

$$\int \frac{1 + 2\cos x}{(2 + \cos x)^2}\,dx = \frac{\sin x}{2 + \cos x} + C$$

137. Conceptual

138. $f'(x) = g(x)$

Put $1 + f^2(x) = t$

$$2f(x)f'(x)\,dx = dt$$

$$\int \frac{1}{2}t\ \ dt = \frac{1}{2}\,\frac{\left(1 + f^2(x)\right)}{2} + c$$

139. Conceptual

140. Conceptual

141. $\int \cos^{-1}\sqrt{\frac{x}{x+a}}\,dx = f(x) + c$

Differentiate

$$\cos^{-1}\sqrt{\frac{x}{x+a}} = f'(x)$$

$$f'(a)=\frac{\pi}{4}$$

142. $$\int \frac{dx}{\left(1+\sqrt{x}\right)^{2022}}$$

put $x=p^2$

$$2\int\left[\frac{p+1}{(1+p)^{2022}}-\frac{1}{(1+p)^{2022}}\right]dp$$

$$\frac{-2}{\left(1+\sqrt{x}\right)^{2021}}\left[\frac{1+\sqrt{x}}{2020}-\frac{1}{2021}\right]+c$$

143. $$\int \frac{x^8+4}{x^4-2x^2+2}dx$$

$$\int \frac{\left(x^4-2x^2+2\right)\left(x^4+2x^2+2\right)}{\left(x^4-2x^2+2\right)}dx$$

$$\frac{x^5}{5}+\frac{2x^3}{3}+2x+c$$

144. Conceptual

145. $$\frac{-1}{\sqrt{2}}\int \frac{(\cos x-\sin x)}{1+(\sin+\cos x)^2}dx$$

$$=\frac{-1}{\sqrt{2}}\tan^{-1}(\sin x+\cos x)+c$$

146. $$\int\left[\tan^{-1}\left(1-x+x^2\right)+\tan^{-1}(x)+\tan^{-1}(1-x)\right]dx$$

$$\int\left[\tan^{-1}\left(1-x+x^2\right)+\cot^{-1}\left(1-x+x^2\right)\right]dx$$

$$\int \frac{\pi}{2}dx=\frac{\pi x}{2}+c$$

147. $$\int \frac{dx}{\sqrt{x^2-x^5}}=\frac{1}{3}\log|f(x)|+c$$

$$\int \frac{x^2}{x^3\sqrt{1-x^3}}dx$$

Put $1-x^3=t^2$

$$=\frac{-2}{3}\int\frac{dt}{1-t^2}$$

$$=\frac{1}{3}\log\left|\frac{1-\sqrt{1-x^3}}{1+\sqrt{1-x^3}}\right|+c$$

$$f\left(\frac{1}{2}\right)=\frac{\sqrt{8}-\sqrt{7}}{\sqrt{8}+\sqrt{7}}$$

148. $$\int\frac{dx}{\sin\left(x-\frac{\pi}{3}\right)\cos x}$$

$$\int\frac{4}{\sin^2 x-\sqrt{3}\cos 2x-\sqrt{3}}$$

$$\int\frac{4\sec^2 x\,dx}{2\tan x-2\sqrt{3}}$$

$$=2\log\left(\tan x-\sqrt{3}\right)+c$$

$$=\log\left(1+x^2\right)\left(x+\frac{x^2}{2}+\frac{1}{2}\right)-\frac{x}{2}\Rightarrow g(x)=\frac{-x^2}{2}$$

149. Conceptual

150. Conceptual

151. Conceptual

152. $$\int\frac{xdx}{\left(1+x^2\right)^{12/15}\left(2+x^2\right)^{18/15}}$$

$$=\int\frac{xdx}{\left(\frac{1+x^2}{2+x^2}\right)^{4/5}\left(2+x^2\right)^2}$$

Put $\frac{1+x^2}{2+x^2}=p$

$$\frac{2xdx}{\left(2+x^2\right)^2}=dp$$

$$\Rightarrow \frac{1}{2}\int\frac{dp}{p^{4/5}}=\frac{5}{2}p^{1/5}+c$$

$$=\frac{5}{2}\left(\frac{1+x^2}{2+x^2}\right)^{1/5}+c$$

$\alpha=\frac{5}{2}\quad n=5$

$n/\alpha=2$

153. $\int(\log x)^2 x^3 dx$

Apply by parts.

$$\Rightarrow \frac{x^4(\log x)^2}{4}-\frac{1}{2}\int\left(\log x\quad x^3\right)dx$$

Again apply

$$\frac{x^4}{4}(\log x)^2-\frac{1}{8}\log x\quad x^4+\frac{1}{8}\quad\frac{x^4}{4}+c$$

$$=\frac{x^4}{32}\left(8(\log x)^2-4\log x+1\right)+c$$

154. $f(x)=\int\left(2x-3+\frac{4}{x}\quad\frac{-5}{x^2}\right)dx$

$$f(x)=x^2-3x+4\log x+\frac{5}{x}+c$$

$f(1)=1\to c=-2$

$f(5)=9+4\log 5$

155. $\int\left(x^{3/2}-\sec x\tan x+3-\frac{2}{x}+\frac{1}{x^2}\right)dx$

$$=\frac{2}{5}x^2\sqrt{x}-\sec x+3x-2log\ x-1/x+c$$

156. $\int(2x-3)\sqrt{3x+2dx}$

Put $3x+2=t^2$

$$\frac{2}{3}\int\left(\frac{2}{3}t^4-\frac{13}{3}t^2\right)dt$$

$$\frac{2}{135}\left(6t^5-65t^3\right)c$$

$$\frac{2}{135}\sqrt{3x+2}\left(54x^2-123x-106\right)+c$$

157. $\int\frac{1+\cos 8x}{\tan 2x-\cot 2x}dx=f(x).\cos g(x)$

$$=-\int\frac{2\sin 4x\ \cot 4x}{2}dx$$

$$\frac{1}{16}\cos 8x+c$$

$f(x)=\frac{1}{16}\quad g(x)=8x$

$$f\left(\frac{1}{4}\right)+g\left(\frac{1}{4}\right)=\frac{33}{16}$$

158. Conceptual

159.

