7. INVERSE TRIGONOMETRIC FUNCTIONS

1. If $\theta=\cot^{-1}(7)+\cot^{-1}(8)+\cot^{-1}(18)$, then $\cot\theta$ is equal to **[AP EAMCET 22-09-20_Shift-1]**
1. 2 2. 3
3. 4 4. 1

2. $If\ Tan^{-1}\left[\frac{1}{1+2}\right]+Tan^{-1}\left[\frac{1}{1+(2)(3)}\right]+Tan^{-1}\left[\frac{1}{1+(3)(4)}\right]+........+Tan^{-1}\left[\frac{1}{1+n(n+1)}\right]=Tan^{-1}\theta\ then\ \theta=$ **(2020)**
1. $\frac{n}{n+1}$ 2. $\frac{n+1}{n+2}$
3. $\frac{n+2}{n+1}$ 4. $\frac{n}{n+2}$

3. Let Z denote the set of integers .Then match the items in List-I, with those of the items in List-II ,then the correct matching is

List-I	List-II
A) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)+\sin^{-1}\left(\frac{1}{3}\right)$	(I) $k\pi\pm(-1)^k\frac{\pi}{6},k\in Z$
B) $\sin^{-1}\left(\frac{(-1)^n}{2}\right),n\in Z$	(II) $k\pi\pm1,k\in Z$
C) $Tan^{-1}\left(\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right)$	(III) $\frac{3}{2}$
D) $\sin^{-1}\lvert\sin x\rvert=\sqrt{\sin^{-1}\lvert\sin x\rvert}\Rightarrow x\in$	(IV) $\frac{3\pi}{8}$
	(V) $\frac{\pi}{2}$

[TS EAMCET 09-09-20_Shift-1]

1. A B C D: V I III II
2. A B C D: IV II V I
3. A B C D: V I IV II
4. A B C D: IV II V III

4. Consider the statements:

I) If $f(x)=\sin\left(\cot^{-1}\left(\cos\left(\tan^{-1}x\right)\right)\right)$, $r_1:r_2$ then $f(0)=\frac{1}{2}$

II) $\sin\left(4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}\right)=1$

Then the correct option among the following is **[TS EAMCET 09-09-20_Shift-2]**
1. Both I and II are false
2. Both I and II are true
3. I is true, but II is false
4. I is false,but II is true

5. If $\tan^{-1}\frac{1}{5}+\frac{1}{2}\sec^{-1}x+\tan^{-1}\frac{1}{8}=\frac{\pi}{8}$, then $x^2=$ **[TS EAMCET 10-09-20_Shift-1]**
1. $\frac{12}{7}$ 2. $\frac{50}{49}$
3. $\frac{13}{12}$ 4. $\frac{1}{2}$

6. If $\sin^{-1}\left(\frac{12}{x}\right)+\sin^{-1}\left(\frac{5}{x}\right)=\frac{\pi}{2}$, then $x=$ **[TS EAMCET 10-09-20_Shift-2]**
1. 5 2. 7
3. 13 4. 17

7. $If\ f(x)=\tan^{-1}\left(\frac{1}{\sin^2+\sin x+1}\right)+\tan^{-1}\left(\frac{1}{\sin^2x+3\sin x+3}\right)+\tan^{-1}\left(\frac{1}{\sin^2x+5\sin x+7}\right)+....+$ up to 10 terms, then $f'(0)=$ **[TS EAMCET 10-09-20_Shift-2]**
1. $\frac{-1}{101}$ 2. $\frac{100}{101}$
3. $\frac{-100}{101}$ 4. 0

8. The number of real roots of the equation $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0$ that are greater than or equal to one are **[TS EAMCET 11-09-20_Shift-1]**
1. 1 2. 2
3. 3 4. 4

9. If $x=\left(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}\right)$, then $\frac{\sin x+\cos x}{\tan x}=$ **[TS EAMCET 11-09-20_Shift-2]**
1. $\frac{12}{\sqrt{10}}$ 2. $\frac{15}{\sqrt{10}}$

