

18. LIMITS

1. $\lim_{x \rightarrow \infty} \left(\frac{2x^2 + 3x + 4}{x^2 - 3x + 5} \right)^{\frac{3|x|+1}{2|x|-1}} =$

[AP EAMCET 17-09-20_Shift-1]

- | | |
|------------------|----------------|
| 1. $\frac{3}{2}$ | 2. $2\sqrt{2}$ |
| 3. 3 | 4. $\sqrt{2}$ |

2. If $a > 0, n \in \mathbb{R}$ then $\lim_{x \rightarrow a} x^n =$

[AP EAMCET 17-09-20_Shift-1]

- | | |
|---------------|---------------|
| 1. na^n | 2. $(n-1)a^n$ |
| 3. na^{n-1} | 4. a^n |

3. $\lim_{x \rightarrow 0} \frac{8}{\sin^8 x} \left\{ 1 - \cos\left(\frac{x^2}{2}\right) - \cos\left(\frac{x^2}{4}\right) + \cos\left(\frac{x^2}{2}\right)\cos\left(\frac{x^2}{4}\right) \right\} =$

[AP EAMCET 17-09-20_Shift-1]

- | | |
|-------------------|-------------------|
| 1. $\frac{1}{16}$ | 2. $\frac{1}{32}$ |
| 3. $\frac{1}{64}$ | 4. $\frac{1}{8}$ |

4. $\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\left(\cos \frac{x}{2} \right) \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)} =$

[AP EAMCET 17-09-20_Shift-2]

- | | |
|-------------------------|--------------------------|
| 1. $\frac{3}{\sqrt{2}}$ | 2. $\frac{-1}{\sqrt{2}}$ |
| 3. $\frac{1}{\sqrt{2}}$ | 4. $\frac{5}{\sqrt{2}}$ |

5. If $\lim_{x \rightarrow \infty} \left(1 + \frac{p}{x} \right)^{qx} = e^9$ where $p, q \in \mathbb{N}$ then $p + q =$

[AP EAMCET 17-09-20_Shift-2]

- | | |
|-------|-------|
| 1. 6 | 2. 9 |
| 3. 81 | 4. 18 |

6. $\lim_{x \rightarrow 0} (1 + 3x)^{2/x} =$

[AP EAMCET 18-09-20_Shift-1]

- | | |
|-------------|--------------|
| 1. 6 | 2. e^6 |
| 3. e^{-6} | 4. $e^{1/6}$ |

7. $\lim_{n \rightarrow \infty} \frac{n!}{(n+1)! - n!} =$

[AP EAMCET 18-09-20_Shift-1]

- | | |
|------|-------|
| 1. 1 | 2. -1 |
| 3. 2 | 4. 0 |

8. $\lim_{x \rightarrow 0} \frac{a^x - 1}{\sin(x)} =$ [AP EAMCET 21-09-20_Shift-1]

- | | |
|--------------|--------------------------|
| 1. $\log(a)$ | 2. $\frac{1}{2} \log(a)$ |
| 3. 0 | 4. 1 |

9. $\lim_{n \rightarrow \infty} \frac{2^2 + 4^2 + 6^2 + \dots + (2n)^2}{n^3} =$

[AP EAMCET 21-09-20_Shift-1]

- | | |
|------------------|------------------|
| 1. $\frac{2}{3}$ | 2. $\frac{4}{3}$ |
| 3. $\frac{3}{2}$ | 4. $\frac{8}{7}$ |

10. $\lim_{x \rightarrow 1} \left((1-x) \tan \left(\frac{\pi x}{2} \right) \right) =$

[AP EAMCET 21-09-20_Shift-2]

- | | |
|--------------------|--------------------|
| 1. $\frac{1}{\pi}$ | 2. $\frac{3}{\pi}$ |
| 3. $\frac{4}{\pi}$ | 4. $\frac{2}{\pi}$ |

11. Find the value of $\lim_{x \rightarrow 0} \frac{\sin(x^m)}{(\sin x)^n}$, given that $n < m$

[AP EAMCET 21-09-20_Shift-2]

- | | |
|------|-------------|
| 1. 2 | 2. 1 |
| 3. 0 | 4. ∞ |

12. If $\lim_{x \rightarrow 3} \left(\frac{x^n - 3^n}{x - 3} \right) = 108$ and $n \in \mathbb{N}$ then the value

of 'n' is [AP EAMCET 22-09-20_Shift-2]

- | | |
|------|------|
| 1. 3 | 2. 6 |
| 3. 5 | 4. 4 |

13. $\lim_{x \rightarrow 0} \frac{\left(1 + \frac{x}{2} \right)^{\frac{5}{7}} - 1}{x} =$

[AP EAMCET 22-09-20_Shift-2]

- | | |
|-------------------|-------------------|
| 1. $\frac{5}{7}$ | 2. $\frac{10}{7}$ |
| 3. $\frac{5}{14}$ | 4. $\frac{5}{17}$ |

14. $\lim_{x \rightarrow \infty} (1 + \frac{2}{x})^{3x} =$ [AP EAMCET 23-09-20_Shift-1]

1. e^6 2. e^3
3. e^2 4. e

15. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} =$

1. 0 2. -1
3. $\frac{1}{2}$ 4. 1

16. $\lim_{n \rightarrow \infty} \left\{ n - \sqrt{n^2 - 4n} \right\} =$

1. 0 2. 2
3. 4 4. 1

17. $f: R^+ \rightarrow R$ be an increasing function such that $f(x) > 0$ for all x , if

$\lim_{x \rightarrow \infty} \frac{f(9x)}{f(3x)} = 1$, then $\lim_{x \rightarrow \infty} \frac{f(6x)}{f(3x)} =$

[TS EAMCET 09-09-20_Shift-1]

1. 1 2. 2
3. $\frac{3}{2}$ 4. $\frac{2}{3}$

18. $\lim_{x \rightarrow 0} \frac{x^4 + x^3 + x^2}{\sin^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right) \cdot \tan^{-1} x} =$

[TS EAMCET 09-09-20_Shift-2]

1. $\frac{1}{\sqrt{2}}$ 2. 0
3. 1 4. $\frac{-1}{\sqrt{2}}$

19. If $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty$ and

$\lim_{x \rightarrow 0} \frac{\log(1+x)^{(1+x)}}{x^2} - \frac{1}{x} = k$, Then $12k =$

[TS EAMCET 10-09-20_Shift-1]

1. 1 2. 3
3. 6 4. 9

20. $\lim_{x \rightarrow 0} \frac{1 - \cos(x^2 + \pi(x+2))}{x^2} =$

[TS EAMCET 10-09-20_Shift-2]

1. $\frac{\pi}{2}$ 2. $\frac{\pi^2}{4}$
3. $\frac{\pi^2}{2}$ 4. $\frac{\pi}{4}$

21. $\lim_{x \rightarrow 0} \frac{1 - \cos(1 - \cos x)}{\sin^4 x} =$

[TS EAMCET 11-09-20_Shift-2]

1. $\frac{1}{2}$ 2. $\frac{1}{4}$
3. $\frac{1}{6}$ 4. $\frac{1}{8}$

22. $\lim_{z \rightarrow 1} \frac{z^3 - 1}{z^6 - 1} =$ [AP EAMCET 19-08-2021_Shift-1]

1. -1 2. 1
3. 2 4. -2

23. $\lim_{n \rightarrow \infty} \frac{n(2n+1)^2}{(n+2)(n^2+3n-1)} =$

[AP EAMCET 20-08-2021_Shift-1]

1. 0 2. 4
3. 2 4. ∞

24. If $\lim_{x \rightarrow \infty} \left(\frac{11x^3 - 3x + 4}{13x^3 - 5x^2 - 7} \right) = \frac{a}{b}$ then the value of

$a+b$ equals

[AP EAMCET 20-08-2021_Shift-2]

1. 11 2. 13
3. 8 4. 24

25. $\lim_{x \rightarrow 1} \frac{(1-x)(1-x^2) \dots (1-x^{2^n})}{\{(1-x)(1-x^2) \dots (1-x^n)\}^2} = _, \forall n \in \mathbb{N}$

[AP EAMCET 20-08-2021_Shift-2]

1. ${}^{2n}P_n$ 2. ${}^{2n}C_n$
3. $(2n)!$ 4. $\frac{(2n)!}{n!}$

26. The limit $\lim_{x \rightarrow 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1} \underline{\hspace{1cm}}$

[AP EAMCET 23-08-2021_Shift-1]

1. exists and is equal to $\sqrt{2}$
2. exists and is equal to $-\sqrt{2}$
3. does not exist
4. exists and is equal to $(1/2)$

27. $\lim_{x \rightarrow 0} \left(\frac{\sin ax}{\tan bx} \right) =$ [APEAMCET 25-08-2021_Shift-1]

1. ab 2. $\frac{a}{b}$
3. $\frac{b}{a}$ 4. 1

$$28. \lim_{x \rightarrow 0} \frac{\tan(x) + 4 \tan(2x) - 3 \tan(3x)}{x^2 \tan(x)} =$$

[AP EAMCET 23-08-2021_Shift-2]

1. 8
2. -8
3. 16
4. -16

$$29. \lim_{x \rightarrow 0} \frac{8}{x^8} \left(1 - \cos\left(\frac{x^2}{2}\right) - \cos\left(\frac{x^2}{4}\right) + \cos\left(\frac{x^2}{2}\right) \cdot \cos\left(\frac{x^2}{4}\right) \right) =$$

[AP EAMCET 23-08-2021_Shift-2]

1. $\frac{1}{4}$
2. $\frac{1}{8}$
3. $\frac{1}{16}$
4. $\frac{1}{32}$

$$30. \text{ If } \lim_{n \rightarrow \infty} \frac{1 - (10)^n}{1 + (10)^{n+1}} = \frac{-\alpha}{10}, \text{ then } \alpha =$$

[AP EAMCET 24-08-2021_Shift-1]

1. 0
2. -1
3. 1
4. 2

$$31. \lim_{x \rightarrow \infty} \left(\frac{x+6}{x+1} \right)^{x+4} =$$

[AP EAMCET 24-08-2021_Shift-1]

