25. MAXIMA AND MINIMA

1. The value of 'k' for which the function $f(x)=k(x+\sin x)+k$ is increasing, is equal to **[AP EAMCET 18-09-20_Shift-1]**

1. $k<0$ 2. $k>0$
3. $k=0$ 4. Data insufficient

2. The interval in which y = ln(ln(x)) is decreasing is **[AP EAMCET 21-09-20_Shift-1]**

1. $(-\infty,0)\cup(2,\infty)$ 2. (0,2)
3. (0,1) 4. (-1,0)

3. Maximum area of the rectangle that can be formed with the fixed perimeter 'p' cm____ **[AP EAMCET 21-09-20_Shift-1]**

1. $\frac{p^2}{8}cm^2$ 2. $\frac{p^2}{16}cm^2$
3. $\frac{p^2}{64}cm^2$ 4. $\frac{p^2}{32}cm^2$

4. IF the tangent of curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ meets the x-axis at A and y-axis at B , Then AB **[AP EAMCET 21-09-20_Shift-2]**

1. 2a 2. 3a
3. a 4. 4a

5. If y=$\frac{(ax-b)}{(x-1)(x-4)}$ has a turning point P(2,-1) then the values of a and b are . **[AP EAMCET 23-09-20_Shift-1]**

1. a=0,b=1 2. a=1,b=0
3. a=-1,b=0 4. a=0,b=-1

6. If the curved surface area of right circular cylinder inscribed in a sphere of radius 22cm is maximum then the height of the cylinder will be__ **[AP EAMCET 23-09-20_Shift-1]**

1. $\frac{11}{\sqrt{2}}$ cm 2. $11\sqrt{2}$ cm
3. $(0.22)\sqrt{2}$ m 4. $(0.11)\sqrt{2}$ cm

7. The difference between the greatest and least values of the function f(x)=-x+sin2x on $[-\frac{\pi}{2},\frac{\pi}{2}]$ is **[AP EAMCET 23-09-20_Shift-1]**

1. $\frac{\sqrt{3}+\sqrt{2}}{2}$ 2. $\frac{\sqrt{3}+\sqrt{2}}{2}+\frac{\pi}{6}$
3. $\frac{\sqrt{3}}{2}+\frac{\pi}{3}$ 4. $\frac{\sqrt{3}}{2}-\frac{\pi}{6}$

8. The two positive numbers with 't', and the sum of their squares is minimum are

1. $\frac{t}{4},\frac{3t}{4}$ 2. $\frac{t}{3},\frac{2t}{3}$
3. $\frac{t}{2},\frac{t}{2}$ 4. $\frac{2t}{5},\frac{3t}{5}$

9. For the function $f(x)=x^{40}-x^{20}$ find the absolute minimum value in the interval [0, 1]

1. $\frac{1}{2}$ 2. $\frac{-1}{4}$
3. 0 4. 1

10. The function $f(x)=x^3-4x^2+4x+3$ defined on [-1,3] has **[TS EAMCET 09-09-20_Shift-1]**

1. minimum value -6 at x=-1
2. minimum value 6 at x=3
3. minimum value 3 at x=2
4. maximum value 9 at x=3

11. The minimum value of the function $f(x)=2x^2-\ln|x|, when\, x\geq 1$ is **[TS EAMCET 09-09-20_Shift-2]**

1. $\frac{1}{2}+\log 2$ 2. 2
3. 4 4. $2+\log 2$

12. If α is such a minimum value for which the inverse of $f(x)=x^2+3x-3$ exists in $[\alpha,\infty)$ and g is the inverse of the f then at $x=\alpha+\frac{5}{2}, \frac{dg}{dx}=$ **[TS EAMCET 10-09-20_Shift-2]**

1. $\sqrt{10}$ 2. $\pm\sqrt{10}$
3. $\pm\frac{2}{\sqrt{10}}$ 4. $\pm\frac{1}{\sqrt{10}}$

13. Match the functions of list - I with the items of list - II

List – I	List – II
A) $3x^4-2x^3-6x^2+6x+1$	I) has minimum value at x=4
B) $x+\frac{1}{x},\forall x<0$	II) has maximum value at x=-1
C) $x^4(7-x)^3$	III) has maximum value at x=4

$D) x^4+(8-x)^4$	IV) is decreasing in $[2,\infty)$
	V) is increasing in $[2,\infty)$

[TS EAMCET 10-09-20_Shift-2]

1. A B C D — IV I II III
2. A B C D — V IV I III
3. A B C D — V II III I
4. A B C D — IV II I V

14. A function y=f(x) with $f(-1)=-249$ has no maximum and has only one minimum at x=5 with f(5)=75, which one of the following is true? **[TS EAMCET 11-09-20_Shift-1]**
1. at some point in $(-1,5)$, f(x) is discontinuous
2. the minimum value cannot be 75 since f(-1)<f(5)=75
3. f(x) is discontinuous at every point of R
4. f(x) is continuous on R

15. Assertion (A): The function $f(x)=x-\log\left(\frac{1+x}{x}\right), x>0$ has no maximum.

Reason (R): If a function $f(x)$ is strictly increasing in an interval (a, b), then at any point in (a, b), $f'(x)\neq 0$

[TS EAMCET 11-09-20_Shift-2]
1. (A) is true, (R) is true and (R) is the correct explanation for A.
2. (A) is true, (R) is true but (R) is the not the correct explanation for A.
3. (A) is true but (R) is false.
4. (A) is false but (R) is true.