3. $\frac{1}{\sqrt{10}}$ 4. $\frac{6\sqrt{2}}{\sqrt{10}}$

10. If $\tan^{-1}\left[\frac{1}{1+1.2}\right]+\tan^{-1}\left[\frac{1}{1+2.3}\right]+$ $...+\tan^{-1}\left[\frac{1}{1+n(n+1)}\right]=\tan^{-1}[x]$, then $x=$

[AP EAMCET 19-08-2021_Shift-1]

1. $\frac{1}{n+1}$ 2. $\frac{n}{n+1}$
3. $\frac{1}{n+2}$ 4. $\frac{n}{n+2}$

11. For how many distinct values of x, the following $\sin\left[2\cos^{-1}\cot\left(2\tan^{-1}x\right)\right]=0$ holds? **[AP EAMCET 19-08-2021_Shift-2]**

1. 8 2. 2
3. 6 4. 4

12. If $x=\sin\left(2Tan^{-1}2\right), y=\cos\left(2Tan^{-1}3\right)$, $z=\sec\left(2Tan^{-1}4\right)$ then ______

[AP EAMCET 20-08-2021_Shift-1]

1. $x<y<z$ 2. $y<z<x$
3. $z<x<y$ 4. $z<y<x$

13. $\tan^{-1}(-2)-\tan^{-1}(3)$ is equal to

[AP EAMCET 20-08-2021_Shift-2]

1. $\frac{3\pi}{4}$ 2. $\frac{-\pi}{6}$
3. $\frac{\pi}{6}$ 4. $\frac{-3\pi}{4}$

14. If $\theta=2\tan^{-1}\frac{1}{8}+2\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}$ and $\tan\frac{\theta}{2}=\sqrt{m}+\sqrt{n}$ where m and n are positive integers such that $m<n$ then $\left(m^n+n^m\right)^{m+n}=$

[AP EAMCET 23-08-2021_Shift-1]

1. 18 2. 27
3. 25 4. 36

15. For what values of x, the following identity is valid and holds? $\tanh^{-1}(x)=\frac{1}{2}\log_e\left(\frac{1+x}{1-x}\right)$

[AP EAMCET 23-08-2021_Shift-1]

1. $(-\infty,\infty)$ 2. $(1,\infty)$
3. $(-\infty,1)$ 4. $(-1,1)$

16. If $\cos^{-1}x=\sin^{-1}(3x)$ then $x=$

[AP EAMCET 24-08-2021_Shift-2]

1. $\frac{\sqrt{10}}{10}$ 2. $\frac{\sqrt{5}}{5}$
3. $\frac{5}{2\sqrt{6}}$ 4. $\frac{-\sqrt{10}}{10}$

17. What is the value of $Sin^{-1}\frac{12}{13}+Cos^{-1}\frac{4}{5}+Tan^{-1}\frac{63}{16}$

[AP EAMCET 25-08-2021_Shift-2]

1. π 2. $\frac{\pi}{2}$
3. $\frac{\pi}{6}$ 4. $\frac{3\pi}{4}$

18. If $\sec^{-1}\frac{x}{a}-\sec^{-1}\frac{x}{b}=\sec^{-1}b-\sec^{-1}a$, then $x=$

[AP EAMCET 23-08-2021_Shift-1]

1. ab 2. $-ab$
3. a^2 4. b^2

19. If $\sin^{-1}\left(\frac{x}{5}\right)+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then $5+x=$

[TS EAMCET 04-08-2021_Shift-2]

1. 6 2. 5
3. 7 4. 8

20. If $Sin^{-1}x<Cos^{-1}x$, then

[TS EAMCET 04-08-2021_Shift-1]

1. $-1\le x<\frac{1}{\sqrt{2}}$ 2. $-\sqrt{3}\le x<-1$
3. $\frac{1}{\sqrt{2}}<x\le 1$ 4. $1<x<\sqrt{3}$

21. $Coth^{-1}(2)+Cosech^{-1}\left(-2\sqrt{2}\right)=$

[TS EAMCET 04-08-2021_Shift-1]

1. $\log\sqrt{\frac{3}{2}}$
2. $\log\sqrt{6}$
3. $\log\frac{3}{\sqrt{2}}$
4. $\log\frac{3}{2}$

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22. $\sin\left(\tan^{-1}\frac{4}{5}+\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{9}-\tan^{-1}\frac{1}{7}\right)=$