1. e^4
2. e^6
3. e^5
4. e

$$32. \lim_{x \rightarrow 0^+} \frac{x \sin^{-1}\left(\frac{2x}{1+x^2}\right)}{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)} =$$

[AP EAMCET 24-08-2021_Shift-1]

1. $\frac{1}{2}$
2. $\frac{1}{3}$
3. $\frac{1}{4}$
4. $\frac{1}{6}$

$$33. \lim_{x \rightarrow (-3)} \left(\frac{\sin^{-1}(x+3)}{x^2+3x} \right) =$$

[AP EAMCET 25-08-2021_Shift-1]

1. 0
2. ∞
3. -3
4. $-\frac{1}{3}$

$$34. \lim_{x \rightarrow \infty} \left(\frac{2 + \sin x}{x^2 + 3} \right) =$$

[AP EAMCET 25-08-2021_Shift-2]

1. 0
2. 1
3. -1
4. ∞

35. If $[.]$ here denotes the greatest integer function,

$$\lim_{x \rightarrow 0} x^7 \left[\frac{1}{x^3} \right] =$$

[AP EAMCET 25-08-2021_Shift-2]

1. 1
2. 0
3. -1
4. Does not exist

$$36. \text{ If } \lim_{x \rightarrow 0} \left\{ 1 + x \log(1+a^2) \right\}^{1/x} = 2a \sin^2 \theta, a > 0$$

and $\theta \in R$, then _____

[AP EAMCET 25-08-2021_Shift-2]

1. $\theta = n\pi \pm \frac{\pi}{2}, (n \in Z)$
2. $\theta = 2n\pi \pm \frac{\pi}{2}, (n \in Z)$
3. $\theta = n\pi + \frac{\pi}{2}, (n \in Z)$
4. $\theta = n\pi \pm \frac{\pi}{4}, (n \in Z)$

$$37. \lim_{x \rightarrow 0} \frac{x^2 2^x - x^2 \sin x - x^2}{3^x + \cos x - 3^x \cos x - 1} =$$

[TS EAMCET 04-08-2021_Shift-2]

1. $\frac{1}{\log 3} (\log 2 - 1)$
2. $\frac{4}{\log 3} (1 - \log 2)$
3. $\frac{4}{\log 3} (\log 2 - 1)$
4. $\frac{2}{\log 3} (\log 2 - 1)$

$$38. \text{ If } \lim_{x \rightarrow 2} \frac{3x^2 - ax + 5b}{x - 2} = 17, \text{ then } ab =$$

[TS EAMCET 04-08-2021_Shift-2]

1. -34
2. -25
3. -22
4. 22

$$39. \lim_{x \rightarrow 0} \frac{(1 - \cos 2x)}{x \tan 2x + \frac{2x}{3} \tan 3x} =$$

[TS EAMCET 04-08-2021_Shift-2]

1. -6
2. $\frac{1}{2}$
3. 0
4. $-\frac{6}{5}$

40. If $f(x) = \tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right)$ then

$$\lim_{x \rightarrow \frac{1}{2}} \frac{2 \left[f(x) - f\left(\frac{1}{2}\right) \right]}{2x-1} =$$

[TS EAMCET 04-08-2021_Shift-2]

- | | |
|--------------------------------|-------------------------|
| 1. $\frac{1}{\sqrt{2}}$ | 2. $\frac{\sqrt{3}}{2}$ |
| 3. $\frac{\sqrt{2}}{\sqrt{3}}$ | 4. $\frac{1}{\sqrt{3}}$ |

41. $\lim_{x \rightarrow 1} \frac{\log x}{(1-x)} =$

[TS EAMCET 04-08-2021_Shift-1]

- | | |
|------|-------------------|
| 1. 1 | 2. -1 |
| 3. 0 | 4. $-\frac{1}{2}$ |

42. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$, where $[x]$ denotes the greatest integer less than or equal to x , then $\lim_{x \rightarrow 0^-} f(x)$

[TS EAMCET 05-08-2021_Shift-1]

1. Exist and equal to 1
2. Exist and equal to $\sin 1$
3. Exist and equal to $-\sin 1$
4. Does not exist

43. $\lim_{x \rightarrow -\infty} \frac{3|x|^3 - x^2 + 2|x| - 5}{-5|x|^3 + 3x^2 - 2|x| + 7} =$

[TS EAMCET 05-08-2021_Shift-1]

- | | |
|------------------|-------------------|
| 1. $\frac{3}{5}$ | 2. $-\frac{5}{7}$ |
| 3. $\frac{5}{7}$ | 4. $-\frac{3}{5}$ |

44. If $f(x) = -(\sin^2 x + \cos^5 x)$, then $\lim_{x \rightarrow 0} \frac{1}{x} f'(x)$

[TS EAMCET 05-08-2021_Shift-2]

1. Exist and is equal to 0
2. Exist and is equal to 7
3. Exist and is equal to 3
4. Does not exist

45. $\lim_{n \rightarrow \infty} P \left(1 + \frac{r}{100n} \right)^n =$

[TS EAMCET 05-08-2021_Shift-2]

- | | |
|--------------------------|---|
| 1. P | 2. $P \left(1 + \frac{r}{100} \right)^r$ |
| 3. $P e^{\frac{r}{100}}$ | 4. $P e^{\frac{r}{100}}$ |

46. If a, b are roots of the equation

$px^2 + qx + r = 0$, then

$$\lim_{x \rightarrow b} \frac{1 - \cos 2(px^2 + qx + r)}{2(px - pb)^2} =$$

[TS EAMCET 06-08-2021_Shift-2]

- | | |
|-------------------------|----------------------|
| 1. $\frac{1}{2}(b-a)^2$ | 2. $(a+b)^2$ |
| 3. $\frac{1}{2}$ | 4. $a^2 - 2ab + b^2$ |

47. If $f: R \rightarrow R$ defined as

$$f(x) = \frac{x^3 + 2x^2 + x + 2}{x^2 + x - 2} \text{ (when } x \neq -2 \text{) is}$$

continuous at $x = -2$, then $f(-2) =$

[TS EAMCET 06-08-2021_Shift-2]

- | | |
|------|-------------------|
| 1. 5 | 2. $-\frac{5}{3}$ |
| 3. 2 | 4. $\frac{3}{5}$ |

48. $\lim_{x \rightarrow 0^-} \frac{\sqrt{\frac{1}{2}(1 - \cos^2 x)}}{x} =$

[TS EAMCET 06-08-2021_Shift-1]

- | | |
|-------------------------|--------------------------|
| 1. $\frac{1}{\sqrt{2}}$ | 2. $-\frac{1}{\sqrt{2}}$ |
| 3. -1 | 4. Does not exist |

49. $\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3} =$

[TS EAMCET 06-08-2021_Shift-1]

- | | |
|-------|-------------------|
| 1. 1 | 2. 0 |
| 3. -1 | 4. Does not exist |

50. Let $f: R^+ \rightarrow R^+$ be a function satisfying
 $f(x) - x = \lambda$ (constant), $\forall x \in R^+$ and
 $f(xf(y)) = f(xy) + x, \forall x, y \in R^+$.

$$\text{Then } \lim_{x \rightarrow 0} \frac{(f(x))^{\frac{1}{3}} - 1}{(f(x))^{\frac{1}{2}} - 1} =$$

[AP EAMCET 04-07-2022_Shift-1]

1. $\frac{1}{3}$ 2. 0
 3. $\frac{2}{3}$ 4. 1

51. If $\lim_{x \rightarrow 0} \frac{|x|}{\sqrt{x^4 + 4x^2 + 5}} = k,$

$$\lim_{x \rightarrow 0} x^4 \sin\left(\frac{1}{3\sqrt{x}}\right) = l, \text{ Then } k + l =$$

[AP EAMCET 04-07-2022_Shift-1]

1. 0 2. 1
 3. -1 4. 5

52. If $\lim_{n \rightarrow \infty} x^n \log_e x = 0$, then $\log_x 12 =$

[AP EAMCET 04-07-2022_Shift-1]

1. Negative
 2. Positive
 3. Zero
 4. Any value between -1 and 1

53. If $[.]$ denotes greatest integer function, then

$$\lim_{x \rightarrow \frac{-3}{5}} \frac{1}{x} \left[\frac{-1}{x} \right] =$$

[AP EAMCET 04-07-2022_Shift-2]

1. $-5/3$ 2. $5/3$
 3. $10/3$ 4. $-10/3$

54. If l, m ($l < m$) are roots of $ax^2 + bx + c = 0$, then

$$\lim_{x \rightarrow \alpha} \frac{|ax^2 + bx + c|}{ax^2 + bx + c} =$$

[AP EAMCET 04-07-2022_Shift-2]

1. $\frac{|a|}{a}, \forall \alpha \in R$ 2. $\frac{-|a|}{a}, \text{ when } \alpha \notin (l, m)$
 3. $\frac{-|a|}{a}, \text{ when } \alpha \in (l, m)$ 4. $\frac{|a|}{a}, \alpha \in (l, m)$

55. Let $f(x) = \begin{cases} \frac{1}{|x|}, & \text{for } |x| > 1 \\ ax^2 + b, & \text{for } |x| \leq 1 \end{cases}$ If $\lim_{x \rightarrow 1} f(x)$ and

$\lim_{x \rightarrow -1} f(x)$ exist, then the possible values for a and b

are [AP EAMCET 04-07-2022_Shift-2]

1. $a=b=1$ 2. $a=-1/2, b=-3/2$
 3. $a=3/2, b=-1/2$ 4. $a=1/2, b=-3/2$

56. $\lim_{x \rightarrow -\infty} \log_e (\cosh x) + x =$

[AP EAMCET 05-07-2022_Shift-1]

1. $\log 2$ 2. $-\log 2$
 3. $\log\left(\frac{1}{2}\right) + 2$ 4. $\log\left(\frac{1}{2}\right) - 2$

57. If a, b, c are three distinct real numbers and

$$\lim_{x \rightarrow \infty} \frac{(b-c)x^2 + (c-a)x + (a-b)}{(a-b)x^2 + (b-c)x + (c-a)} = \frac{1}{2} \text{ then } a + 2c =$$

[AP EAMCET 05-07-2022_Shift-1]