16. The stationary points of the curve $y=8x^2-x^4-4$ are____

[AP EAMCET 19-08-2021_Shift-1]
1. $(0,-4),(2,12),(-2,12)$
2. $(0,4),(-2,12),(1,2)$
3. $(0,-4),(-1,12),(2,12)$
4. $(0,4),(-1,2),(1,2)$

17 Which statement among the following is true?

a) the function $f(x)=x|x|$ is strictly increasing on $R-\{0\}$.

b) The function $f(x)=\log_{(1/4)}x$ is strictly increasing on $(0,\infty)$.

c) A one-one function is always an increasing function.

d) $f(x)=x^{1/3}$ is strictly decreasing on R

[AP EAMCET 19-08-2021_Shift-1]
1. a
2. b
3. C
4. d

18. For which value(s) of 'a', $f(x)=-x^3+4ax^2+2x-5$ is decreasing for every 'x' ?

[AP EAMCET 19-08-2021_Shift-1]
1. $(1,2)$
2. $(3,4)$
3. R
4. No value of 'a'

19 The maximum value of $f(x)=\sin(x)$ in the interval $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ is_____

[AP EAMCET 19-08-2021_Shift-2]
1. 0
2. -1
3. 1
4. $\sqrt{2}$

20 If $f''(x)$ is a positive function for all $x\in R$, $f'(3)=0$ and $g(x)=f(\tan^2(x)-2\tan(x)+4)$ for $0<x<\frac{\pi}{2}$, then the interval in which $g(x)$ is increasing is_____

[AP EAMCET 19-08-2021_Shift-2]
1. $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$
2. $\left(0,\frac{\pi}{4}\right)$
3. $\left(0,\frac{\pi}{3}\right)$
4. $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

21. Find the positive value of 'a' for which the equality $2\alpha+\beta=8$ holds. Where 'α' and 'β' are the points of maximum and minimum respectively of the function $f(x)=2x^3-9ax^2+12a^2x+1$

[AP EAMCET 20-08-2021_Shift-1]
1. 0
2. 2
3. 1
4. $\frac{1}{4}$

22. Find the minimum value of $2x+3y$ when $xy=6$. **[AP EAMCET 20-08-2021_Shift-2]**

1. 9
2. 12
3. 8
4. 6

23. If $g(x)=\frac{1}{6}f(3x^2-1)+\frac{1}{2}f(1-x^2), \forall x \in R$, where $f''(x)>0, \forall x \in R$. Then $g(x)$ is increasing in the interval___

[AP EAMCET 20-08-2021_Shift-2]

1. $\left(\frac{-1}{\sqrt{2}},0\right)\cup\left(\frac{1}{\sqrt{2}},\infty\right)$
2. $\left(\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$
3. $(-1,0)\cup(1,2)$
4. $\left(-\infty,\frac{-1}{\sqrt{2}}\right)\cup\left(\frac{1}{\sqrt{2}},\infty\right)$

24. If the function $f(x)=2x^3-9ax^2+12a^2x+1$ attains its maximum and minimum at 'p' and 'q' respectively such that $p^2=q$, then 'a' equals__**[AP EAMCET 20-08-2021_Shift-2]**

1. 0
2. 1
3. 2
4. -1

25. The absolute minimum value of x^4-x^2-2x+5 is

[AP EAMCET 23-08-2021_Shift-1]

1. 5
2. 3
3. 7
4. Does not exist

26. If the area of a circular sector of perimeter 60m is to be maximized, then its radius must be ______m

[AP EAMCET 23-08-2021_Shift-2]

1. 20
2. 15
3. 10
4. 5

27. If $f(x)=kx^3-9x^2+9x+3(k>0)$ is increasing for all x, then _____

[AP EAMCET 24-08-2021_Shift-1]

1. $k\le 3$
2. $k\ge 3$
3. $0<k<1$
4. $1<k<3$

28. The maximum value of $y=x(\log x)^2$ is

[AP EAMCET 24-08-2021_Shift-1]_____

1. e^{-2}
2. $2e^{-2}$
3. $3e^{-2}$
4. $4e^{-2}$

29. For the function $f(x)=x^3-6x^2-12x-3,\ x=2$ is a ______

[AP EAMCET 24-08-2021_Shift-2]