$$\int\frac{\cos\left(\log\left(\frac{2x+3}{3-2x}\right)\right)}{(3-2x)^2}dx$$

$$put\ \frac{2x+3}{3-2x}=e^t$$

$$\frac{dx}{(3-2x)^2}=\frac{e^t dt}{12}$$

$$\frac{1}{12}\int e^t \cos t dt$$

$$=\frac{e^t}{24}(\cos t+\sin t)+c$$

$$g(x)=\log\left(\frac{2x+3}{3-2x}\right)$$

$$g(x)=\log\left(\frac{2x+3}{3-2x}\right)$$

$$g(1)=\log 5$$

$$f(x)=\frac{2x+3}{3-2x}\Rightarrow f(1)=5$$

160. $$\int\frac{3x^6-2x^4+x^2-2}{1+x^2}dx$$

$$\int\left(3x^4-5x^2-\frac{8}{1+x^2}+6\right)dx$$

$$=\frac{3x^5}{5}-\frac{5x^3}{3}-8\tan^{-1}x+6x+c$$

161. Conceptual

162. $$f(x)=\int\frac{16x^7+5x^{10}}{\left(x^3+2+3x^8\right)^2}dx \quad x\ge 0$$

$$=\int\frac{16x^{-9}+5x^{-6}}{\left(3+2x^{-8}+x^{-5}\right)^2}dx$$

Put $3+2x^{-8}+x^{-5}=t$

$$=\int\frac{-dt}{t^2}=\frac{1}{t}+c$$

$$f(x)=\frac{x^8}{3x^8+2+x^3}+c$$

$$f(0)=1\rightarrow c=1$$

$$f(-1)=\frac{5}{4}$$

163. $$\int f(x)dx=g(x)+c$$

$$f(x)=g'(x)$$

$$\frac{d}{dx}\left[\log\left(1+g^2(x)\right)+c\right]$$

$$=\frac{2g(x)\ f(x)}{1+g^2(x)}$$

164. $$f(x)=\int\left[\sec^2x-2+\cos\sec^2x+\frac{4(\sin^3x+\cos^3x}{4\sin^2x\cos^2x}\right]dx$$

$$f(x)=\tan x-\cot x-2x+\sec x-\cos ecx+c$$

$$f\left(\frac{\pi}{4}\right)=0\rightarrow c=\frac{\pi}{2}$$

$$3\left[f(\frac{\pi}{6})+2\right]=\frac{\pi}{2}$$

165. $$\int\sqrt{4\cos^2x-5\sin^2x}\ \ \cos x\ dx$$

$$=\int\sqrt{4-9\sin^2x}\cos x\ \ dx$$

Put sinx =t

$$=\int\sqrt{4-9t^2}dt$$

$$\frac{\sin x}{2}\sqrt{4-9\sin^2x}+\frac{2}{3}\sin^{-1}\left(\frac{3\sin x}{2}\right)+c$$

166. $$\int\left(2^x+\frac{1}{x^2}-\frac{1}{x}-\sqrt{1+\sin x}\right)dx$$

$$=\frac{2^x}{\log 2}-\frac{1}{x}-\log x+\int(\sin x+\cos x)dx$$

$$=\frac{2^x}{\log 2}-\log x-\frac{1}{x}+\sin x-\cos x+c$$

167. $$\tan\alpha=\frac{4}{3}$$

$$\int \frac{dx}{3\cos x-4\sin x}$$

$$=\frac{1}{5}\log\left|\sec(x+\alpha)+\tan(x+\alpha)\right|+c$$

$$=\frac{1}{5}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}+\frac{\alpha}{2}\right)\right|+c$$

168. Conceptual

169. $$\int \frac{2-3\sin^2 x}{1+\cos 2x}dx$$

$$=\int\left(\frac{3}{2}-\frac{1}{2}\sec^2 x\right)dx$$

$$=\frac{3x}{2}-\frac{1}{2}\tan x+c$$

$$f\left(\frac{\pi}{4}\right)=1\rightarrow c=\frac{3}{8}(4-\pi)$$

170. $$\int\left(\frac{\sin x+\sin 2x}{1+\cos x+\cos 2x}\right)^2 dx$$

$$=\int\left(\sec^2 x-1\right)dx$$

$$=\tan x-x+c$$

171. $$=\frac{1}{2}\int\frac{\left(1+x^2\right)+\left(1-x^2\right)}{x^4+3x^2+1}dx$$

$$\frac{1}{2}\left[\int\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+5}dx-\int\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2+1}dx\right]$$

$$\frac{1}{2}\left[\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)-\tan^{-1}\left(\frac{x^2+1}{x}\right)\right]$$