1. b 2. $2b$
 3. $3b$ 4. $4b$

58. $\lim_{x \rightarrow -\infty} \frac{3|x| - x}{|x| - 2x} - \lim_{x \rightarrow 0} \frac{\log(1+x^3)}{\sin^3 x} =$

[AP EAMCET 05-07-2022_Shift-1]

1. $\frac{1}{3}$ 2. $-\frac{1}{4}$
 3. 2 4. $-\frac{5}{3}$

59. $\lim_{n \rightarrow \infty} \sqrt{2} \left[\frac{(2+\sqrt{2})^n + (2-\sqrt{2})^n}{(2+\sqrt{2})^n - (2-\sqrt{2})^n} \right] =$

[AP EAMCET 05-07-2022_Shift-2]

1. $2 + \sqrt{2}$ 2. $2 - \sqrt{2}$
 3. 1 4. $\sqrt{2}$

60. If $a > 0$, $[.]$ denotes greatest integer function

$$\text{and } \lim_{x \rightarrow a^-} \left(\frac{|x|^3}{a} - \left[\frac{x}{a} \right]^3 \right) = k, \lim_{x \rightarrow a^+} \left(\frac{|x|^3}{a} - \left[\frac{x}{a} \right]^3 \right) = l$$

then [AP EAMCET 05-07-2022_Shift-2]

1. $k=l$ 2. $k-l=1$
 3. $l-k=1$ 4. $l=a^2, k \text{ does not exist}$

$$61. \lim_{n \rightarrow \infty} \frac{A + e^{nx}}{x + Ae^{nx}} =$$

[AP EAMCET 05-07-2022_Shift-2]

1. A/x , when $x < 0$ 2. 1, when $x > 0$
3. 0, when $\forall x \in R$ 4. A , when $x = 0$

$$62. \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r+2}{r(r+1)(r+3)} =$$

[AP EAMCET 06-07-2022_Shift-1]

1. $\frac{29}{36}$ 2. $\frac{1}{36}$
3. $\frac{5}{36}$ 4. $\frac{23}{36}$

$$63. \text{ If } a > 0, b > 0 \text{ then } \lim_{n \rightarrow \infty} \left(\frac{a + b^{1/n} - 1}{a} \right)^n =$$

[AP EAMCET 06-07-2022_Shift-1]

1. a^b 2. b^a
3. $b^{1/a}$ 4. $a^{1/b}$

$$64. \lim_{x \rightarrow \frac{\pi}{6}} \frac{3 \sin x - \sqrt{3} \cos x}{6x - \pi}$$

[AP EAMCET 06-07-2022_Shift-1]

1. $\frac{-1}{\sqrt{3}}$ 2. $\frac{1}{\sqrt{3}}$
3. $\frac{1}{\sqrt{2}}$ 4. $\frac{-1}{\sqrt{2}}$

$$65. \lim_{n \rightarrow \infty} \sum_{r=1}^n \cot^{-1} \left(r^2 + \frac{3}{4} \right) =$$

[AP EAMCET 06-07-2022_Shift-2]

1. $\cot^{-1} 2$ 2. $\cot^{-1} \frac{1}{3}$
3. $\tan^{-1} 2$ 4. $\tan^{-1} \frac{1}{3}$

$$66. \text{ If } n > 0 \text{ and } \lim_{x \rightarrow 0} \frac{((a-n)nx - \tan x) \sin nx}{x^2} = 0,$$

then minimum value of a is

[AP EAMCET 06-07-2022_Shift-2]

1. 1 2. 2
3. 3 4. -1

$$67. \lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right)^{\frac{x}{x+1-e^x}} =$$

[AP EAMCET 06-07-2022_Shift-2]

1. e 2. e^{-1}
3. e^2 4. e^{-2}

$$68. \text{ If } \lim_{x \rightarrow 0^+} x^2 \left(\frac{\frac{1}{e^x} - e^{-\frac{1}{x}}}{\frac{1}{e^x} + e^{-\frac{1}{x}}} \right) = k \text{ and}$$

$$\lim_{x \rightarrow 0^-} x^2 \left(\frac{\frac{1}{e^x} - e^{-\frac{1}{x}}}{\frac{1}{e^x} + e^{-\frac{1}{x}}} \right) = l, \text{ then}$$

[AP EAMCET 07-07-2022_Shift-1]

1. $k=1$ 2. $k=1, l=-1$
3. $k=-1, l=1$ 4. $k \neq l \neq \pm 1$

$$69. \text{ If } f(x) = \frac{e^x - 1}{\frac{1}{e^x} + 1}, \text{ then}$$

[AP EAMCET 07-07-2022_Shift-1]

1. $\lim_{x \rightarrow 0} f(x) = 0$ 2. $\lim_{x \rightarrow \infty} f(x) = 1$
3. $\lim_{x \rightarrow 0} f(x) = -1$ 4. $\lim_{x \rightarrow \infty} f(x) = 0$

$$70. \lim_{x \rightarrow 0} \frac{\sqrt{11+|x|} - 6\sqrt{2+|x|}}{6 - 2\sqrt{2+|x|}} =$$

[AP EAMCET 07-07-2022_Shift-1]

1. -1 2. $-\frac{1}{2}$
3. $\frac{\sqrt{11-6\sqrt{2}}}{3-\sqrt{2}}$ 4. $\frac{1}{2}$

$$71. \lim_{x \rightarrow 0} x^3 \left\{ \sqrt{x^2 + \sqrt{x^4 + 1}} - \sqrt{2}x \right\} =$$

[AP EAMCET 07-07-2022_Shift-2]

1. $\sqrt{2}$ 2. $\frac{1}{2\sqrt{2}}$
3. $\frac{1}{4\sqrt{2}}$ 4. $\frac{1}{\sqrt{2}}$

$$72. \lim_{x \rightarrow -a} \frac{x^7 + a^7}{x + a} = 7 \Rightarrow a =$$

[AP EAMCET 07-07-2022_Shift-2]

1. ± 7 2. ± 6
3. ± 1 4. ± 2

$$73. \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \tan \frac{x}{2}}{1 + \tan \frac{x}{2}} \cdot \frac{1 - \sin x}{(\pi - 2x)^3} =$$

[AP EAMCET 07-07-2022_Shift-2]

1. $\frac{1}{32}$
2. 0
3. $\frac{1}{16}$
4. $\frac{1}{8}$

74. If $\lim_{n \rightarrow \infty} x_n$ exists and is finite,

$$x_1 = 2, x_{n+1} = \frac{a + bx_n}{b + cx_n} \forall n \in N \text{ and } c > b > a > 0$$

then $\lim_{n \rightarrow \infty} x_n =$

[AP EAMCET 08-07-2022_Shift-1]

1. $\sqrt{ab/c}$
2. $\sqrt{a/c}$
3. $\sqrt{c/b}$
4. $\sqrt{a/b}$

75. The integral value of n for which

$$\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n} \text{ is a finite non zero real number is}$$

[AP EAMCET 08-07-2022_Shift-1]

1. 4
2. 3
3. 2
4. 1

$$76. \lim_{x \rightarrow 0} \frac{x^2 \log(\cos x)}{\log(1 + x^2)} =$$

[AP EAMCET 08-07-2022_Shift-1]

1. 0
2. 1
3. -1
4. ∞

77. If $[.]$ denotes greatest integer function, then

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{[\sin x] - [\cos x] + 1}{2} =$$

[AP EAMCET 08-07-2022_Shift-2]

1. 0
2. $-\frac{1}{2}$
3. $\frac{1}{2}$
4. 1

$$78. \text{ If } A \neq 0 \text{ and } x > 0, \text{ then } \lim_{n \rightarrow \infty} \frac{\cos x - e^{nx}}{1 - Ae^{nx}} =$$

[AP EAMCET 08-07-2022_Shift-2]

1. Does not exist
2. 1
3. $\frac{\cos x}{A}$
4. $\frac{1}{A}$

$$79. \text{ Let } f(x) = \lim_{y \rightarrow \infty} y \left(x^{\frac{1}{y}} - 1 \right) \text{ and}$$

$$2022 f\left(\frac{1}{x}\right) + P f(x) = f(x^2), \text{ then } P =$$

[AP EAMCET 08-07-2022_Shift-2]

1. 2020
2. 2021
3. 2023
4. 2024

$$80. \lim_{x \rightarrow 3^-} \frac{x^3 - 3x^2 - 4x + 12}{2x^3 - 7x^2 + 2x + 3} =$$

[TS EAMCET 18-07-2022_Shift-1]

1. 0
2. ∞
3. $\frac{5}{14}$
4. $\frac{6}{13}$

$$81. \lim_{x \rightarrow 0} \frac{2^{2x} - 2^{x+1} + 2 - \cos 2x}{x^2} =$$

[TS EAMCET 18-07-2022_Shift-1]

1. $2 + \log 2$
2. $2 + (\log 2)^2$
3. $2 + (\log 4)^2$
4. $2 + \log 4$

$$82. \lim_{x \rightarrow 0} \frac{\tan^2(\pi \sec^4 x)}{\pi^2 x^4} =$$

[TS EAMCET 19-07-2022_Shift-I]

1. 0
2. 4
3. 1
4. 16

$$83. \lim_{x \rightarrow 0} \left(\frac{4!}{x^8} \left(1 - \cos \frac{x^2}{3} - \cos \frac{x^2}{4} + \cos \frac{x^2}{3} \cos \frac{x^2}{4} \right) \right) =$$

[TS EAMCET 19-07-2022_Shift-I]

1. 8
2. $\frac{1}{6}$
3. $\frac{1}{24}$
4. $\frac{2}{3}$

84. Let $f(x) = \sin x, g(x) = \cos x, h(x) = x^2$ then

$$\lim_{x \rightarrow 1} \frac{f(g(h(x))) - f(g(h(1)))}{x-1} =$$

[TS EAMCET 19-07-2022_Shift-1]

1. 0 2. $-2 \sin 1 \cos(\cos 1)$
 3. ∞ 4. $-2 \sin 1 \cos 1$

85.
$$\lim_{x \rightarrow 2} \left[(x^2 - 4x + 4) \cos\left(\frac{2}{x-2}\right) + \frac{x^2 - 4}{x^3 - 2x - 4} \right] =$$

[TS EAMCET 19-07-2022_Shift-2]