1. Point of maxima
2. Point of minima
3. Point of inflection
4. Not a critical point

30. Find the absolute maximum of $x^{40}-x^{20}$ on the interval [0, 1]

[AP EAMCET 25-08-2021_Shift-1]

1. $\frac{-1}{4}$
2. 0
3. $\frac{1}{4}$
4. $\frac{1}{2}$

31. The maximum area of a rectangle that can be formed with a fixed perimeter of 20 units is ____ sq.units

[AP EAMCET 25-08-2021_Shift-1]

1. 30
2. 25
3. 20
4. 15

32. If $f(x)=x^5-5x^4+5x^3-10$ has its local maxima and minima at $x=a$ and $x=b$ respectively, then $2a+b=$

[AP EAMCET 25-08-2021_Shift-1]

1. 5
2. 4
3. 7
4. 3

33. If the function $f(x)=a\sin(x)+\frac{1}{3}\sin(3x)$ attains maximum value at $x=\frac{\pi}{3}$, then 'a' equals _____ **[AP EAMCET 25-08-2021_Shift-2]**

1. 3
2. $\frac{1}{3}$

3. 2 4. $\frac{1}{2}$

34. The absolute maximum value of the function $f(x)=2x^3-9x^2+12x+1$ on $[0,2]$ is

[TS EAMCET 04-08-2021_Shift-2]

1. 8 2. 1
3. 6 4. 5

35. Let $R^* = R-\left\{(2k-1)\frac{\pi}{2} / k \in I\right\}$. The function $f: R^* \to R$ is defined as $f(x)=\tan x - x$, then $f(x)$ is

TS EAMCET 05-08-2021_Shift-2]

1.an increasing function
2.a decreasing function
3.minimum at $x=0$
4.periodic function

36. If a cubic function $f(x)=ax^3+bx^2-\frac{18}{5}x+\frac{19}{10}$ has maximum value 10 at $x=-3$ and has minimum value $-\frac{5}{2}$ at $x=2$, then $f(1)=$

[TS EAMCET 05-08-2021_Shift-2]

1. -10 2. $\frac{-6}{5}$
3. 6 4. $\frac{28}{5}$

37 The maximum value of $f(x)=\frac{x}{1+4x+x^2}$ is

[AP EAMCET 04-07-2022_Shift-1]

1. $1/4$ 2. $1/5$
3. $1/6$ 4. $1/7$

38. The minimum value of $f(x)=x+\frac{4}{x+2}$ is

[AP EAMCET 04-07-2022_Shift-1]

1. -1 2. -2
3. 1 4. 2

39 The condition that $f(x)=ax^3+bx^2+cx+d$ has no extreme value is

[AP EAMCET 04-07-2022_Shift-1]

1. b^2-4ac 2. $b^2=3ac$
3. $b^2<3ac$ 4. $b^2>3ac$

40. Two particles P and Q located at the points $P(t,t^3-16t-3)$, $Q(t+1,t^3-6t-6)$ are moving in a plane, the minimum distance between the points in their motion is

[AP EAMCET 04-07-2022_Shift-2]

1. 1 2. 5
3. 169 4. 49

41 If a, b > 0 then, minimum value of $y=\frac{b^2}{a-x}+\frac{a^2}{x}, 0<x<a$ is

[AP EAMCET 05-07-2022_Shift-1]

1. $\frac{(a-b)^2}{a}$ 2. $\frac{(a+b)^2}{a}$
3. a^2+b^2 4. a^2-b^2

42 The point on the curve $y=x^2+4x+3$ which is closest to the line $y=3x+2$

[AP EAMCET 05-07-2022_Shift-1]

1. $\left(\frac{1}{2},\frac{5}{4}\right)$ 2. $\left(\frac{-1}{2},\frac{5}{4}\right)$
3. $\left(2,\frac{-5}{3}\right)$ 4. $\left(2,\frac{5}{3}\right)$

43. The maximum area of a right angled triangle with hypotenuse *h is*

[AP EAMCET 06-07-2022_Shift-1]

1. $h^2/2\sqrt{2}$ 2. $h^2/2$
3. $h^2/\sqrt{2}$ 4. $h^2/4$

44. The value of '*a*' *for* which the function $f(x)=a\sin x+\frac{1}{3}\sin 3x$ has an extremum value at $x=\frac{\pi}{3}$ is

[AP EAMCET 06-07-2022_Shift-1]

1. 1 2. -1

3. 0 4. 2

45. Two sides of a triangle are given. If the area of the triangle is maximum then the angle between the given sides is

[AP EAMCET 06-07-2022_Shift-2]