172. $$\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}=\text{A}+\text{B}\frac{18e^{2x}}{9e^{2x}-4}$$

$$\Rightarrow\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}=\text{A}+\frac{\text{B}18e^x}{9e^x-4e^{-x}}$$

$$\Rightarrow 4e^x+6e^{-x}=\text{A}\left(9e^x-4e^{-x}\right)+18\text{B}e^x$$

Coifficient of e^{-x} *is*

$6=-4\text{A}\ \Rightarrow \boxed{\text{A}=\frac{-3}{2}}$

Coifficient of e^x *is*

$4=9\text{A}+18\text{B}$

$$\Rightarrow 4=9\left(\frac{-3}{2}\right)+18\text{B}$$

$$\Rightarrow 8=-27+36\text{B}$$

$$\Rightarrow 36\text{B}=35\Rightarrow\boxed{\text{B}=\frac{35}{36}}$$

173. $$f(x)=\frac{1}{\sqrt{2}}\text{Tan}^{-1}\left[\frac{x}{\sqrt{2}}\right]+c$$

$$0=\frac{1}{\sqrt{2}}\text{Tan}^{-1}\left[\frac{\sqrt{2}}{2}\right]+c$$

$$0=\frac{1}{\sqrt{2}}\left[\frac{\pi}{4}\right]+c$$

$$\Rightarrow c=\frac{-\pi}{4\sqrt{2}}$$

$$\therefore f(x)=\frac{1}{\sqrt{2}}Tan^{-1}\left[\frac{x}{\sqrt{2}}\right]-\frac{\pi}{4\sqrt{2}}$$

$$\therefore f(0)=\frac{-\pi}{4\sqrt{2}}$$

174. $I=\int e^{x}\left(\frac{2+2\sin x\cos x}{2\cos^{2}x}\right)$

$=\int e^{x}\left(\sec^{2}x+\text{Tan}x\right)dx=e^{x}.\text{Tan}x+c$

175. $Put\,Tan\frac{x}{2}=t$

$I=\int\frac{dx}{4+5\cos x}$

$=\int\frac{1}{4+5\left(\frac{1-t^{2}}{1+t^{2}}\right)}\left(\frac{2dt}{1+t^{2}}\right)$

$=\frac{2dt}{9-t^{2}}=2\int\frac{dt}{3^{2}-t^{2}}$

$=2\left[\frac{1}{2(3)}\ln\left|\frac{3+t}{3-t}\right|\right]+c$

$=\frac{1}{3}\ln\left|\frac{3+Tan\frac{x}{2}}{3-Tan\frac{x}{2}}\right|+c$

176. $f(x)=\int\frac{5x^{8}+7x^{6}}{\left(x^{2}+2x^{7}+1\right)^{2}}dx$

$=\int\frac{\left(5x^{8}+7x^{6}\right)}{x^{14}\left(\frac{1}{x^{5}}+2+\frac{1}{x^{7}}\right)^{2}}dx$

$=\int\frac{\left(\frac{5}{x^{6}}+\frac{7}{x^{8}}\right)dx}{\left(\frac{1}{x^{5}}+2+\frac{1}{x^{7}}\right)^{2}}$

$=\int\frac{-dt}{t^{2}}$ where $t=\frac{1}{x^{5}}+2+\frac{1}{x^{7}}$

$=\frac{1}{t}+c$

$=\frac{x^{7}}{x^{2}+2x^{7}+1}+c$

$\therefore f(0)=0\Rightarrow c+0=0\Rightarrow c=0$

$f(x)=\frac{x^{7}}{x^{2}+2x^{7}+1}\Rightarrow f(1)=\frac{1}{4}$

177. $I=\int(\log x)^{2}x^{3}.dx$

$=(\log x)^{2}.\frac{x^{4}}{4}-\int\frac{x^{4}}{4}.2(\log x)\frac{dx}{x}$

$=(\log x)^{2}\frac{x^{4}}{4}-\frac{1}{2}\int(\log x)^{1}.x^{3}.dx$

$=(\log x)^{2}.\frac{x^{4}}{4}-\frac{1}{2}\left[(\log x)\frac{x^{4}}{4}-\int\frac{x^{4}}{4}\times\frac{1}{x}dx\right]$

$=(\log x)^{2}\frac{x^{4}}{4}-\frac{1}{2}\left[(\log x)\frac{x^{4}}{4}-\frac{x^{4}}{16}\right]+c$

178. $Put\log x=t$

$\Rightarrow\frac{1}{x}dx=dt$

$I=\int\frac{dx}{x\left[(\log x)^{2}+4\log x-1\right]}$

$=\int\frac{dt}{t^{2}+4t-1}=\int\frac{dt}{(t+2)^{2}-\left(\sqrt{5}\right)^{2}}$

$=\frac{1}{2\sqrt{5}}\ln\left|\frac{t+2-\sqrt{5}}{t+2+\sqrt{5}}\right|+c$

$=\frac{1}{2\sqrt{5}}\ln\left|\frac{\log x+\left(2-\sqrt{5}\right)}{\log x+\left(2+\sqrt{5}\right)}\right|+c$

Given

$A=\frac{1}{2\sqrt{5}},B=2-\sqrt{5}\;\&\;c=2+\sqrt{5}$

179. Given $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}\ dx$

$$\int \frac{x^2-1}{x^5\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}\ dx$$

$$\int \frac{\left(\frac{1}{x^3}-\frac{1}{x^5}\right)dx}{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}$$

$$let\ 2-\frac{2}{x^2}+\frac{1}{x^4}=t^2$$

180. $\int (\log(\log x)+(\log x)^2)dx$

$\log x = t$

$x = e^t$

$$\int e^t\left(\log t+\frac{1}{t^2}\right)dt$$

$$\int e^t\left(\log t+\frac{1}{t}-\frac{1}{t}+\frac{1}{t^2}\right)dt$$

$$=e^t\left(\log t-\frac{1}{t}\right)+C$$

$$=x\left(\log(\log x)-\frac{1}{\log x}\right)+C$$

181 $\int 3^{-\log_9 x^2}\,dx$

$$\int 3^{-\frac{2}{2}\log_3 x}\,dx$$

$$\int 3^{\log_3 \frac{1}{x}}\,dx=\log|x|+c$$

182. $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sqrt{x}\left(\sin^{-1}\sqrt{x+\cos^{-1}\sqrt{x}}\right)}dx$

$$=\frac{2}{\pi}\int\frac{\sin^{-1}\sqrt{x}-\left(\frac{\pi}{2}-\sin^{-1}\sqrt{x}\right)}{\sqrt{x}}dx$$