1. 0 2. ∞
 3. 1 4. $\frac{2}{5}$

86.
$$\lim_{x \rightarrow 0} \frac{\tan 2x - 2 \tan x}{(1 - \cos x)(2^x - 1)} =$$

[TS EAMCET 19-07-2022_Shift-2]

1. $\frac{2}{\log 2}$ 2. $\frac{1}{\log 4}$
 3. $4 \log 2$ 4. $\frac{4}{\log 2}$

87. Let $A = (a_{ij})$ be an $n \times n$ matrix defined by

$$a_{ij} = \begin{cases} k^i, & \forall i = j \\ 0, & \text{otherwise} \end{cases} \text{ . If } m = \text{trace of } A$$

and $\lim_{k \rightarrow 1} \frac{n-m}{1-k} = 171$ then the value of n is

[TS EAMCET 18-07-2022_Shift-2]

1. 18 2. 23
 3. 35 4. 42

88.
$$\lim_{x \rightarrow \infty} x^3 \left[\sqrt{x^2 + \sqrt{x^4 + 1}} - \sqrt{2x} \right] =$$

[TS EAMCET 18-07-2022_Shift-2]

1. 0 2. 1
 3. $\frac{1}{4\sqrt{2}}$ 4. $\frac{3}{2\sqrt{2}}$

89. Let $[x]$ denote the greatest integer less than or equal to x and $f(x) = 2x - [2x]$. If

$$\lim_{x \rightarrow 2^-} f(x) = l_1 \text{ and } \lim_{x \rightarrow 2^+} f(x) = l_2 \text{ then } l_1 + l_2 =$$

[TS EAMCET 20-07-2022_Shift-1]

1. 1 2. 2
 3. 0 4. 4

90.
$$\lim_{x \rightarrow 0} \frac{(2^x - 1)(1 + \sin x)^{\frac{2}{\sin x}}}{\log(1 + 2x)} =$$

[TS EAMCET 20-07-2022_Shift-1]

1. $e^2 \log 4$ 2. $e \log \sqrt{2}$
 3. $e^2 \log 2$ 4. $e^2 \log \sqrt{2}$

91.
$$\lim_{x \rightarrow 2} \frac{x^3 - x^2 - x - 2}{2x^3 - 3x^2 - 3x + 2} =$$

[TS EAMCET 20-07-2022_Shift-2]

1. 0 2. ∞
 3. $\frac{5}{7}$ 4. $\frac{7}{9}$

92.
$$\lim_{x \rightarrow 0} \frac{4[\sin(2022x) - \sin(2020x)]}{x[\cos(2022x) + 2\cos(2021x) + \cos(2020x)]} =$$

[TS EAMCET 20-07-2022_Shift-2]

1. 1 2. 2
 3. 2020 4. 2021

93. Let $f(x) = |x - 3| + |x + 5|$ and

$$A = \left\{ a \in \mathbb{R} / \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \text{ exists} \right\}.$$

Then the number of real numbers which are in $(-\infty, -3) \cup (5, \infty)$ but not in A is

[16th May 2023 Shift 1]

1. 2 2. 0
 3. 1 4. 3

94.
$$\lim_{x \rightarrow 1} \left(\lim_{y \rightarrow \infty} y(ex)^{1/y} - 1 \right) =$$

[16th May 2023 Shift 1]

1. e 2. 0
 3. 1 4. -1

95. Assertion (A): $\lim_{x \rightarrow 0} \frac{1}{x} = \infty$

Reason (R) : As the value of x

decreases, the value of $\frac{1}{x}$ increases

[16th May 2023 Shift 2]

1. Both A, R are true, and R is correct explanation of A
 2. A, R are both true and R is not correct explanation of A
 3. A is true and R is false
 4. A is false and R is true

96. If $f(x) = \begin{vmatrix} 1 & 0+x & 36+x^2 \\ 0 & x-3 & 3x^2-27 \\ 0 & 2x-4 & 8x^2-32 \end{vmatrix}$,
then $\lim_{x \rightarrow 1} \frac{f(x)}{f(-x)} =$ [17th May 2023 Shift 1]

1. 2 2. -1
3. 0 4. 1

97. $f(x)$ is differentiable on R and $f'(m) \neq 0, m \in R$.

If $\lim_{x \rightarrow m} \frac{x f(m) - m f(x)}{x - m} + f'(m) = f(m)$, then $m =$
[17th May 2023 Shift 1]

1. 0 2. -1
3. 1 4. 2

98. If $\lim_{x \rightarrow 2} \frac{1 + \sqrt{1 + 4 \log_2 x}}{2 + (2x + \sin^2 x + 2 \cos x)(2x - 4)} = m$,

then $m(m-1) =$ [17th May 2023 Shift 2]

1. 0 2. $\log_2 e$
3. 1 4. $\frac{1 + \sqrt{3}}{2}$

99. If $f(x) = \frac{1 - x + \sqrt{9x^2 + 10x + 1}}{2x}$, then $\lim_{x \rightarrow -1^-} f(x) =$

[18th May 2023 shift -1]

1. 1 2. -1
3. 0 4. -1/5

100. $\lim_{x \rightarrow 9} \frac{(2.5)^{81-x^2} - (0.4)^{x+9}}{x+9} =$

[18th May 2023 Shift 2]

1. $18 \log(2.5) + \log(0.4)$ 2. $\log(2.5) - \log(0.4)$
3. $18(\log(2.5) + \log(0.4))$ 4. $-19 \log(0.4)$

101. Let $S_n = 1 + 3x + 9x^2 + 27x^3 \dots n$ terms and

$-\frac{1}{3} < x < \frac{1}{3}$. If $\lim_{n \rightarrow \infty} S_n = f(x)$, then

[18th May 2023 Shift 2]

1. 0 2. $\frac{1}{3}$
3. 1 4. -1

102. Let $[.]$ denote the greatest integer function.

Assertion (A): $\lim_{x \rightarrow \infty} \frac{[x]}{x} = 1$

Reason (R): $f(x) = x-1$, $g(x) = [x]$, $h(x) = x$ and

$\lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{h(x)}{x} = 1$

[18th May 2023 Shift 2]

1. A is true, R is true; R is correct explanation of A

2. A, R are true; R is not the correct explanation of A

3. A is true, R is false

4. A is false, R is true

103. Let $f(x) = \sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}}$. If $\lim_{x \rightarrow 1^-} f(x) = l$

And $\lim_{x \rightarrow m} f(x) = 5/2$, then the set of all

possible finite values of l and m is

[19th May 2023 Shift 1]

1. $\{0, 1\}$ 2. $\left\{0, \frac{1}{3}, \frac{2}{3}\right\}$
3. $\left\{0, \frac{2}{5}, \frac{3}{5}\right\}$ 4. $\left\{\frac{1}{5}, \frac{4}{5}\right\}$

104. $\lim_{x \rightarrow 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3} =$

[12th MAY 2023 SHIFT-1]

1. $\frac{1}{10}$ 2. $-\frac{1}{10}$
3. $\frac{2}{5}$ 4. $-\frac{2}{5}$

105. If a, b, c and k are non-zero real numbers and

$\lim_{x \rightarrow \infty} x(a^{1/x} + b^{1/x} + c^{1/x} - 3k^{1/x}) = 0$, then $k =$

[12th MAY 2023 SHIFT-1]

1. 0 2. $(abc)^{1/3}$
3. $(abc)^{-1/3}$ 4. 1

106. $\lim_{x \rightarrow 1} (1-x) \tan\left(\frac{\pi}{2}x\right) =$

[12th MAY 2023 SHIFT -2]

1. $\frac{\pi}{2}$ 2. $\frac{2}{\pi}$
3. 1 4. 0

107. If $f(9)=9$ and $f'(9)=4$, then

$$\lim_{x \rightarrow 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3} =$$

[12TH MAY 2023 SHIFT -2]

1. 3 2. 4
3. 6 4. 9

108. $\lim_{x \rightarrow 2^+} ([x]^2 - [x] - 2) + \lim_{x \rightarrow 3^-} ([x]^2 - 4[x] + 3) =$

[13TH MAY 2023 SHIFT-1]

1. 39 2. 33
3. 28 4. 44

109. $\lim_{x \rightarrow 0} \frac{(3^{2x} - \sqrt{x+1}) \sin 5x}{1 - \cos 4x} =$

[13TH MAY 2023 SHIFT-1]

1. $\frac{3}{5}(\log 18 - 1)$ 2. $\frac{5}{16} \log \left(\frac{81}{e} \right)$
3. $\frac{4}{15}(\log 81 - 1)$ 4. $\frac{16}{5}[\log(27) - 1]$

110. $\lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=1}^n (k^2 x) =$

[EAPCET 14-05-23 SHIFT-1]

1. x 2. $\frac{x}{2}$
3. $\frac{x}{3}$ 4. $\frac{x}{4}$

111. The quadratic equation whose roots are

$$l = \lim_{\theta \rightarrow 0} \left(\frac{3 \sin \theta - 4 \sin^3 \theta}{\theta} \right) \text{ and}$$

$$m = \lim_{\theta \rightarrow 0} \left(\frac{2 \tan \theta}{\theta(1 - \tan^2 \theta)} \right) \text{ is}$$

[EAPCET 14-05-23 SHIFT-1]

1. $x^2 - 5x + 6 = 0$ 2. $x^2 + 5x + 6 = 0$
3. $x^2 - 5x - 6 = 0$ 4. $x^2 + 5x - 6 = 0$

112. $\lim_{x \rightarrow 2} \frac{\sqrt[3]{6+x} - \sqrt[3]{10-x}}{x-2} =$

[EAPCET 13-05-23 SHIFT-2]

1. $1/8$ 2. $1/4$
3. $1/2$ 4. $1/16$

113. $\lim_{x \rightarrow 0} \frac{\tan^4 x - \sin^4 x}{x^6} =$

[EAPCET 13-05-23 SHIFT-2]