1. 45^0 2. 30^0

3. 60^0 4. 90^0

46 a,b,c,d,e,f,g,h be distinct elements in the set{-7,-5,-3,-2,2,4,6,13} The minimum value of $(a+b+c+d)^2+(e+f+g+h)^2$ is

[AP EAMCET 06-07-2022_Shift-2]

1. 30 2. 32

3. 34 4. 40

47. The perimeter of a sector is constant . If its area is to be maximum, the sectorical angle should be **[AP EAMCET 07-07-2022_Shift-1]**

1. $\frac{\pi}{6}^c$ 2. $\frac{\pi}{4}^c$

3. 4^c 4. 2^c

48 The set of all x for which $\sin x \le x$ *is*

[AP EAMCET 07-07-2022_Shift-1]

1. $\left(0,\frac{\pi}{2}\right)$ 2. $\left(-\frac{\pi}{2},\pi\right)$

3. $\left(-\frac{\pi}{2},0\right)$ 4. $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

49. The function

$f(x)=2x^3-9ax^2+12a^2x+1(a>0)$ attains its maximum and minimum at p and q respectively and $p^2=q$, then a=

[AP EAMCET 07-07-2022_Shift-1]

1. 1 2. 2

3. $\frac{1}{2}$ 4. 3

50. The minimum distance of a point on the curve $y=x^2-4$ from the origin is

[AP EAMCET 07-07-2022_Shift-2]

1. $\frac{\sqrt{15}}{2}$ 2. $\frac{\sqrt{19}}{2}$

3. $\sqrt{\frac{15}{2}}$ 4. $\sqrt{\frac{19}{2}}$

51. If an open cylinder of given surface area has maximum volume then its radius is

[AP EAMCET 07-07-2022_Shift-2]

1. Height of the cylinder
2. Height of the cylinder/2
3. 2 times Height of the cylinder
4. 3 times Height of the cylinder

52 If x+y=k, $x>0, y>0$ then x^2+y^2 is minimum, if

[AP EAMCET 07-07-2022_Shift-2]

1. x>y 2. x<y

3. x=y 4. x=2y

53. Maximum area of the rectangle inscribed in a circle of radius 10 cms is

[AP EAMCET 08-07-2022_Shift-1]

1. 100 2. 200

3. 250 4. 150

54 At $x=0, f(x)=\cos x-1+\frac{x^2}{2}-\frac{x^3}{3}$

[AP EAMCET 08-07-2022_Shift-1]

1. Has a minimum value
2. Has a maximum value
3. Has no extremum value
4. Is not defined

55. The minimum value of $\left(1+\frac{1}{\sin^n\alpha}\right)\left(1+\frac{1}{\cos^n\alpha}\right)$ is

[AP EAMCET 08-07-2022_Shift-1]

1. 1 2. 2

3. $(1+2^n)^2$ 4. $\left(1+2^{n/2}\right)^2$

56. The least intercept made by a tangent to the ellipse $\frac{x^2}{64}+\frac{y^2}{49}=1$ with coordinate axes

[AP EAMCET 08-07-2022_Shift-2]

1. 40 2. 10

3. 15 4. 100

57 Let a function $f(x)$ be continuous in an interval [a, b]. Let $\delta > 0$ be a very small real number. Let $c \in (a,b)$ be such that $f(c-\delta) < f(c)$ and $f(c+\delta) < f(c)$ for every $\delta > 0$. Let $(f(\alpha-\delta)-f(\alpha))(f(\alpha+\delta)-f(\alpha)) < 0 \ \forall \alpha \in (a,b)$ and $\alpha \neq c$. Then

[TS EAMCET 18-07-2022_Shift-1]

1. $f(x)$ has a local maximum at c and a local minimum at α
2. $f(x)$ has a local maximum at α and a local minimum at c
3. $f(x)$ has only one local maximum at c
4. $f(x)$ has only one local minimum at c

58. The local maximum value of tile function $f(x) = -(x-2)^3(x+2)^2$ is

[TS EAMCET 18-07-2022_Shift-2]

1. 0 2. $\frac{12^3 \cdot 8^2}{5^5}$

3. 125 4. $\frac{2^9 \cdot 3^2}{5^6}$

59. Let $\sqrt{3}$ be the radius and $\frac{\pi}{3}$ be the semi vertical angle of the given cone. Then the height of the right circular cylinder of maximum volume that can be inscribed in the given cone is

[TS EAMCET 19-07-2022_Shift-I]

1. 3 2. $\frac{\sqrt{3}}{2}$

3. $\frac{2}{\sqrt{3}}$ 4. $\frac{1}{3}$

60. Let $f(x) = \begin{cases} 1+6x-3x^2, & x \le 1 \\ x+\log_2(b^2+7), & x > 1 \end{cases}$

Then the set of all possible values of b such that f(1) is the maximum value of f(x) is