$$=\frac{2}{\pi}\int\frac{2\sin^{-1}\sqrt{x}-\frac{\pi}{2}}{\sqrt{x}}\ dx=\frac{4}{\pi}\int\frac{\sin^{-1}\sqrt{x}}{\sqrt{x}}dx-\int\frac{1}{\sqrt{x}}dx$$

$$=\frac{4}{\pi}\int\frac{\sin^{-1}\sqrt{x}}{\sqrt{x}}dx-2\sqrt{x}+c$$

$$\sqrt{x}=t\Rightarrow\frac{1}{2\sqrt{x}}dx=dt\Rightarrow\frac{1}{\sqrt{x}}dx=2dt$$

$$=\frac{8}{\pi}\int\sin^{-1}t\,dt-2\sqrt{x}+c$$

$$=\frac{8}{\pi}\left(\sqrt{x}\sin^{-1}\sqrt{x}-\sqrt{1-x}\right)-2\sqrt{x}+c$$

183 $\int\frac{3x+2}{4x^2+4x+5}dx=A\log(4x^2+4x+5)+B\mathrm{Tan}^{-1}(x+\frac{1}{2})+c$

$3x+2=A\frac{dy}{dx}(4x^2+4x+5)+B$

$\Rightarrow 3x+2=A(8x+4)+B$

compare like terms $8A=3\Rightarrow A=\frac{3}{8}$

$4A+B=2\Rightarrow 4\left(\frac{3}{8}\right)+B=2\Rightarrow\frac{3}{2}+B=2$

$B=2-\frac{3}{2}=\frac{1}{2}$

$$\int\frac{3x+2}{4x^2+4x+5}=\int\frac{\frac{3}{8}\frac{dy}{dx}(4x^2+4x+5)}{4x^2+4x+5}dx+\frac{1}{2}\int\frac{1}{4x^2+4x+5}dx$$

$$=\frac{3}{8}\log(4x^2 4x+5)+\frac{1}{8}\int\frac{1}{x^2+x+\frac{5}{4}}dx$$

$$=\frac{3}{8}\log(4x^2 4x+5)+\frac{1}{8}Tan^{-1}\left(x+\frac{1}{2}\right)+c$$

$$\therefore A=\frac{3}{8}, B=\frac{1}{8}$$

184 $\int e^x(\log x+\frac{1}{x^2})dx$

$=\int e^x(\log x-\frac{1}{x}+\frac{1}{x}+\frac{1}{x^2})dx$

$=\int e^x\left[(\log x-\frac{1}{x})+\left(\frac{1}{x}+\frac{1}{x^2}\right)\right]dx$

$=e^x\left(\log x-\frac{1}{x}\right)+c$

185

$$\int\frac{\sqrt{x^4+x^{-4}+2}}{x^3}dx=\int\frac{\sqrt{x^4+\frac{1}{x^4}+2}}{x^3}dx$$

$$=\int\frac{x^2+\frac{1}{x^2}}{x^3}dx=\int\left(\frac{1}{x}+\frac{1}{x^5}\right)dx=\log x-\frac{1}{4x^4}+C$$

186. $\int e^{\sin x}\frac{(x\cos^3 x-\sin x)}{\cos^2 x}dx=\int\left(x\cos xe^{\sin x}-e^{\sin x}\sec x\tan x\right)dx$

Integrating by parts

$\left[xe^{\sin x}-\int e^{\sin x}1\,dx\right]-\left[e^{\sin x}\sec x-\int e^{\sin x}\cos x\sec x\,dx\right]$

$xe^{\sin x}-e^{\sin x}\sec x$

$e^{\sin x}(x-\sec x)+c$

187 $f^1(x)=a\cos x+b\sin x$ .... (1)

$f^1(0)=4,\ f(0)=3,\ f\left(\frac{\pi}{2}\right)=5,\ f(x)=?$

From (1) $\Rightarrow f^1(0)=a\cos 0+b\sin 0=a$

$\boxed{4=a}$

$f^1(x)=a\cos x+b\sin x$

I.O.B.S

$f(x)=a\sin x-b\cos x+k$

$\Rightarrow x=0, f(0)=a(0)-b(1)+k$

$3=-b+k$

$\Rightarrow x=\frac{\pi}{2}, f\left(\frac{\pi}{2}\right)=a(1)-b(0)+k$

$5=a+k$

$\boxed{k=1}$, $\boxed{b=-2}$

$\therefore f(x)=a\sin x-b\cos x+k$

sub a, b, k values in (1)

$\therefore f(x)=4\sin x+2\cos x+1$

188 $\int f(x)dx=\psi(x)$ .... (1)

$\Rightarrow\int x^5 f(x^3)dx=\int x^3x^2f(x^3)dx$, put $x^3=t$

$=\frac{1}{3}\int t.f(t)dt$, $3x^2dx=dt$

$=\frac{1}{3}\left[t\int f(t)dt-\int\left(\frac{dt}{dt}\int f(t)dt\right)dt\right]$

$=\frac{1}{3}\left[x^3\psi(t)-\int 1.\psi(t)3x^2dx\right]$

$=\frac{1}{3}\left[x^3\psi(x^3)-3\int\psi(x^3)x^2dx\right]$

$=\frac{1}{3}x^3\psi(x^3)-\int\psi(x^3)x^2\,dx$

189 $\int\frac{\sin^2\alpha-\sin^2 x}{\cos x-\cos\alpha}dx=f(x)+Ax+B$

$$\int\frac{\sin^2\alpha-\sin^2 x}{\cos x-\cos\alpha}dx=\int\frac{(\sin\alpha+\sin x)(\sin\alpha-\sin x)}{-2\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}dx$$

$$=\int\frac{\left(\cancel{2}\sin\left(\cancel{\frac{\alpha+x}{2}}\right)\cos\left(\frac{\alpha-x}{2}\right)\right)\left(2\cos\left(\frac{\alpha+x}{2}\right)\cancel{\sin}\left(\frac{\alpha-x}{2}\right)\right)}{\cancel{-2}\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=\int 2\cos\left(\frac{\alpha-x}{2}\right)\cos\left(\frac{\alpha+x}{2}\right)dx=\int(\cos\alpha+\cos x)dx$$