1. $1/2$ 2. $5/2$
3. 2 4. 4

KEY

1)	2	2)	4	3)	2	4)	3	5)	1
6)	2	7)	4	8)	1	9)	2	10)	4
11)	3	12)	4	13)	3	14)	1	15)	3
16)	2	17)	1	18)	3	19)	3	20)	3
21)	4	22)	3	23)	2	24)	4	25)	2
26)	3	27)	2	28)	4	29)	4	30)	3
31)	3	32)	2	33)	4	34)	1	35)	2
36)	1	37)	4	38)	4	39)	2	40)	4
41)	2	42)	2	43)	4	44)	3	45)	3
46)	4	47)	2	48)	2	49)	1	50)	3
51)	1	52)	1	53)	1	54)	3	55)	3
56)	2	57)	3	58)	1	59)	4	60)	2
61)	1	62)	1	63)	3	64)	2	65)	3
66)	2	67)	2	68)	1	69)	4	70)	4
71)	3	72)	3	73)	1	74)	2	75)	2
76)	1	77)	4	78)	4	79)	4	80)	3
81)	2	82)	2	83)	3	84)	4	85)	4
86)	4	87)	1	88)	3	89)	1	90)	4
91)	4	92)	2	93)	3	94)	3	95)	4
96)	3	97)	3	98)	3	99)	2	100)	4
101)	2	102)	1	103)	4	104)	2	105)	2
106)	2	107)	2	108)	1	109)	2	110)	3
111)	1	112)	2	113)	3				

SOLUTIONS

1. $\lim_{x \rightarrow \infty} \left(\frac{2x^2 + 3x + 4}{x^2 - 3x + 5} \right)^{\frac{3x+1}{2x-1}} = (2)^{3/2} = 2\sqrt{2}$

2. $\lim_{x \rightarrow a} x^n = a^n$

3. $\lim_{x \rightarrow 0} \frac{8}{\sin^8 x} \left\{ \left(1 - \cos \left(\frac{x^2}{2} \right) \right) \left(1 - \cos \left(\frac{x^2}{4} \right) \right) \right\}$
 $\lim_{x \rightarrow 0} \frac{8}{\sin^8 x} \left\{ 4 \sin^2 \left(\frac{x^2}{4} \right) \sin^2 \left(\frac{x^2}{8} \right) \right\}$

$$\lim_{x \rightarrow 0} 32 \frac{x^8}{\sin^8 x} \left(\lim_{x \rightarrow 0} \left(\frac{\sin \left(\frac{x^2}{4} \right)}{\frac{x^2}{4}} \right)^2 \frac{1}{16} \right)$$

$$\left(\lim_{x \rightarrow 0} \left(\frac{\sin \left(\frac{x^2}{8} \right)}{\frac{x^2}{8}} \right)^2 \frac{1}{64} \right)$$

$$= 32 \cdot \frac{1}{16} \cdot \frac{1}{64} = \frac{1}{32}$$

4. $\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\left(\cos \frac{x}{2} \right) \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)}$

Now $\frac{1 - \sin \frac{x}{2}}{\cos \frac{x}{2} \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)}$

$$= \frac{\sin^2 \frac{x}{4} + \cos^2 \frac{x}{4} - 2 \sin \frac{x}{4} \cos \frac{x}{4}}{\left(\cos^2 \frac{x}{4} - \sin^2 \frac{x}{4} \right) \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)}$$

$$= \frac{\left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)^2}{\left(\cos \frac{x}{4} + \sin \frac{x}{4} \right) \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right) \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)}$$

$$= \frac{1}{\left(\cos \frac{x}{4} + \sin \frac{x}{4} \right)}$$

$$\therefore \lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\left(\cos \frac{x}{2} \right) \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)} = \lim_{x \rightarrow \pi} \frac{1}{\left(\cos \frac{x}{4} + \sin \frac{x}{4} \right)} = \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

5. $\lim_{x \rightarrow \infty} \left(1 + \frac{p}{x} \right)^{qx} = e^9$

$$\Rightarrow \lim_{x \rightarrow \infty} \left[\left(1 + \frac{p}{x} \right)^{\frac{x}{p}} \right]^{pq} = e^9$$

$$\Rightarrow e^{pq} = e^9$$

$$\Rightarrow pq = 9 \text{ is possible for } p = 3, q = 3$$

$$\therefore p + q = 6$$

6. $\lim_{x \rightarrow 0} (1 + 3x)^{2/x} = 1^\infty$

$$\lim_{x \rightarrow 0} (1 + 3x)^{2/x} = e^{\lim_{x \rightarrow 0} \frac{2}{x} (1 + 3x - 1)}$$

$$= e^{\lim_{x \rightarrow 0} \frac{2}{x} (3x)} = e^6$$

7. $\lim_{n \rightarrow \infty} \frac{n!}{n! \left[\frac{(n+1)!}{n!} - 1 \right]} = \lim_{n \rightarrow \infty} \frac{1}{n+1-1}$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

8. $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} \cdot \frac{x}{\sin x} = \log a \cdot (1) = \log a$

9. $\lim_{n \rightarrow \infty} \frac{2^2 + 4^2 + 6^2 + \dots + (2n)^2}{n^3} = \lim_{n \rightarrow \infty} \frac{2^2 (1^2 + 2^2 + \dots + (n)^2)}{n^3}$

$$= 4 \lim_{n \rightarrow \infty} \frac{n(n+1)(2n+1)}{6n^3} = 4 \left(\frac{2}{6} \right) = \frac{4}{3}$$

10. $\lim_{x \rightarrow 1} \left((1-x) \tan \left(\frac{\pi x}{2} \right) \right)$

$$\text{let } 1-x = y \Rightarrow x = 1-y$$

$$\Rightarrow \lim_{y \rightarrow 0} \left(y \tan \left(\frac{\pi (1-y)}{2} \right) \right) \Rightarrow \lim_{y \rightarrow 0} \left(y \cot \frac{\pi y}{2} \right)$$

$$\Rightarrow \lim_{y \rightarrow 0} \frac{\frac{\pi y}{2}}{\left(\tan \frac{\pi y}{2} \right) \frac{\pi}{2}} \Rightarrow \frac{2}{\pi} (1) = \frac{2}{\pi}$$

11. Given: $\lim_{x \rightarrow 0} \frac{\sin(x^m)}{(\sin x)^n} = \lim_{x \rightarrow 0} \frac{x^m \sin(x^m)}{x^m \frac{(\sin x)^n}{x^n} x^n}$

$$= \lim_{x \rightarrow 0} \frac{x^m}{x^n} \left(\frac{\sin(x^m)}{x^m} \right) \cdot \frac{1}{\left(\frac{\sin x}{x} \right)^n}$$

$$= \lim_{x \rightarrow 0} x^{m-n} (1) = 0 \quad (\because n < m)$$

$$12. \lim_{x \rightarrow 3} \left(\frac{x^n - 3^n}{x - 3} \right) = 108$$

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$$

$$n \cdot 3^{n-1} = 108 \Rightarrow m \cdot 3^n = 324$$

$$\text{put } n = 4$$

$$13. \lim_{x \rightarrow 0} \frac{\left(1 + \frac{x}{2}\right)^{\frac{5}{7}} - 1}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{\left(1 + \frac{x}{2}\right)^{\frac{5}{7}} - 1}{\left(1 + \frac{x}{2}\right) - 1} \times \frac{1}{2}$$

$$\text{put } 1 + \frac{x}{2} = y \Rightarrow \lim_{y \rightarrow 1} \frac{y^{\frac{5}{7}} - 1}{y - 1} \times \frac{1}{2}$$

$$= \frac{5}{7} \cdot \frac{1}{2} = \frac{5}{14}$$

$$14. \lim_{x \rightarrow \infty} 3x \left(1 + \frac{2}{x} - 1\right) = e^6$$

$$15. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$16. \lim_{n \rightarrow \infty} \left\{ n - \sqrt{n^2 - 4n} \right\}$$

$$= \lim_{n \rightarrow \infty} \left\{ n - \sqrt{n^2 - 4n} \times \frac{n + \sqrt{n^2 - 4n}}{n + \sqrt{n^2 - 4n}} \right\}$$

$$= \lim_{n \rightarrow \infty} \frac{4n}{n + \sqrt{n^2 - 4n}} = \frac{4}{2} = 2$$

$$17. f(x) > 0, 3x < 6x < 9x, 1 < \lim_{x \rightarrow \infty} \frac{f(6x)}{f(3x)} < 1$$

$$\text{From sandwich theorem } \lim_{x \rightarrow \infty} \frac{f(6x)}{f(3x)} = 1$$

$$18. \lim_{x \rightarrow 0} \frac{x^4 + x^3 + x^2}{\sin^{-1} \frac{x}{\sqrt{1+x^2}} \cdot \tan^{-1} x}$$

$$\lim_{x \rightarrow 0} \frac{(x^2 + x + 1)\sqrt{1+x^2}}{\left(\frac{\sin^{-1} \frac{x}{\sqrt{1+x^2}}}{\frac{x}{\sqrt{1+x^2}}} \right) \left(\frac{\tan^{-1} x}{x} \right)} = 1$$

$$19. \lim_{x \rightarrow 0} \frac{(1+x) \log(1+x)}{x^2} - \frac{1}{x}$$

$$\lim_{x \rightarrow 0} \frac{(1+x) \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \right) - \frac{1}{x}}{x^2}$$

$$\lim_{x \rightarrow 0} (1+x) \left(\frac{1}{x} - \frac{1}{2} + \frac{x}{3} - \frac{x^2}{4} + \dots \right) - \frac{1}{x}$$

$$\lim_{x \rightarrow 0} \left\{ \frac{1}{x} - \frac{1}{2} + \frac{x}{3} - \frac{x^2}{4} + \dots + 1 - \frac{x}{2} + \frac{x^2}{3} - \dots - \frac{1}{x} \right\}$$

$$= 1 - \frac{1}{2} + 0 - 0 + \dots$$

$$= 1 - \frac{1}{2} = \frac{1}{2} \quad \therefore k = \frac{1}{2}$$

$$12k = 6$$

$$20. \frac{0}{0} \text{ form}$$

Apply L.H rule 2 times

$$= \frac{\pi^2}{2}$$

$$21. \lim_{x \rightarrow 0} \frac{1 - \cos(1 - \cos x)}{x^4} \cdot \lim_{x \rightarrow 0} \frac{1}{\frac{\sin^4 x}{x^4}}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos(2 \sin^2 \frac{x}{2})}{x^4} = \lim_{x \rightarrow 0} \frac{2 \sin^2(\sin^2 \frac{x}{2})}{x^4} = \frac{1}{8}$$