[TS EAMCET 05-08-2021_Shift-1]

1. $\frac{1}{2}$
2. $\frac{1}{\sqrt{2}}$
3. $\frac{\sqrt{3}}{2}$
4. 1

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23. If

$f(n)=\tan$

$\left[\tan^{-1}\frac{1}{1+2}+\tan^{-1}\frac{1}{1+6}+\tan^{-1}\frac{1}{1+12}+...+\tan^{-1}\frac{1}{1+n(n+1)}\right]$

then $f(2021)=$

[TS EAMCET 05-08-2021_Shift-2]

1. $\frac{2020}{2022}$
2. $\frac{2022}{2024}$
3. $\frac{2021}{2023}$
4. $\frac{2019}{2021}$

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24. $2\sin^{-1}x+\sin^{-1}\left(2x\sqrt{1-x^2}\right)+3\cos^{-1}x$

$-\cos^{-1}\left(4x^3-3x\right)=$

[TS EAMCET 06-08-2021_Shift-2]

1. $4\sin^{-1}x$, when $x\in[-1,1]$
2. π, when $x\in\left[-1,-\frac{1}{\sqrt{2}}\right]$
3. $-\pi$, when $x\in\left[\frac{-1}{2},\frac{1}{2}\right]$
4. $4\sin^{-1}x+2\cos^{-1}\left(4x^3-3x\right), x\in\left[\frac{1}{\sqrt{2}},1\right]$

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25. $2\tan^{-}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)=$

[TS EAMCET 06-08-2021_Shift-1]

1. $\tan^{-1}\left(\frac{49}{29}\right)$
2. $\frac{\pi}{2}$
3. 0
4. $\frac{\pi}{4}$

KEY

1)	2	2)	4	3)	3	4)	1	5)	2
6)	3	7)	3	8)	2	9)	1	10)	4
11)	3	12)	4	13)	4	14)	2	15)	4
16)	1	17)	1	18)	1	19)	4	20)	1
21)	1	22)	4	23)	3	24)	2	25)	4

SOLUTIONS

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1. $\theta=\cot^{-1}\left(\frac{56-1}{8+7}\right)+\cot^{-1}(18)$

$=\cot^{-1}\left(\frac{11}{3}\right)+\cot^{-1}(18)=\cot^{-1}(3)$

$\cot\theta=3$

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2. $\tan^{-1}\left(\frac{1}{1+2}\right)+\tan^{-1}\left(\frac{1}{1+(2)(3)}\right)+.....$

$+\tan^{-1}\left(\frac{1}{1+n(n+1)}\right)=\tan^{-1}\theta$

$\Rightarrow\tan^{-1}(2)-\tan^{-1}(1)+\tan^{-1}(3)-\tan^{-1}(2)+....$

$......+\tan^{-1}(n+1)-\tan^{-1}(n)=\tan^{-1}(\theta)$

$\Rightarrow\tan^{-1}(n+1)-\tan^{-1}(1)=\tan^{-1}(\theta)$

$\Rightarrow\tan^{-1}\left(\frac{n}{2+n}\right)=\tan^{-1}(\theta)$

$\therefore\theta=\frac{n}{n+2}$

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3. $(A)\ \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)+\sin^{-1}\left(\frac{1}{3}\right)$

$=\sin^{-1}\left(\frac{8}{9}+\frac{1}{9}\right)=\frac{\pi}{2}$

$(B)\ \sin^{-1}\left(\frac{(-1)^n}{2}\right)=\sin^{-1}\left(\frac{1}{2}\right)$ *if n even*

$=\sin^{-1}\left(\frac{-1}{2}\right)$ *if n odd*

$k\pi\pm(-1)^n\frac{\pi}{6}$

$(c)\ \tan^{-1}\left(\sqrt{2}+1\right)=\tan^{-1}\left(\tan 67\frac{1}{2}^{0}\right)=\frac{3\pi}{8}$

$(D)\ \sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$

is holds for $x \in k\pi \pm 1$

4. Conceptual

5. $\tan^{-1}\left(\dfrac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\cdot\frac{1}{8}}\right)+\dfrac{1}{2}\sec^{-1}x=\dfrac{\pi}{8}$