$=x\cos\alpha+\sin x+C$

$=f(x)+Ax+B$

Here, $B=C,\ A=\cos\alpha,\ f(x)=\sin x$

190 $\int\left(x^{3m}+x^{2m}+x^m\right)\left(2x^{2m}+3x^m+6\right)^{\frac{1}{m}}dx$

$$\boxed{\int f'(x)\left(f(x)\right)^n dx=\frac{f(x)^{n+1}}{n+1}+C}$$

$$=\int\frac{\left(x^{3m}+x^{2m}+x^m\right)}{x}(x)^{\frac{m}{m}}\left(2x^{2m}+3x^m+6\right)^{\frac{1}{m}}dx$$

$$=\int\frac{x^{3m}+x^{2m}+x^m}{x}\left[x^m\left(2x^{2m}+3x^m+6\right)\right]^{\frac{1}{m}}dx$$

$$=\int\frac{x^{3m}+x^{2m}+x^m}{x}\left[2x^{3m}+3x^{2m}+6x^m\right]^{\frac{1}{m}}dx$$

$$=\frac{6m}{6m}\int\frac{x^{3m}+x^{2m}+x^m}{x}\left[2x^{3m}+3x^{2m}+6x^m\right]^{\frac{1}{m}}dx$$

$$=\frac{1}{6m}\frac{\left[2x^{3m}+3x^{2m}+6x^m\right]^{\frac{1}{m}+1}}{\frac{1}{m}+1}+C$$

$$=\frac{1}{6(m+1)}\left[2x^{3m}+3x^{2m}+6x^m\right]^{\frac{1}{m}+1}+C$$

191 $\int \frac{dx}{(2ax+x^2)^{\frac{3}{2}}}$

$$\int \frac{dx}{\left((x+a)^2-a^2\right)^{3/2}}$$

$x+a=t \Rightarrow dx=dt$

$$=\int \frac{dt}{\left(t^2-a^2\right)^{3/2}}$$

$Put\, t = a\sec\theta \Rightarrow dt = a\sec\theta\ \tan\theta\ d\theta$

$$=\int \frac{a\ Sec\theta Tan\theta\, d\theta}{(a^2\sec^2\theta - a^2)^{3/2}}$$

$$=\int \frac{a\ Sec\theta\ Tan\theta\ d\theta}{a^3\, Tan^3\theta} = \frac{1}{a^2}\int \frac{\cos\theta\, d\theta}{\sin^2\theta}$$

$put\ \sin\theta = u$

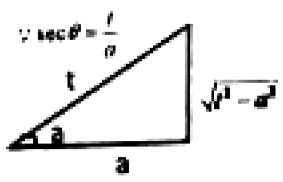
$$=\frac{1}{a^2}\int \frac{du}{u^2} = \frac{1}{a^2}\left(-\frac{1}{u}\right)+c$$

$$=-\frac{1}{a^2}\left(\frac{1}{u}\right)+c$$

$$=-\frac{1}{a^2}\left(\frac{1}{\sin\theta}\right)+c$$

$$=-\frac{1}{a^2}\left(\frac{t}{\sqrt{t^2-a^2}}\right)+c$$

$$=-\frac{1}{a^2}\left(\frac{x+a}{\sqrt{2ax+a^2}}\right)+c$$



192 $f(x) = \lim_{n\to\infty}\left(\frac{x^{\frac{1}{n}} - x^{\frac{1}{n+1}}}{\frac{1}{n^2}}\right) = \left(\frac{0}{0}\right)$

Using $L-H$ *rule*

and D.W.R to n

$$=\lim_{n\to\infty}\frac{\frac{-x^{\frac{1}{n}}}{n^2}\log x + \frac{x^{\frac{1}{n+1}}}{(n+1)^2}\log x}{\frac{-2}{n^3}}$$

$$=\log x\left\{\lim_{n\to\infty}\left(\frac{1}{n^2}-\frac{1}{(n+1)^2}\right)\times\frac{n^3}{2}\right\}$$

$$=\log x\left\{\lim_{n\to\infty}\left(\frac{(n+1)^2-n^2}{n^2(n+1)^2}\right)\frac{n^3}{2}\right\}$$

$$=\log x\left\{\lim_{n\to\infty}\frac{2n^2+n}{2(n+1)^2}\right\} = \log x$$

$\therefore f(x) = \log x$

$\therefore \int xf(x)\,dx = \int x.(\log x)\,dx$

$$=\frac{x^2}{2}\log x - \frac{x^2}{4}+c$$

193 $\int \frac{1+\cos x - 2\cos x}{\cos x(1+\cos x)}dx = \int\frac{dx}{\cos x} - 2\int\frac{1}{1+\cos x}dx$

$$=\log|\sec x+\tan x| - 2\int\frac{1-\cos x}{\sin^2 x}dx$$

$$=\log|\sec x+\tan x| - 2\int\left(\mathrm{cosec}^2 x - co\sec x.\cot x\right)dx$$

$$=\log|\sec x+\tan x| - 2(\mathrm{cosec}x - \cot x)+c$$

194 $g(x)=\int f(x)dx=\int\left(1+2^x\log 2\right)dx$

$g(x)=x+2^x+c \quad ---(1)$

$\frac{1}{2}=-1+2^{-1}+c\Rightarrow c=1$

$g(x)=x+2^x+1$ meets y axis $\therefore$ x=0

g(0) = 0+1+1 $\therefore$ g(0) = 2

Curve meet y-axis at (0, 2)