[TS EAMCET 19-07-2022_Shift-2]

1. $[-1,1]$ 2. $[0,1]$

3. $[0, 2]$ 4. $[-1,0]$

61. If x and y are two positive integers such that $x+2y=10$ *and* x^2y^3 is maximum then $x^2+2y^3 =$

[TS EAMCET 20-07-2022_Shift-1]

1. 34 2. 137

3. 43 4. 70

62. The absolute maximum value of the function $f(x) = 2x^3-3x^2-36x+9$ defined on $[-3, 3]$ is **[TS EAMCET 20-07-2022_Shift-2]**

1. 36 2. 53

3. 63 4. 72

63 If $A = \{x / 9x \ge x^2+20\}$ and $f : A \to R$ is defined by $f(x) = 2x^3-15x^2+36x-48$, then the maximum value of $f(x)$ is

[15th May 2023 Shift 1]

1. -20 2. 7

3. 20 4. -16

64. If (2,a) and (b,19) are two stationary points of the curve $y = 2x^3-15x^2+36x+c$, then a+b+c=

[15th May 2023 Shift 2]

1. -20 2. 15

3. -12 4. 24

65. If $f(x) = px^3+qx^2+rx+t$ attains local minimum and local maximum values at x = –2 and x=2 respectively and p is a root of $9x^2-1=0$, *then* $3p+q+r=$

[16th May 2023 Shift 1]

1. 4 2. 3

3. -5 4. -8

66 Given that the solid obtained by rotating a rectangle about one of its side is a cylinder. If the perimeter of a rectangle is 48 cm and volume of the cylinder formed by rotating it is maximum, then the dimensions of that rectangle is **[16th May 2023 Shift 1]**

1. 14,10 2. 20,4

3. 18,6 4. 8,6

67. The sum of the global minimum and global maximum values of the function $f(x)=\frac{4}{3}x^3-4x in[0,2]$ is

[16th May 2023 Shift 2]

1. 0
2. $\frac{8}{3}$
3. $-\frac{8}{3}$
4. 1

68. A(1,15),b(3,–12),C(6,12) are three consecutive turning points of a continuous curve y=f(x). If f(x)=0 only for $x=\alpha\, and\, x=\beta\, then\left|\beta-\alpha\right|<$

[16th May 2023 Shift 2]

1. 27
2. 2
3. 5
4. 24

69. The maximum value of x^4y^4 when $a^2x^4+b^2y^4=c^6$ is **[17th May 2023 Shift 1]**

1. $\frac{c^{12}}{16a^4b^4}$
2. $\frac{c^{12}}{4a^2b^2}$
3. $\frac{c^6}{(a+b)^{12}}$
4. $\frac{c^6}{a^4+b^4}$

70. The function $f(x)=x^2+\frac{54}{x}$

[17th May 2023 Shift 1]

1. Is increasing and has minimum value 27 in the interval $(0,\infty)$
2. Is decreasing and has neither maximum nor minimum in the interval $(-\infty,0)$
3. has maximum value 27 in the interval $(-\infty,\infty)$
4. Is increasing and has neither maximum nor minimum values in the interval $(-\infty,\infty)$

71. An extreme value of f(x)= $\frac{4}{\sin x}+\frac{1}{1-\sin x}$ in $(0,\frac{\pi}{2})$ is

[17th May 2023 Shift 2]

1. 9
2. 8
3. 2/3
4. –7/2

72. If the height of a cone of greatest volume that can be inscribed in a sphere of radius R , then the ratio of the volume of the cone to the volume of the sphere is

[19th May 2023 Shift 1]

1. 8:27
2. 27:64
3. 8:125
4. 4:5

73. If the function $f(x)=\frac{x}{5}+\frac{5}{x},(x\neq0)$ attains its relative maximum value at $x=\alpha$ then $\sqrt{\alpha^2+2\alpha-6}=$ **[12TH MAY 2023 SHIFT-1]**

1. 10
2. 6
3. 5
4. 3

74. x and y are two positive integers such that $2x+3y=50.If\ x^2y^3$ is maximum for x=α and y=β ,then $\frac{\alpha}{2}+\frac{\beta}{5}=$

[13TH MAY 2023 SHIFT-1]

1. 10
2. $\frac{10}{3}$
3. 5
4. 7

75 If the function $f(x)=xe^{-x},x\in R$ attains its maximum value $\beta\, at\, x=\alpha$ then $(\alpha,\beta)=$

[14-05-23 SHIFT-1]

1. $\left(2,\frac{1}{e}\right)$
2. $\left(1,\frac{1}{e}\right)$
3. $\left(1,\frac{-1}{e}\right)$
4. $\left(\frac{1}{e},1\right)$

76 If the absolute maximum and absolute minimum values of the function $f(x)=x^3-2x^2+x-3$ defined on [0,2] are M and m respectively, then M+ m =