$2\tan^{-1}\left(\dfrac{1}{3}\right)+\sec^{-1}x=\dfrac{\pi}{4}$

$2\tan^{-1}\left(\dfrac{1}{3}\right)+\tan^{-1}\sqrt{x^2-1}=\dfrac{\pi}{4}$

$\tan^{-1}\left(\dfrac{\frac{2}{3}}{1-\frac{1}{9}}\right)+\tan^{-1}\sqrt{x^2-1}=\dfrac{\pi}{4}$

$\tan^{-1}\left(\dfrac{\frac{3}{4}+\sqrt{x^2-1}}{1-\frac{3}{4}\sqrt{x^2-1}}\right)=\dfrac{\pi}{4}$

$3+4\sqrt{x^2-1}=4-3\sqrt{x^2-1}$

$7\sqrt{x^2-1}=1$

$\sqrt{x^2-1}=\dfrac{1}{7}$

$x^2=\dfrac{1}{49}+1$

$x^2=\dfrac{50}{49}$

6. $\sin^{-1}\left(\dfrac{12}{x}\right)+\sin^{-1}\left(\dfrac{5}{x}\right)=\dfrac{\pi}{2}$

$\dfrac{12}{x}\dfrac{\sqrt{x^2-25}}{x}+\dfrac{5}{x}\dfrac{\sqrt{x^2-144}}{x}=1$

$12\sqrt{x^2-25}+5\sqrt{x^2-144}=x^2$

By option verification

$x=13$

$169=169$

7. $t_n=\tan^{-1}\left[\dfrac{1}{1+(\sin x+n)(\sin x+(n+1))}\right]$

$=\tan^{-1}[\sin x+(n+1)]-\tan^{-1}(\sin x+n)$

$f(x)=Tan^{-1}(\sin x+1)-Tan^{-1}(\sin x)+\dots\dots\dots$

$\dots\dots +Tan^{-1}(\sin x+10)-Tan^{-1}(\sin x+9)$

$=Tan^{-1}(\sin x+10)-Tan^{-1}(\sin x)$

$f'(x)=\dfrac{\cos x}{1+(\sin x+10)^2}-\dfrac{\cos x}{1+\sin^2 x}$

$f'(0)=-\dfrac{100}{101}$

8. Given $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0$

Let, $\tan^{-1}x=\theta\Rightarrow x=\tan\theta$

$\Rightarrow\sin\left[2\cos^{-1}\{\cot 2\theta\}\right]=0$

$\Rightarrow\sin\left[2\cos^{-1}\left[\dfrac{1-\tan^2\theta}{2\tan\theta}\right]\right]=0$

$\Rightarrow\sin\left[2\cos^{-1}\left(\dfrac{1-x^2}{2x}\right)\right]=0$

$\Rightarrow\sin\left[\sin^{-1}\left[\sqrt{1-\left[2\left(\dfrac{1-x^2}{2x}\right)^2-1\right]^2}\right]\right]=0$

On simplifying

$\Rightarrow 2\left(\dfrac{1-x^2}{2x}\right)^2-1=\pm1$

$\Rightarrow\left(\dfrac{1-x^2}{2x}\right)=\pm1$ *and* $x=\pm1$

$\Rightarrow x=-1\pm\sqrt{2},\ 1\pm\sqrt{2},\ \pm1$

No. of solutions greater than are equal to one are '2'

9. $x=\tan^{-1}\left\{\dfrac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{40}}\right\}\Rightarrow\tan x=\dfrac{1}{3},\sin x=\dfrac{1}{\sqrt{10}}$

$\cos x=\dfrac{3}{\sqrt{10}}$

10. $\tan^{-1}(2)-\tan^{-1}(1)+\tan^{-1}(3)+\dots.\tan^{-1}(n+1)-\tan^{-1}(n)$

$\tan^{-1}(n+1)-\tan^{-1}(1)$

$x=\dfrac{n}{n+2}$

11. Given $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0$

Let, $\tan^{-1}x=\theta \Rightarrow x=\tan\theta$

$\Rightarrow \sin\left[2\cos^{-1}\left\{\cot 2\theta\right\}\right]=0$

$$\Rightarrow \sin\left[2\cos^{-1}\left[\frac{1-\tan^2\theta}{2\tan\theta}\right]\right]=0$$

$$\Rightarrow \sin\left[2\cos^{-1}\left(\frac{1-x^2}{2x}\right)\right]=0$$

$$\Rightarrow \sin\left[\sin^{-1}\left[\sqrt{1-\left[2\left(\frac{1-x^2}{2x}\right)^2-1\right]^2}\right]\right]=0$$