195 $I=\int\frac{x^3}{\sqrt{1+x^2}}dx=\int\frac{x^2(xdx)}{\sqrt{1+x^2}}$

$PUT \quad 1+x^2=t^2$

$2xdx=2t\,dt$

$xdx=tdt$

$x^2=t^2-1$

$=\int\frac{t^2-1}{\cancel{t}}\cancel{t}dt=\frac{t^3}{3}-t+c$

$=\frac{1}{3}\left(1+x^2\right)^{3/2}-\left(1+x^2\right)^{\frac{1}{2}}+c$

$\therefore A=\frac{1}{3} \qquad B=-1$

$A+B=\frac{1}{3}-1=\frac{-2}{3}$

196 $\int\frac{dx}{\sin(x-a)\cos(x-b)}$

$=\frac{1}{\cos(b-a)}\int\frac{\cos((x-a)-(x-b))}{\sin(x-a)\cos(x-b)}dx$

$=\frac{1}{\cos(b-a)}\int\frac{\cos(x-a)\cos(x-b)+\sin(x-a)\sin(x-b)}{\sin(x-a)\cos(x-b)}dx$

$=\frac{1}{\cos(b-a)}\int(\cot(x-a)+\tan(x-b))dx$

$=\frac{1}{\cos(b-a)}\left[\log|\sin(x-a)|-\log|\cos(x-b)|\right]+c$

$=\frac{1}{\cos(b-a)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+c$

197 $I_4+I_6=\int\left(\tan^4x+\tan^6x\right)dx$

$=\int\tan^4x\sec^2x\,dx$

$=\frac{\tan^5x}{5}+c$

198 $\int\frac{\sqrt{x}}{\sqrt{1-x^3}}dx=\int\frac{\sqrt{x}}{\sqrt{1-\left(x^{3/2}\right)^2}}dx$

$x^{3/2}=t\Rightarrow\frac{3}{2}x^{\frac{1}{2}}dx=dt$

$\Rightarrow\int\frac{\frac{2}{3}dt}{\sqrt{1-t^2}}=\frac{2}{3}\sin^{-1}t+c$

$=\frac{2}{3}\sin^{-1}\left(x^{3/2}\right)+c$

$\therefore f(x)=x^{3/2};g(x)=\sin^{-1}x$

199 $\frac{x^2}{\left(x^2-1\right)\left(x^2+1\right)}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{Cx+D}{x^2+1}$

By solving we get $A=\frac{1}{4},B=-\frac{1}{4},C=0,D=\frac{1}{2}$

$\int\left(\frac{1}{4(x-1)}-\frac{1}{4(x+1)}+\frac{1}{2\left(x^2+1\right)}\right)dx$

$=\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|+\frac{1}{2}\tan^{-1}x+c$

200 $\int \frac{dx}{1+\sin x} = \int \frac{1-\sin x}{\cos^2 x} dx$

$= \text{Tan x} - \text{sec x}$

$= \frac{-(1-\sin x)}{\cos x}$

$= -\left(\frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}} \right)$

$= -\left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}} \right)$

$= -\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$= \tan\left(\frac{x}{2} - \frac{\pi}{4}\right) \theta = \frac{\pi}{4}$

201 $\int \frac{8^1.8^x + 4.4^x}{4^x} dx$

$\int 8.\left(\frac{8}{4}\right)^x dx + \int 4\, dx$

$8\,\frac{(2)^x}{\log 2} + 4x$

202 Given $f(x) = \frac{x}{\left(1+nx^n\right)^{1/n}}$

Now $\int x^{n-2}.\text{f}(x)dx = \int \frac{x^{n-1}}{\left(1+nx^n\right)^{1/n}} dx$

Let $1+nx^n = t^n$

$\Rightarrow x^{n-1}.dx = \frac{t^{n-1}}{n} dt$

$= \int \frac{t^{n-1}}{nt}.dt$

$= \frac{1}{n}\int t^{n-2}.dt = \frac{1}{n}\left(\frac{t^{n-1}}{n-1}\right) + c$

$= \frac{1}{n(n-1)}\left(1+nx^n\right)^{1-\frac{1}{n}} + c$

203 Given $\int \frac{1+x\cos x}{x\left(1-x^2\left(e^{\sin x}\right)^2\right)} dx$

put x.esinx=t

Diff w.r to 'x'

$\frac{(1+x\cos x)dx}{x} = \frac{dt}{t}$

$= \int \frac{dt}{t\left(1-t^2\right)} = \log t - \frac{1}{2}\log(1-t) - \frac{1}{2}\log(1+t) + c$

$= \frac{1}{2}\log\left(\frac{t^2}{t^2-1}\right) + c$

$= \frac{1}{2}\log\left|\frac{\left(xe^{\sin x}\right)^2}{\left(xe^{\sin x}\right)^2 - 1}\right| + c$

204. $\int \frac{1}{(x-1)\sqrt{x+2}}dx$

put $x+2=t^2 \Rightarrow x=t^2-2$

$\Rightarrow x-1=t^2-3$

$1.dx=2t.dt$

$=\int \frac{1}{(t^2-3)(\cancel{t})}.2\cancel{t}.dt$

$=2\int \frac{1}{(t)^2-(\sqrt{3})^2}dt$

$=\frac{1}{\sqrt{3}.\log}\left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right|+c$

205 $e^{-2x}\left[\frac{1-\sin 2x}{2\cos^2 x}\right]$

$\frac{e^{-2x}}{2}\left[\sec^2 x-2Tanx\right]$

$e^{-2x}\left[\frac{\sec^2 x}{2}-Tan\ x\right]$

$\int e^{-2x}\left(\frac{1}{2}\sec^2 x-Tanx\right)dx$

$-2x=t$

$-x=\frac{t}{2}$

$-dx=\frac{dt}{2}$

$=\frac{e^{-2x}}{2}Tanx+C$

206 $\int \frac{\cancel{(\sin^4 x+\cos^4 x)}(\sin^4 x-\cos^4 x)}{\cancel{\sin^4 x+\cos^4 x}}dx$