[13-05-23 SHIFT-2]

1. -4
2. -104/27
3. 2
4. -2

KEY

1)	2	2)	3	3)	2	4)	3	5)	2
6)	3	7)	3	8)	3	9)	2	10)	3
11)	2	12)	4	13)	3	14)	1	15)	1
16)	1	17)	1	18)	4	19)	3	20)	4
21)	2	22)	2	23)	1	24)	3	25)	2
26)	2	27)	2	28)	4	29)	3	30)	2
31)	2	32)	1	33)	3	34)	3	35)	1
36)	2	37)	3	38)	4	39)	3	40)	1
41)	2	42)	2	43)	4	44)	4	45)	4
46)	2	47)	4	48)	1	49)	2	50)	1
51)	1	52)	3	53)	2	54)	3	55)	4
56)	3	57)	3	58)	2	59)	4	60)	1
61)	4	62)	2	63)	2	64)	2	65)	2
66)	2	67)	1	68)	3	69)	2	70)	2
71)	1	72)	1	73)	4	74)	4	75)	2
76)	1								

SOLUTIONS

1. $f(x)=k(x+\sin x)+k$

$f'(x)=k(1+\cos x)$

$\because 1+\cos x>0$

If f(x) is increasing then

$f'(x)>o \text{ if } k>0$

2 Conceptual

3 $2(l+b)=p$

$l+b=\dfrac{p}{2}$

$b=\dfrac{p}{2}-l$

A = lb

$f(l)=l\left(\dfrac{p}{2}-l\right)$

$f'(l)=\dfrac{p}{2}-2l$

$f'(l)=0\Rightarrow l=\dfrac{p}{4}$

$b=\dfrac{p}{4}, l=b$

$f''(l)=-2$

Area is maximum at l=b

Require area $A=\left(\dfrac{p}{4}\right)^2=\dfrac{p^2}{16}cm^2$

4. Given $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$

$Let\ P\left(a\cos^3\theta, a\sin^3\theta\right)\ \&\ m=\dfrac{-\sin\theta}{\cos\theta}$

Eq. of tangent is $\dfrac{x}{a\cos\theta}+\dfrac{y}{a\sin\theta}=1$

$Let\ A(a\cos\theta,0)\ \&\ B=(0,a\sin\theta)$

be intercept points

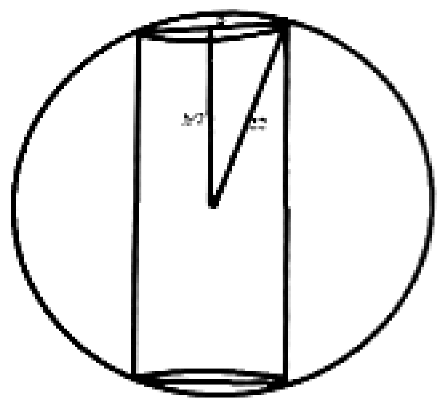
Now AB length $=\sqrt{a^2\cos^2\theta+a^2\sin^2\theta}=a$

5. $f'(x)=\dfrac{a(x-1)(x-4)-(ax-b)(2x-5)}{(x-1)^2(x-4)^2}$

At (2, -1), $f'(2)=0\Rightarrow b=0$

$f(2)=-1\Rightarrow a=1$

6. From diagram,



$R^2=484-\dfrac{h^2}{4}$

$S=2\pi Rh\Rightarrow S^2=4\pi^2R^2h^2$

$=4\pi^2\left(484-\dfrac{h^2}{4}\right)h^2$

$f(h)=4\times 484\pi^2h^2-\pi^2h^4$

$f'(h)=0$

$h=\sqrt{968}cm$

$h=0.22\sqrt{2}m$

$f''(h)<0$ at $h=0.22\sqrt{2}m$

'S' is maximum at $h=0.22\sqrt{2}m$

7. $f'(x)=0$

$\Rightarrow x=\pm\dfrac{\pi}{6}$

$f''\left(-\dfrac{\pi}{6}\right)>0$

$$f''\left(\frac{\pi}{6}\right)<0$$

$$f\left(\frac{-\pi}{6}\right)=\frac{\pi}{6}-\frac{\sqrt{3}}{2}$$

$$f\left(\frac{\pi}{6}\right)=\frac{-\pi}{6}+\frac{\sqrt{3}}{2}$$

Minimum value is $\frac{-\pi}{2}$

Maximum value is $\frac{-\pi}{6}+\frac{\sqrt{3}}{2}$

Therefore, req. difference is $\frac{\pi}{3}+\frac{\sqrt{3}}{2}$

8. Let x, y be two positive numbers.

$x+y=t \Rightarrow y=t-x$

Sum of squares = $x^2+y^2=2x^2+t^2-2tx$

$$f(x)=2x^2+t^2-2tx$$

$f'(x)=0 \Rightarrow 4x-2t=0$

$x=\frac{t}{2},\ y=\frac{t}{2}$

$\therefore\ f''(x)=4>0$ minimum.