On simplifying

$$\Rightarrow 2\left(\frac{1-x^2}{2x}\right)^2-1=\pm 1$$

$$\Rightarrow \left(\frac{1-x^2}{2x}\right)=\pm 1 \text{ and } x=\pm 1$$

$\Rightarrow x=-1\pm\sqrt{2},\ 1\pm\sqrt{2},\ \pm 1$

No. of solutions are '2'

12. *Let* $\tan^{-1}(2)=\alpha$

$\tan\alpha=2$

$$x=\sin 2\alpha=2\sin\alpha\cos\alpha=2\left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right)=\frac{4}{5}$$

$$y=\cos 2\beta=2\cos^2\beta-1=\frac{-3}{5}$$

$$z=\sec 2\gamma=\frac{-5}{3}$$

13. conceptual

14. $\theta=2\left(\tan^{-1}\left(\frac{1}{8}\right)+\tan^{-1}\left(\frac{1}{5}\right)\right)+\tan^{-1}\left(\frac{1}{7}\right)$

$$\theta=2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)$$

$$\theta=\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{1}{7}\right)$$

$$\theta=\tan^{-1}(1)=\frac{\pi}{4}$$

$$\tan\frac{\theta}{2}=\tan\frac{\pi}{8}=\sqrt{2}-1$$

$m=2, n=1$

15. $x\in(-1,1)$

16. $\cos^{-1}x=\frac{\pi}{2}-\cos^{-1}3x$

$$\cos^{-1}x+\cos^{-1}3x=\frac{\pi}{2}$$

$$\cos^{-1}(x(3x)-\sqrt{1-x^2}.\sqrt{1-9x^2})=\frac{\pi}{2}$$

$$3x^2-\sqrt{1-x^2}.\sqrt{1-9x^2}=0$$

simplify

17. $\sin^{-1}\left(\frac{12}{5}\right)+\cos^{-1}\left(\frac{4}{5}\right)$

$$=\tan^{-1}\frac{12}{5}+\tan^{-1}\frac{3}{4}$$

$$=\tan^{-1}\left(\frac{63}{-16}\right)$$

$$=\pi-\tan^{-1}\left(\frac{63}{16}\right)$$

18. $\sec^{-1}\frac{x}{a}=\theta=\cos^{-1}\frac{a}{x}$

$$\sec\theta=\frac{x}{a}$$

$$\cos^{-1}\left(\frac{a}{x}\right)-\cos^{-1}\left(\frac{b}{x}\right)=\cos^{-1}\left(\frac{1}{b}\right)-\cos^{-1}\left(\frac{1}{a}\right)$$

$$\cos^{-1}\left(\frac{ab}{x^2}+\sqrt{1-\frac{a^2}{x^2}}.\sqrt{1-\frac{b^2}{x^2}}\right)=\cos^{-1}\left(\frac{1}{ab}+\sqrt{1-\frac{1}{b^2}}.\sqrt{1-\frac{1}{a^2}}\right)$$

simplify

19. Let $\operatorname{cosec}^{-1}\frac{5}{4}=\alpha$

$$\operatorname{cosec}\alpha=\frac{5}{4}$$

$$\sin\alpha=\frac{4}{5} \qquad \cos\alpha=\frac{3}{5}\Rightarrow\alpha=\cos^{-1}\frac{3}{5}$$

$$\sin^{-1}\left(\frac{x}{5}\right)+\alpha=\frac{\pi}{2}$$

$$\sin^{-1}\frac{x}{5}+\cos^{-1}\frac{3}{5}=\frac{\pi}{2}$$

$x=3$

$x+5=8$

20. $\mathrm{Sin}^{-1}x<\mathrm{Cos}^{-1}x$

$$\mathrm{Sin}^{-1} x < \frac{\pi}{2} - \mathrm{Sin}^{-1} x \ \&$$

$$2\mathrm{Sin}^{-1} x < \frac{\pi}{2} \Rightarrow \mathrm{Sin}^{-1} x < \frac{\pi}{4}$$