$=\int -\cos 2x dx$

$=-\frac{1}{2}\sin 2x+\underline{c}$

207 By using integrating by parts, we get

$I=x^2\left(\frac{-1}{xTanx+1}\right)+2\int \frac{x}{xTanx+1}\ dn$

$=\frac{-x^2}{xTanx+1}+2\log|x\sin x+\cos x|+c$

Hell $f(x)=2\log|x\sin x+\cos x|+c$

208 $\int \sin(100x+x)\,(\sin x)^{99}.dx$

$=\int(\sin(100x).\cos x+\cos 100x.\sin x)(\sin x)^{99}dx$

$=\frac{\sin(100x).(\sin x)^{100}}{100}+c$

$\therefore\ \frac{\lambda}{\mu}=\frac{100}{100}=1$

209. $\int e^x\left(\sin^2 2x-8\cos 4x\right)dx$

$=\int e^x\left(\frac{1-\cos 4x}{2}-8\cos 4x\right)dx$

$=\frac{1}{2}\int e^x(1-17\cos 4x)dx$

$=\frac{e^x}{2}-\frac{17}{2}\int e^x.\cos 4x\,dx$

$=\frac{e^x}{2}-\frac{\cancel{17}}{2}.\frac{e^x}{\cancel{17}}(\cos 4x-\sin 4x)+c$

$=e^x\left(\frac{1}{2}-\frac{1}{2}(\cos 4x-\sin 4x)\right)+c$

Here $f(x)=\frac{1}{2}-\frac{1}{2}(\cos 4x-\sin 4x))$

$f\left(\frac{\pi}{4}\right)=\frac{1}{2}\quad -\frac{1}{2}(-1-0)=\frac{1}{2}+\frac{1}{2}=1$

210 Given $In=\int \frac{\sin nx}{\sin x}\ dx$

Now $I_{n+1}-I_{n-1}=\int \frac{\sin(n+1)x-\sin(n-1)x}{\sin x}\ dx$

$=\int \frac{2\cos nx.\cancel{\sin x}}{\cancel{\sin x}}\ dx$

$=\frac{2}{n}.\sin nx+c$

211 1) $\int \frac{\sin^2 x}{\cos^4 x} dx = \int \tan^2 x.\sec^2 x\, dx = \frac{\tan^3 x}{3} + c$

2) $\int \frac{\sin^4 x}{\cos^2 x} dx = \int \tan^2 x \sin^2 x\, dx$

$= \int (\sec^2 x - 1)\sin^2 x\, dy$

$= \tan x + \frac{\sin 2x}{4}\ \frac{-3x}{2} + c$

3) $\int \frac{\sin^3 x}{\cos^2 x} dx = \int \tan^2 x \sin x\, dx = \cos x + \sec x + c$

4) $\int \frac{\sin^3 x}{\cos^3 x} dx = \int \tan^3 x\, dx$

$= \int \tan^2 x \tan x\, dx = \frac{\tan^2 x}{2} + \log|\cos x| + c$

212 $\int \frac{x}{(a+x)^5} dx$

$Let (a+x) = t \Rightarrow x = t - a$

$dx = dt$

$\int \frac{t-a}{t^5} dt$

Here $k = 1, f(x) = \frac{a}{4} - \frac{(a+x)}{3}$

$\therefore \frac{f(-a)}{ak} = \frac{1}{4}$

213. $\int x^4 (\log x)^3 dx = x^5$

$\left[A(\log x)^3 + B(\log x)^2 + C(\log x) + D\right] + C$

$A = \frac{1}{5}\ \ B = \frac{-3}{25}\ C = \frac{6}{125}\ D = \frac{-6}{125}$

$A + B + C + 5D = \frac{2}{25}$

214 $\int \frac{\sec^2 x\, dx}{16\sec^2 x - 7Tan^2 x}$

$\int \frac{\sec^2 x\, dx}{16 + 9\tan^2 x}$

Put tan x=t

$\sec^2 x dx = dt$

$\frac{1}{9}\int \frac{dt}{\frac{16}{9} + t^2}$

$= \frac{1}{9}\frac{1}{4/3} Tan^{-1}\left(\frac{t}{4/3}\right) + c$

$= \frac{1}{12} Tan^{-1}\left(\frac{3\tan x}{4}\right) + c$

215. $\sec x + \tan x = t$......(1)

$(\sec x + \tan x)(\sec x - \tan x) = 1$

$\sec x - \tan x = \frac{1}{t}$

$(\sec x\, tanx + \sec^2 x) dx = dt$

$\sec x[Tanx + \sec x] dx = dt$

$\int \frac{\sec^2 x}{t^2} \times \frac{dt}{\sec x}$

$(1) + (2)$

$2\sec x = t + \frac{1}{t} \Rightarrow \sec x = \frac{1}{2}\left[\frac{t^2+1}{t}\right]$

$\Rightarrow \frac{1}{2}\int \frac{(t^2+1)}{t^4} dt$

$\Rightarrow \frac{1}{2}\left(\int \frac{1}{t^2} dt + \int t^{-4} dt\right)$

$\frac{1}{2}\left[\frac{-1}{\sec x + \tan x} - \frac{1}{3(\sec x + \tan x)^3}\right]$