9 $f(x)=x^{40}-x^{20},\ [0,1]$

$$f'(x)=40x^{39}-20x^{19}$$

$f'(0)=0,\quad f'(1)=20>0$

Now $f'(x)=0 \Rightarrow x=0,\ x=\left(\frac{1}{2}\right)^{1/20}$

$$f\left(\left(\frac{1}{2}\right)^{1/20}\right)=\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)=\frac{-1}{4}$$

$\therefore$ absolute minimum value $=\frac{-1}{4}$

10. $f(x)=x^3-4x^2+4x+3$

$$f'(x)=3x^2-8x+4$$

$f'(x)=0,\quad x=2,\frac{2}{3}$

$f''(x)=6x-8$

$f''(2)=4>0$

$f(x)$ has minimum value at $x=2$

Minimum value = 3

11. $f(x)=2x^2-\ln|x|,\ x\ge 1$

$f'(x)=4x-\frac{1}{x}$

$f'(x)=0 \Rightarrow 4x^2-1=0 \Rightarrow x=\pm\frac{1}{2}$

$f'(x)>0$ for $x\in\left(-\infty,\frac{-1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

$f'(x)<0$ for $x\in\left(-\frac{1}{2},\frac{1}{2}\right)$

For $x\ge 1$, $f(x)$ is increasing function

Minimum value $f(1)=2$

12 Let $f(x)=g^{-1}(x)=y$

$$x=f^{-1}(y)$$

$$x^2+3x-3=y$$

$$\left(x+\frac{3}{2}\right)^2=\frac{\sqrt{4y+21}}{2}$$

$$x=-\frac{3}{2}\pm\frac{\sqrt{4y+21}}{2}$$

$$f^{-1}(y)=g(y)$$

$g(x)=-\frac{3}{2}\pm\frac{\sqrt{4x+21}}{2}$ exists

If $4x+21\ge 0$

i.e.., $x\ge -\frac{21}{4}$

g(x) exists in $\left[-\frac{21}{4},0\right)$

$x=\alpha+\frac{5}{2}=-\frac{11}{4}$

$\frac{dg}{dx}=0\pm\frac{4}{2(2)\sqrt{4x+21}}=\pm\frac{1}{\sqrt{4x+21}}$ at

$x=-\frac{11}{4}\Rightarrow\frac{dg}{dx}=\pm\frac{1}{\sqrt{10}}$

13 Conceptual

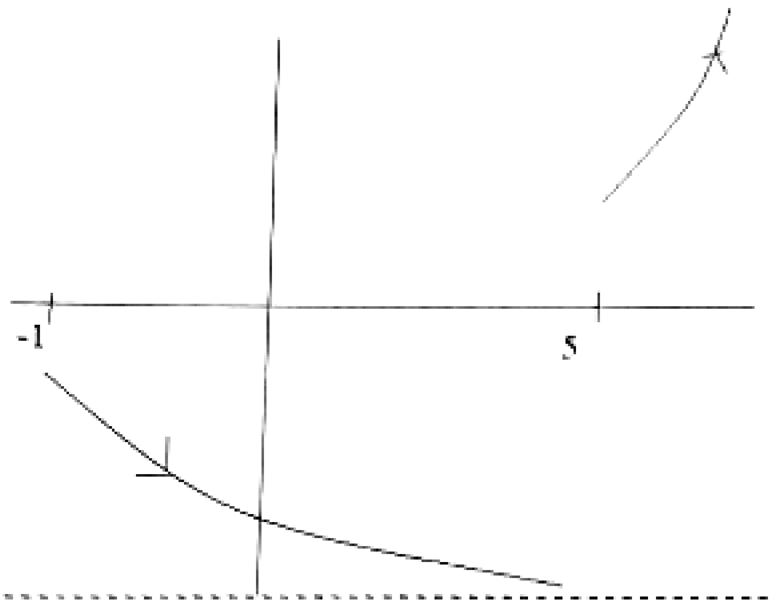
14. Given, function $y=f(x)$

And also $f(-1)=-249$

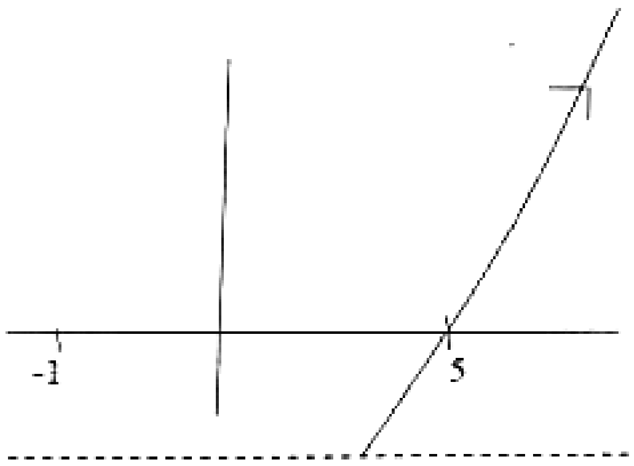
$f(5)=75$

Has one minimum value at x=5

Case (i):