$$x < \frac{1}{\sqrt{2}} \to (1)$$

Domain of $Sin^{-1}x$ & $Cos^{-1}x$

$$x \in [-1,1] \to (2)$$

from $(1) \& (2)$

$$1 \le x < \frac{1}{\sqrt{2}}$$

21. $\mathrm{Tanh}^{-1}\left(\frac{1}{2}\right) + \mathrm{Sinh}^{-1}\left(\frac{-1}{2\sqrt{2}}\right)$

$$\frac{1}{2}\log_e\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) + \log_e\left(\frac{-1}{2\sqrt{2}} + \sqrt{\left(\frac{-1}{2\sqrt{2}}\right)^2 + 1}\right)$$

$$\frac{1}{2}\log_2(3) + \log_e\left(\frac{-1}{2\sqrt{2}} + \sqrt{\frac{1}{8}+1}\right)$$

$$\log_e 3^{\frac{1}{2}} + \log_e\left(\frac{-1}{2\sqrt{2}} + \frac{3}{2\sqrt{2}}\right)$$

$$= \log_e\sqrt{3} + \log_e\left(\frac{2}{2\sqrt{2}}\right) = \log_e\left(\sqrt{3}\right) + \left[\log 1 - \log\sqrt{2}\right]$$

$$= \log_e\left(\sqrt{3}\right) - \log\left(\sqrt{2}\right) = \log_e\left(\sqrt{\frac{3}{2}}\right)$$

22. $\sin\left(\tan^{-1}\left(\frac{\frac{4}{5}+\frac{1}{9}}{1-\frac{4}{5}\frac{1}{9}}\right)\right) + \tan^{-1}\left(\frac{\frac{4}{3}-\frac{1}{7}}{1+\frac{4}{3}\frac{1}{7}}\right)$

$$\sin\left(\tan^{-1}\left(\frac{41}{41}\right) + \tan^{-1}\left(\frac{25}{25}\right)\right)$$

$$\sin\left(\frac{\pi}{2}\right) = 1$$

23. $f(x) = \tan\left(\tan^{-1}\left(\frac{2-1}{1+2(1)}\right)\right) + \tan^{-1}\left(\frac{3-2}{1+3(2)}\right)$

$$+\ldots.\tan^{-1}\left(\frac{n+1-n}{1+(n+1)n}\right)$$

$$= \tan\begin{pmatrix}\tan^{-1}(2) - \tan^{-1}(1) + \tan^{-1}(3) - \tan^{-1}(2) \\ + \ldots + \tan^{-1}(n+1) - \tan^{-1}(n)\end{pmatrix}$$

$$= \tan\left(\tan^{-1}(n+1) - \tan^{-1}(1)\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{n+1-1}{1+(n+1)1}\right)\right) = \tan\left(\tan^{-1}\left(\frac{n}{n+2}\right)\right) = \frac{n}{n+2}$$

$$f(2021) = \frac{2021}{2021+2} = \frac{2021}{2023}$$

24. $2\sin^{-1} x + \sin^{-1}\left(2x\sqrt{1-x^2}\right) + 3\cos^{-1} x$

$$-\cos^{-1}\left(4x^3 - 3x\right)$$

Put $x = \cos\theta$

$$2\sin^{-1} x + \sin^{-1}\sin 2\theta + 3\theta - 3\theta$$

$$2.\sin^{-1} x + 2\theta$$

$$2\left(\sin^{-1} x + \cos^{-1} x\right)$$

$$2\frac{\pi}{2} = \pi$$

25. $2\tan^{-}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$$\mathrm{Tan}^{-1}\left(\frac{3}{4}\right) + \mathrm{Tan}^{-1}\left(\frac{1}{7}\right)$$

$$\mathrm{Tan}^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}.\frac{1}{7}}\right) = \mathrm{Tan}^{-1}(1) = \pi/4$$