$= \frac{-1 - 3(\sec x + \tan x)^2}{6(\sec x + \tan x)^3} + c$

216 $put\ \tan\frac{x}{2} = t$

217 $$\frac{1}{(x-2)(x^2+1)}=\frac{A}{x-2}+\frac{Bx+c}{x^2+1}$$

$$1=A(x^2+1)+(Bx+c)(x-2)$$

Put $\boxed{x=2} \Rightarrow A=\frac{1}{5}$

$A-2c=1 \Rightarrow \boxed{c=\frac{-2}{5}}$

Comparing Coefficient of x we get

$\boxed{B=-\frac{1}{5}}$

$$\int\frac{1}{(x-2)(x^2+1)}=\int\frac{1/5}{x-2}dx+\int\frac{-\frac{1}{5}x-\frac{2}{5}}{x^2+1}dx$$

$$=\frac{1}{5}\log|x-2|-\frac{1}{5}\int\frac{x+2}{x^2+1}\,\mathrm{dx}$$

$$\frac{1}{5}\log|x-2|-\frac{1}{10}\log|x^2+1|-\frac{1}{5}2Tan^{-1}(x)+c$$

$$=\frac{1}{5}\left[\log\frac{|x-2|}{\sqrt{1+x^2}}-2Tan^{-1}x\right]+c$$

218. $$\int\frac{x^{49}Tan^{-1}(x^{50})}{1+x^{100}}\,dx$$

$x^{50}=t$

$50\,x^{49}\,dx=dt$

$$\frac{1}{50}\int\frac{Tan^{-1}(\mathrm{t})}{1+t^2}\,dt$$

$$=\frac{1}{50}\,\frac{(Tan^{-1}x^{50})^2}{2}+C$$

$$k=\frac{1}{100}$$

219 $$\int(Logx)^3\,x^5\,dx$$

$$=\frac{x^6}{A}\left[B(Logx)^3+C(\log x)^2+D(Logx)-1\right]+K$$

$\log x_e=t$

$x=e^t$

$dx\;=e^t\;dt$

$$\int t^3\,e^{6t}\,dt$$

$$t^3\frac{e^{6t}}{6}-3t^2\,\frac{e^{6t}}{36}+6t\,\frac{e^{6t}}{216}-\frac{e^{6t}}{216}+k$$

$$\frac{e^{6t}}{6}\left[t^3-\frac{1}{2}t^2+\frac{t}{6}-\frac{1}{36}\right]+k$$

$$\frac{e^{6t}}{216}\left[36t^3-18t^2+6t-1\right]+k$$

$$\frac{x^6}{216}\left[36(\log x)^3-18\,(\log x)^2+6(\log x)-1\right]+k$$

$A=216\,,\;B=36\,,\;\;C=-18,\;D=6$

$A-(B+C+D)=216\;-36+18-6$

$=\;192$

220 $$\int\frac{1}{(x^2+1)(x^2+4)}\,dx$$

$$=\frac{1}{3}\int \frac{x^2+4-\left(x^2+1\right)}{\left(x^2+1\right)\left(x^2+4\right)}\,dx$$

$$\frac{1}{3}\int\frac{1}{x^2+1}\,dx-\frac{1}{3}\int\frac{1}{x^2+4}dx$$

$$\frac{1}{3}\,Tan^{-1}x-\frac{1}{6}\,Tan^{-1}\left(x/2\right)+C$$

221. $$\int\frac{1}{\left(x-1\right)^{3/4}\left(x+2\right)^{5/4}}\,dx$$

$$=\int\frac{1}{\left(\frac{x-1}{x+2}\right)^{3/4}\left(x+2\right)^2}\,dx$$

$$=\frac{x-1}{x+2}=t$$

$$=\frac{3}{\left(x+2\right)^2}\,dx=dt$$

$$=\frac{1}{3}\int\frac{1}{t^{3/4}}\,dt$$

$$=\frac{4}{3}\,t^{1/4}+C=\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$$

222 $$\int\frac{1}{\left(x+\frac{2}{x}\right)\sqrt{x^4+4x^2+3}}dx$$

$$\int\frac{x\,dx}{\left(x^2+2\right)\sqrt{\left(x^2+2\right)^2-1}}$$

$$x^2+2=t$$

$$x\,d\,x=\frac{dt}{2}$$

$$=\frac{1}{2}\int\frac{dt}{t\sqrt{t^2-1}}$$

$$=\frac{1}{2}\sec^{-1}t=\frac{1}{2}\sec^{-1}\left(x^2+2\right)+c$$

223 $$\frac{3\pi}{2}<x<\frac{5\pi}{2}\Rightarrow\frac{3\pi}{4}<\frac{x}{2}<\frac{5\pi}{4}$$

s + c < 0, s – c > 0

$$\int\left(\pm\left(\sin x/2-\cos x/2\right)\pm\left(\sin x/2+\cos x/2\right)\right)dx$$

$$\int\left(\left(\sin x/2-\cos x/2\right)-\left(\sin x/2+\cos x/2\right)\right)dx$$

$$\int\left(\cancel{\sin x/2}-\cos x/2-\cancel{\sin x/2}-\cos x/2\right)dx$$

$$-\int 2\cos x/2$$

$$-4\sin\left(x/2\right)$$

$$f\left(\frac{\pi}{3}\right)-f\left(0\right)=-4\left(\frac{1}{2}\right)-0=-2$$

224 $$\int\frac{2\sin 2x-3\cos x}{2\sin^2 x-3\sin x+4}dx$$

$$2\sin^2 x-3\sin x+4=t$$

$$\left(2\sin 2x-3\cos x\right)dx=dt$$

$$\int\frac{dt}{t}=\log t+c$$

$$=\log\left|2\sin^2 x-3\sin x+4\right|$$

$$f\left(x\right)=\log\left|2\sin^2 x-3\sin x+4\right|$$

$$f\left(\frac{\pi}{2}\right)=\log\left|2-3+4\right|=\log\left|3\right|$$

$$f\left(0\right)=\log 4$$

$$f\left(\frac{\pi}{2}\right)-f\left(0\right)=\log 3-\log 4=\log\frac{3}{4}$$

225. Conceptual