$$22. \frac{\left(\frac{1}{3}\right)(1)^{-2/3}}{\frac{1}{6}(1)^{-5/2}} = 2$$

$$23. \text{Coefficient of } n^3, \frac{4}{1} = 4$$

$$24. \frac{a}{b} = \frac{11}{13} \Rightarrow a + b = 24$$

$$25. \text{Put } x = 1 + h$$

$$26. \lim_{x \rightarrow 1} \frac{|\sqrt{2} \sin(x-1)|}{x-1} \text{ does not exist}$$

$$27. \text{IPE}$$

$$28. \text{Conceptual}$$

$$29. \lim_{x \rightarrow 0} \frac{8 \left\{ \left(1 - \cos^2 \frac{x^2}{2} \right) \left(1 - \cos^2 \frac{x^2}{4} \right) \right\}}{x^8}$$

$$8 \left(\frac{\left(\frac{1}{2} \right)^2}{2} \right) \left(\frac{\left(\frac{1}{4} \right)^2}{2} \right) = \frac{1}{32}$$

$$30. \lim_{x \rightarrow \infty} \frac{10^n \left\{ \left(\frac{1}{10} \right)^n - 1 \right\}}{10^n \left\{ \left(\frac{1}{10} \right)^n + 10 \right\}} = \frac{-\infty}{10}$$

$$\Rightarrow \frac{-1}{10} = \frac{-a}{10} \Rightarrow a = 10$$

$$31. e^{\lim_{x \rightarrow \infty} \left(\frac{x+6}{x+1} - 1 \right)} (x+4)$$

$$e^{\lim_{x \rightarrow \infty} \frac{5(x+4)}{x+1}} = e^5$$

$$32. \text{ Put } x = \tan \theta$$

$$33. \text{ CONCEPTUAL}$$

$$34. \text{ Deg Nr} < \text{deg Dr}$$

$$35. \lim_{x \rightarrow 0} x^7 \left[\frac{1}{x^3} \right] = 0$$

$$36. e^{\lim_{x \rightarrow 0} x \log(1+a^2)} \frac{1}{x} = 2a \sin^2 \theta \Rightarrow \frac{1+a^2}{2a} = \sin^2 \theta$$

$$\sin^2 \theta \in [0, 1]$$

$$\Rightarrow \frac{1+a^2}{2a} \geq 1 \Rightarrow \sin^2 \theta = 1 \Rightarrow \theta = 2n\pi \pm \frac{\pi}{2}$$

$$37. \lim_{x \rightarrow 0} \frac{x^2 2^x - x^2 \sin x - x^2}{3^x + \cos x - 3^x \cos x - 1}$$

$$= \lim_{x \rightarrow 0} \frac{x^2 (2^x - \sin x - 1)}{(1 - \cos x)(3^x - 1)}$$

$$= \lim_{x \rightarrow 0} \frac{\left(\frac{2^x - 1}{x} \right) - \left(\frac{\sin x}{x} \right)}{\left(\frac{1 - \cos x}{x^2} \right) \left(\frac{3^x - 1}{x} \right)} = \frac{\log 2 - 1}{\frac{1}{2} \times \log 3}$$

$$= \frac{2}{\log 3} (\log 2 - 1)$$

$$38. \text{ Conceptual}$$

$$39. \lim_{x \rightarrow 0} \frac{(1 - \cos 2x)}{x \tan 2x + \frac{2x}{3} \tan 3x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{(1 - \cos 2x)}{x^2}}{x \tan 2x + \frac{2x}{3} \tan 3x}$$

$$= \lim_{x \rightarrow 0} \frac{\left(\frac{1 - \cos 2x}{x^2} \right)}{\frac{\tan 2x}{2} + \frac{2}{3} \cdot \frac{\tan 3x}{3}} = \frac{4/2}{2 + \frac{2}{3} \times 3} = \frac{2}{4} = \frac{1}{2}$$

$$40. \text{ Given } f(x) = \tan^{-1} \left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right]$$

Put

$$x = \cos \theta \quad x \rightarrow \frac{1}{2}, \theta = \cos^{-1} x \quad \theta \rightarrow \frac{\pi}{3}$$

$$\Rightarrow f(x) = \tan^{-1} \left[\tan \left(\frac{\pi}{4} - \frac{\theta}{2} \right) \right] = \frac{\pi}{4} - \frac{\theta}{2}$$

$$\Rightarrow f(x) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x \Rightarrow f\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

Now

$$\lim_{x \rightarrow 1/2} \frac{2[f(x) - f(1/2)]}{2x - 1} = \lim_{x \rightarrow 1/2} \frac{\frac{\pi}{4} - \frac{1}{2} \cos^{-1} x - \frac{\pi}{6}}{x - 1/2}$$

$$= \lim_{x \rightarrow 1/2} \frac{\frac{\pi}{4} - \frac{1}{2} \cos^{-1} x}{x - \frac{1}{2}} = \frac{1}{\sqrt{3}} \text{ (using L.H. rule)}$$

$$41. \lim_{x \rightarrow 1} \frac{\log x}{(1-x)}, \text{ By L.H Rule } \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{-1} = -1$$

$$42. f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \Rightarrow x \in I - \{0\} \\ 0, & [x] = 0 \Rightarrow x \in (R - I) \cup \{0\} \end{cases}$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{\sin[x]}{[x]} = \lim_{x \rightarrow 0^-} \frac{\sin(-1)}{(-1)} = \sin 1$$

$$43. \lim_{x \rightarrow -\infty} \frac{3|x|^3 - x^2 + 2|x| - 5}{-5|x|^3 + 3x^2 - 2|x| + 7}$$

$$= \lim_{x \rightarrow -\infty} \frac{-3x^3 - x^2 - 2x - 5}{5x^3 + 3x^2 + 2x + 7} = \frac{-3}{5}$$

$$44. f(x) = -[\sin^2 x + \cos^5 x]$$

$$f'(x) = -[\sin 2x + 5\cos^4 x(-\sin x)]$$

$$= -\sin 2x + 5\cos^4 x \sin x$$

$$\lim_{x \rightarrow 0} \frac{1}{x} f'(x) = \lim_{x \rightarrow 0} \left[\frac{-\sin 2x + 5\cos^4 x \sin x}{x} \right]$$

$$= \lim_{x \rightarrow 0} \left[\frac{-\sin 2x}{x} \right] + \lim_{x \rightarrow 0} \left[\frac{5\cos^4 x \sin x}{x} \right]$$

$$= -2 + 5 = 3$$

$$45. \lim_{n \rightarrow \infty} P \left[1 + \frac{r}{100n} \right]^n \Rightarrow P e^{\lim_{n \rightarrow \infty} \left[1 + \frac{r}{100n} - 1 \right] n}$$

$$P e^{\lim_{n \rightarrow \infty} \left[\frac{r}{100n} \right] n} = P e^{\frac{r}{100}}$$

$$46. \text{ Let } a, b \text{ are roots of } px^2 + qx + r = 0$$

$$a + b = \frac{-q}{p}, ab = \frac{r}{p}$$

$$\lim_{x \rightarrow b} \frac{1 - \cos 2(px^2 + qx + r)}{2(px - pb)^2}$$

$$\lim_{x \rightarrow b} \frac{1 - \cos 2(p(x-a)(x-b))}{2p^2(x-b)^2}$$

$$\lim_{x \rightarrow b} \frac{2\sin^2 p(x-a)(x-b)}{2(p(x-b))^2(x-a)^2} \times (x-a)^2$$

$$\lim_{x \rightarrow b} (x-a)^2 = (b-a)^2 = b^2 - 2ab + a^2$$

$$47. f(x) = \frac{x^3 + 2x^2 + x + 2}{x^2 + x - 2}$$

$$\lim_{x \rightarrow -2} \frac{-8 + 4 - 2 + 2}{4 - 2 - 2} = \frac{-4}{0} = \infty$$

By LH-Rule

$$\lim_{x \rightarrow -2} \frac{3x^2 + 4x + 1}{2x + 1} = \frac{-5}{3}$$

$$48. \lim_{x \rightarrow 0^-} \frac{\sqrt{\frac{1}{2}(1 - \cos^2 x)}}{x} = \lim_{x \rightarrow 0^-} \frac{\sqrt{\frac{1}{2}(\sin^2 x)}}{x}$$

$$= \lim_{x \rightarrow 0^-} \frac{1}{\sqrt{2}} \cdot \left(\frac{-\sin x}{x} \right) \text{ As } x \rightarrow 0^- \Rightarrow x < 0 \Rightarrow |x| = -x$$

$$= \frac{-1}{\sqrt{2}} \cdot \left(\lim_{x \rightarrow 0^-} \frac{\sin x}{x} \right) = \frac{-1}{\sqrt{2}}$$

49. Apply L-Hospital rule '3' times, we get

$$\lim_{x \rightarrow 0} \frac{2\sin x - \sin 2x}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{2\cos x - 2\cos 2x}{3x^2} = \lim_{x \rightarrow 0} \frac{-2\sin x + 4\sin 2x}{6x}$$

$$= \lim_{x \rightarrow 0} \frac{-2\cos x + 8\cos 2x}{6} = \frac{6}{6} = 1$$

(or)

$$\text{Method 2: } \frac{2-1}{1} = 1$$

50. Conceptual

51. Conceptual

52. If $0 < x < 1$ then $x^n = 0$
 $\log_x^{12} = \text{negative}$

53.

$$54. \lim_{x \rightarrow \alpha} \frac{|a(x-l)(x-m)|}{a(x-l)(x-m)} = \frac{|a|}{a} \text{ if } l < \alpha < m$$

55. For $x=1$, $a+b=1$ (verify)

$$56. \lim_{x \rightarrow -\infty} \log(\cosh x) + x$$

$$= \lim_{x \rightarrow -\infty} \log \left(\frac{e^x + e^{-x}}{2} \right) + x$$

$$= \lim_{x \rightarrow -\infty} \log(e^x + e^{-x}) + x - \log 2$$

$$= \lim_{x \rightarrow -\infty} \log e^{-x} + x - \log 2$$

$$= -\log 2$$

$$57. \frac{b-c}{a-b} = \frac{1}{2} \Rightarrow a+2c=3b$$

$$58. x \rightarrow -\infty \Rightarrow x < 0$$

$$\frac{-3x-x}{-x-2x} - \lim_{x \rightarrow 0} \frac{\log(1+x^3)}{\sin^3 x} \times \frac{x^3}{x^3}$$

$$= \frac{4}{3} - 1 = \frac{1}{3}$$

59. Take $(2 + \sqrt{2})^n$ as common

$$\text{Let } 2 + \sqrt{2} = a, 2 - \sqrt{2} = b$$

$$\lim_{n \rightarrow \infty} \left[\frac{1 + \left(\frac{b}{a}\right)^n}{1 - \left(\frac{b}{a}\right)^n} \right] = 1 \left(\because 0 < \frac{b}{a} < 1 \right)$$

60. Let $a=2$

$$\lim_{x \rightarrow -2^-} \frac{8}{2} - 0 = 4 = k$$

$$\lim_{x \rightarrow -2^+} \left(\frac{8}{2} - 1 \right) = 3 = l$$

$$k - l = 1$$

61. When $x < 0 \Rightarrow nx < 0$

$$\Rightarrow e^{nx} < 0$$

$$n \rightarrow \infty, e^{nx} \rightarrow 0$$

62. Apply partial fractions

$$63. \lim_{n \rightarrow \infty} n \left[\frac{1}{a+b^n-1} \right] = e^{\lim_{n \rightarrow \infty} n \left(\frac{1}{b^n-1} \right)}$$

$$1^\infty, e$$

$$\lim_{n \rightarrow 0} \left(\frac{1}{b^n-1} \right) \cdot \frac{1}{a}$$

$$= e^{\frac{1}{n}} = e^{(\log b)^{\frac{1}{a}}} = b^{\frac{1}{a}}$$

64. Apply L.H rule

$$65. \tan^{-1} \left(\frac{\left(r + \frac{1}{2}\right) - \left(r - \frac{1}{2}\right)}{1 + \left(r + \frac{1}{2}\right)\left(r - \frac{1}{2}\right)} \right) = \tan^{-1} \left(r + \frac{1}{2} \right) - \tan^{-1} \left(r - \frac{1}{2} \right)$$

$$\text{Apply } \sum_{r=1}^n \Rightarrow \tan^{-1} \left(n + \frac{1}{2} \right) - \tan^{-1} \left(\frac{1}{2} \right)$$

$$= \frac{\pi}{2} - \tan^{-1} \left(\frac{1}{2} \right) = \cot^{-1} \left(\frac{1}{2} \right) = \tan^{-1} 2$$

$$66. \lim_{x \rightarrow 0} \left(\frac{(a-n)nx - \tan x}{x} \right) \frac{\sin nx}{x}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{(a-n)nx}{x} - n$$

$$= n[(a-n) - 1] \text{ by verify}$$

$$a = 2 \text{ is minimum}$$

$$67. \lim_{x \rightarrow a} g(x)[f(x)-1]$$

$$\text{Apply } e^{\lim_{x \rightarrow a} g(x)[f(x)-1]}$$

$$68. k = \lim_{n \rightarrow \infty} x^2 \left(\frac{\frac{1}{e^x} - 0}{\frac{1}{e^x} + 0} \right) = 0$$

$$l = \lim_{x \rightarrow 0^-} x^2 \left(\frac{0 - e^{\frac{-1}{x}}}{0 + e^{\frac{-1}{x}}} \right) = 0$$

$$k = l$$

$$69. k = \lim_{x \rightarrow \infty} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} = \frac{1-1}{1+1} = 0 \left[x \rightarrow \infty \Rightarrow \frac{1}{x} \rightarrow 0 \right]$$

$$\left[\frac{x}{e^{\frac{1}{x}}} \rightarrow 1 \right]$$

$$70. \text{ Put } x = 0 \frac{\sqrt{11-6\sqrt{2}}}{6-2\sqrt{2}} = \frac{3-\sqrt{2}}{2(3-\sqrt{2})} = \frac{1}{2}$$

71. Put $x = 0$

Ans. 0 (option 3)

72. Put $x = -a$, LH rule

$$7x^6 = 7, (-a)^6 = 1 \Rightarrow a = \pm 1$$

$$73. \lim_{x \rightarrow \frac{\pi}{2}} \tan \left(\frac{\pi}{7} - \frac{x}{2} \right) \cdot \frac{[1 - \sin x]}{(\pi - 2x)^3}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan\left(\frac{\pi}{2} - x\right) \left[1 - \cos\left(\frac{\pi}{2} - x\right)\right]}{\left(\frac{\pi}{2} - x\right) \left(\frac{\pi}{2} - x\right)^2} \times \frac{1}{8} = \frac{1}{32}$$

74. Let $x_n = l$

$$\therefore l = \frac{a + bl}{b + cl} \Rightarrow bl + cl^2 = a + bl$$

$$l^2 = \frac{a}{c} \Rightarrow l = \sqrt{\frac{a}{c}} (\because x_n > 0 \forall n \in \mathbb{N})$$

75. Apply $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$$

$$76. \lim_{x \rightarrow 0} \frac{x^2 \log(\cos x)}{\log(1 + x^2)} = \lim_{x \rightarrow 0} \frac{x^2}{\log(1 + x^2)} \cdot \lim_{x \rightarrow 0} \log(\cos x)$$

$$= 1 \times \log 1 = 0$$

$$77. x \rightarrow \frac{\pi}{2} + \Rightarrow [\sin x] = 1, [\cos x] = 0$$

$$\therefore \frac{1 - 0 + 1}{2} = 1$$

$$78. \lim_{n \rightarrow \infty} \left(\frac{e^{-nx} \cos x - 1}{e^{-nx} - A} \right), e^{-nx} \rightarrow 0 \Rightarrow \frac{0 - 1}{0 - A} = \frac{1}{A}$$

$$79. f(x) = \lim_{y \rightarrow \infty} \frac{\left(\frac{1}{x^y} - 1\right)}{\frac{1}{y}}, f(x) = \log x$$

$$\therefore 2022(-\log x) + p \log x = 2 \log x$$

$$p \log x = (2024) \log x \Rightarrow p = 2024$$

80. $\frac{0}{0}$ form apply L.H. Rule

$$81. \lim_{x \rightarrow 0} \frac{2^{2x} - 2^{x+1} + 1 + 1 - \cos 2x}{x^2} =$$

$$\lim_{x \rightarrow 0} \frac{2^{2x} - 2^{x+1} + 1}{x^2} + \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = \left(\frac{0}{0} \text{ form}\right) + 2$$

$$= \lim_{x \rightarrow 0} \frac{(2^x - 1)^2}{x^2} + 2 = (\log e)^2 + 2$$

82. Apply L.H rule

$$83. \lim_{x \rightarrow 0} \frac{4!}{x^8} \left(1 - \cos \frac{x^2}{3}\right) \left(1 - \cos \frac{x^2}{4}\right)$$

$$\lim_{x \rightarrow 0} 4! \left(\frac{1 - \cos \frac{x^2}{3}}{x^4} \right) \left(\frac{1 - \cos \frac{x^2}{4}}{x^4} \right)$$

$$= 24 \times \frac{1}{18} \times \frac{1}{32} = \frac{1}{24}$$

$$84. \lim_{x \rightarrow 1} \frac{\sin(\cos x^2) - \sin(\cos 1)}{x - 1}$$

Apply L.H. Rule

$$85. 0 \times \text{Finite} + \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^3 - 2x - 4}$$

$$= 0 + \lim_{x \rightarrow 2} \frac{2x}{3x^2 - 2} = \frac{4}{8} = \frac{1}{2}$$

$$86. \lim_{x \rightarrow 0} \frac{2 \tan x \left(1 - \frac{1}{\tan^2 x} - 1\right)}{\frac{(1 - \cos x)(2^x - 1)}{x^2}}$$

$$= \frac{2}{\left(\frac{1}{2}\right) \log 2} \lim_{x \rightarrow 0} \frac{\tan^3 x}{x^3} = \frac{4}{\log 2}$$

$$87. \lim_{k \rightarrow 1} \frac{n - (k + k^2 + k^3 + \dots + k^n)}{1 - k} = 171$$

$$\lim_{k \rightarrow 1} \frac{(1 - k) + (1 - k^2) + \dots + (1 - k^n)}{1 - k} = 171$$

$$\Rightarrow \lim_{k \rightarrow 1} 1 + (1 + k) + \dots + (1 + k + \dots + k^{n-1}) = 171$$

$$\Rightarrow \sum n = 171 \therefore n = 18$$

$$88. \lim_{x \rightarrow \infty} \frac{x^3 (x^2 + \sqrt{x^4 + 1} - 2x^2)}{\sqrt{x^2 + \sqrt{x^4 + 1} + \sqrt{2}x}}$$

$$\lim_{x \rightarrow \infty} \frac{x^3(x^2 + \sqrt{x^4 + 1} - 2x^2)}{x\sqrt{1 + \sqrt{1 + \frac{1}{x^4}} + \sqrt{2}}}$$

$$\lim_{x \rightarrow \infty} \frac{x^2(1)}{(2\sqrt{2})(\sqrt{x^2 + 1} + x^e)} = \frac{1}{2\sqrt{2}(2)} = \frac{1}{4\sqrt{2}}$$

$$89. l_1 = 4 - 3 = 1, l_2 = 4 - 4 = 0, l_1 + l_2 = 1$$

$$90. \lim_{x \rightarrow 0} \left(\frac{2^x - 1}{x} \right) \frac{x}{\log(1 + 2x)} = \frac{\log 2}{2} = \log \sqrt{2}$$

$$\lim_{x \rightarrow 0} (1 + \sin x)^{\frac{2}{\sin x}} = e^{\frac{2}{\sin x}(\sin x)} = e^2$$

$$\therefore e^2 \log \sqrt{2}$$

$$91. \left(\frac{0}{0} \right) \text{ L.H. Rule}$$

$$92. \lim_{x \rightarrow 0} \frac{4.2 \cos\left(\frac{4042x}{2}\right) \sin(x)}{x[\cos(2022x) + 2\cos(2021x) + \cos x]}$$

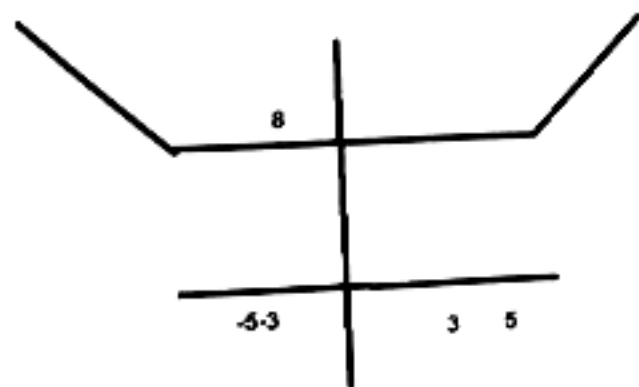
$$= \frac{8 \times 1}{1 + 2 + 1} = \frac{8}{4} = 2$$

$$93. F(X) = |X - 3| + |X + 5|$$

$$X \leq -5 \quad 3 - X - 5 - X = -2 - 2X$$

$$-5 \leq X \leq 3 \quad 3 - X + X + 5 = 8$$

$$X \geq 3 \quad X - 3 + X + 5 = 2X + 2$$



F is not differentiable at $x = -5$ but $-5 \in A$

No of real no's = 1

$$94. \text{ Given}$$

$$\lim_{x \rightarrow 1} \left(\lim_{y \rightarrow \infty} y(ex)^{1/y} - 1 \right) =$$

$$\lim_{y \rightarrow \infty} \frac{e^{\frac{1}{y}} - 1}{\frac{1}{y}} = 1$$

$$95. \text{ A is true}$$

$$\left(\because \text{As } x \rightarrow 0, \frac{1}{x} \rightarrow \infty \right)$$

R is true

$$\left(\because \text{value of } x \text{ decreases, the value of } \frac{1}{x} \text{ increases} \right)$$

$$96.$$

$$f(x) = \begin{vmatrix} 1 & 6+x & 36+x^2 \\ 0 & x-3 & 3x^2-27 \\ 0 & 2x-4 & 8x^2-32 \end{vmatrix}$$

$$f(1) = \begin{vmatrix} 1 & 7 & 37 \\ 0 & -2 & -24 \\ 0 & -2 & -24 \end{vmatrix} = 0$$

$$f(-1) = \begin{vmatrix} 1 & 5 & 37 \\ 0 & -4 & -24 \\ 0 & -6 & -24 \end{vmatrix} \neq 0$$

$$\lim_{x \rightarrow 1} \frac{f(1)}{f(-1)} = \frac{0}{\text{finite value}} = 0$$

$$97.$$

$$\lim_{x \rightarrow m} \frac{xf(m) - mf(x)}{x - m} + f'(m) = f(m)$$

By applying L.H Rule, it

$$\lim_{x \rightarrow m} \frac{f(m) - mf'(x)}{1 - 0} + f'(m) = f(m)$$

$$f(m) - mf'(m) + f'(m) = f(m)$$

$$f'(m)(-m + 1) = 0$$

$$\therefore f'(m) \neq 0, m = 1$$

$$98.$$

$$\lim_{x \rightarrow 2} \frac{1 + \sqrt{1 + 4 \log_2 x}}{2 + (2x + \sin^2 x + 2 \cos x)(2x - 4)} = m$$

$$m = \frac{1 + \sqrt{5}}{2 + 0}$$

$$m(m - 1) = \left(\frac{\sqrt{5} + 1}{2} \right) \left(\frac{\sqrt{5} - 1}{2} \right) = 1$$

$$\begin{aligned}
 99. \quad & \lim_{x \rightarrow -1^-} \frac{1-x+\sqrt{9x^2+10x+1}}{2x} \\
 &= \frac{1+1+\sqrt{9-10+1}}{2(-1)} \\
 &= \frac{2}{-2} = -1
 \end{aligned}$$

100. Using L.H. Rule

$$\begin{aligned}
 & \lim_{x \rightarrow 9} \frac{(2.5)^{81-x^2} \log(2.5)(-2x) - (0.4)^{x+9} \log(0.4)}{1} \\
 &= 18 \log 2.5 - \log 0.4 \\
 &= 18 \log 2.5 + 18 \log 0.4 - 19 \log 0.4 \\
 &= 18 \log 1 - 19 \log 0.4 \\
 &= -19 \log 0.4
 \end{aligned}$$

$$101. S_n = 1 + 3x + 9x^2 + 27x^3 + \dots + n \text{ terms}$$

$$= 1 + (3x) + (3x)^2 + (3x)^3 + \dots$$

$$\text{These are in G.P as } \frac{-1}{3} < x < \frac{1}{3} \quad |3x| < 1$$

$$\begin{aligned}
 f(x) &= \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} 1 + 3x + (3x)^2 + \dots + (\infty) \\
 &= \lim_{n \rightarrow \infty} \frac{1}{1-3x}
 \end{aligned}$$

$$\text{It is not defined at } 1-3x=0, \quad x = \frac{1}{3}$$

$$102. A: [x] = x, x \rightarrow \infty \Rightarrow \lim_{x \rightarrow \infty} \frac{x}{x} = 1 \quad (\text{conceptual})$$

$$R: x-1 \leq [x] < x$$

$$\lim_{x \rightarrow \infty} \frac{x-1}{x} \leq \lim_{x \rightarrow \infty} \frac{[x]}{x} < \lim_{x \rightarrow \infty} \frac{x}{x}, \quad \lim_{x \rightarrow \infty} \frac{[x]}{x} = 1$$

$$103. \lim_{x \rightarrow m} f(x) = 5/2 \Rightarrow \sqrt{\frac{m}{1-m}} + \sqrt{\frac{1-m}{m}} = \frac{5}{2}$$

$$t + \frac{1}{t} = \frac{5}{2}, t = 2 \text{ (or)} \frac{1}{2}$$

$$\frac{m}{1-m} = 4, m = \frac{4}{5} \text{ or } \frac{m}{1-m} = \frac{1}{4}, m = \frac{1}{5}$$

$$\frac{m}{1-m} = \frac{1}{4} \Rightarrow m = \frac{1}{5}$$

$$104. \lim_{x \rightarrow 1} \frac{2x^{3/2} - 2x - 3\sqrt{x} + 3}{2x^2 + x - 3}$$

apply LH rule

$$105. \lim_{x \rightarrow \infty} x \left(a^{\frac{1}{x}} + b^{\frac{1}{x}} + c^{\frac{1}{x}} - 3k^{\frac{1}{x}} \right) = 0$$

$$\lim_{x \rightarrow \infty} \frac{\left(a^{\frac{1}{x}} + b^{\frac{1}{x}} + c^{\frac{1}{x}} - 3k^{\frac{1}{x}} \right)}{\frac{1}{x}} = 0$$

$$\lim_{1/x \rightarrow 0} \left(\frac{a^{\frac{1}{x}} - k^{\frac{1}{x}}}{\frac{1}{x}} \right) + \lim_{1/x \rightarrow 0} \left(\frac{b^{\frac{1}{x}} - k^{\frac{1}{x}}}{\frac{1}{x}} \right) + \lim_{1/x \rightarrow 0} \left(\frac{c^{\frac{1}{x}} - k^{\frac{1}{x}}}{\frac{1}{x}} \right) = 0$$

$$\log a + \log b + \log c - (\log k + \log k + \log k) = 0$$

$$k^3 = \log(abc)$$

$$k = (abc)^{\frac{1}{3}}$$

106. IPE

$$107. \text{ Given that } f(9) = 9, f'(9) = 4$$

$$\lim_{x \rightarrow 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3} = 4$$

By L - hospital rule

$$108. 4 + 2 - 2 + 16 + 16 + 3$$

$$109. \lim_{x \rightarrow 0} \left(\frac{(3^{2x} - \sqrt{x+1})(3^{2x} + \sqrt{x+1})}{(3^{2x} + \sqrt{x+1})} \right) \times \frac{\frac{\sin 5x}{x^2}}{\frac{1 - \cos 4x}{x^2}}$$

$$\lim_{x \rightarrow 0} \frac{3^{4x} - x - 1}{x} \cdot \lim_{x \rightarrow 0} \frac{1}{(3^{2x} + \sqrt{x+1})} \frac{\frac{\sin 5x}{x}}{\frac{1 - \cos 4x}{x^2}}$$

$$= (\log 81 - \log e) \cdot \frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16} \log \left(\frac{81}{e} \right)$$

$$\begin{aligned} 110. &= \lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=1}^n (k^2 x) \\ &= \lim_{n \rightarrow \infty} \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6} x \\ &= \frac{2}{6} x = \frac{x}{3} \end{aligned}$$

$$111. \quad l = \lim_{\theta \rightarrow 0} \frac{3 \sin \theta - 4 \sin^3 \theta}{\theta} = 3$$

$$m = \lim_{\theta \rightarrow 0} \frac{2 \tan \theta}{\theta(1 - \tan^2 \theta)} = 2$$

required Quadratic eqn is $x^2 - 5x + 6 = 0$

112.

$$\lim_{x \rightarrow 2} \frac{\sqrt[3]{6+x} - \sqrt[3]{10-x}}{x-2}$$

$$\lim_{x \rightarrow 2} \frac{1}{3} (6+x)^{-2/3} + \frac{1}{3} (10-x)^{-2/3}$$

$$\frac{1}{3} (8)^{-2/3} + \frac{1}{3} (8)^{-2/3}$$

$$\frac{1}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$$

$$113. \quad \lim_{x \rightarrow 0} \frac{\tan^4 x - \sin^4 x}{x^6}$$

$$\lim_{x \rightarrow 0} \frac{\frac{\sin^4 x}{\cos^4 x} - \sin^4 x}{x^6}$$

$$\lim_{x \rightarrow 0} \frac{\sin^4 x (\sec^4 x - 1)}{x^6}$$

$$\lim_{x \rightarrow 0} \frac{\sin^4 x}{x^4} \cdot \lim_{x \rightarrow 0} \frac{1 - \cos^4 x}{x^2} \cdot \lim_{x \rightarrow 0} \frac{1}{\cos^4 x}$$

$$= 1 \times \frac{4}{2} \times 1 = 2$$