Case (ii)



15. $f(x)=x-\log(1+x)+\log x, x>0$

$$f'(x)=1-\frac{1}{1+x}+\frac{1}{x}$$

$$f'(x)=\frac{x^2+x+1}{x^2+x}=1+\frac{1}{x^2+x}>0$$

$f'(x)>0$

16. Let $f(x)=8x^2-x^4-4 \Rightarrow f'(x)=16x-4x^3$

$f'(x)=0, x=0,2,-2$

$f(0)=-4, f(2)=12, f(-2)=12$

$\therefore$Stationary points are $(0,-4),(2,12),(-2,12)$

17. $f(x)=\begin{cases} x^2, x>0 \\ -x^2, x<0 \end{cases}$, $f'(x)=\begin{cases} 2x, x>0 \\ -2x, x<0 \end{cases}$

$\therefore f(x)$ is increasing $R-\{0\}$.

18. $f(x)=-x^3+4ax^2+2x-5$

$f'(x)<0 \Rightarrow -3x^2+8ax+2<0$

$\Rightarrow 3x^2-8ax-2>0$

The coefficient of x^2 is positive, in equality is positive.

$\therefore \Delta<0 \Rightarrow 64a^2+24<0 \Rightarrow a<\sqrt{\frac{-3}{8}}=\sqrt{\frac{3}{8}}i$

$\therefore$ No value of 'a'.

19 $f(x)=\sin x \Rightarrow f'(x)=\cos x, f''(x)=-\sin x$

$$f'(x)=0 \Rightarrow \cos x=0 \Rightarrow x=(2n+1)\frac{\pi}{2} \Rightarrow x=-\frac{\pi}{2},\frac{\pi}{2}$$

$$f''\left(\frac{-\pi}{2}\right)=-\left(\sin\frac{-\pi}{2}\right)=1>0$$

$f(x)$ has minimum value at $x=\frac{-\pi}{2}$,

Minimum value = -1

$$f''\left(\frac{\pi}{2}\right)=-\sin\frac{\pi}{2}=-1<0$$

$f(x)$ has maximum value at $x=\frac{\pi}{2}$

Max.value =1.

20. Given $f''(x)>0 \Rightarrow f'(x)$ is increasing function.

$f'(3)=0 \Rightarrow f'(3+h)=0$

$g'(x)>0 \Rightarrow f'\left\{\left((\tan x-1)^2+3\right)(\tan x-1)\right\}>0$

$\tan x-1>0$ or $\tan x>1 \Rightarrow x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

21. $f'(x)=6x^2-18ax+12a^2$

$f''(x)=12x-18a$

$f'(x)=0 \Rightarrow x=a, x=2a$

$f''(a)=-6a<0$

$f(x)$ has max. value at $x=a \therefore \alpha=a$

$f''(2a)=6a>0$

$f(x)$ has min .value at $x=2a \therefore \beta=2a$

$2\alpha+\beta=8 \Rightarrow a=2$

22. Let $f(x)=2x+3y$ and $xy=6 \Rightarrow y=\frac{6}{x}$

$$f(x)=2x+\frac{18}{x}\Rightarrow f'(x)=2-\frac{18}{x^2}\Rightarrow f''(x)=\frac{36}{x^3}$$

$$f'(x)=0\Rightarrow x=3,-3$$

$$f''(3)=\frac{4}{3}>0$$

$\therefore f(x)$ has min.value at $x=3, y=2$.

Min.value $=12$.

23. $g'(x)=\frac{1}{6}f'\left[\left(3x^2-1\right)(6x)\right]+\frac{1}{2}f'\left[\left(1-x^2\right)(-2x)\right]$

$$g'(x)=x\left[f'\left(3x^2-1\right)-f'\left(1-x^2\right)\right]>0$$

$$x>0, 3x^2-1>1-x^2\Rightarrow x^2>\frac{1}{2}$$

$$\Rightarrow\left(x+\frac{1}{\sqrt{2}}\right)\left(x-\frac{1}{\sqrt{2}}\right)>0$$

$$x\in\left(-\infty,\frac{-1}{\sqrt{2}}\right)\cup\left(\frac{1}{\sqrt{2}},\infty\right)$$

If $x<0, 3x^2-1<1-x^2\Rightarrow x^2<\frac{1}{2}\Rightarrow x<\frac{1}{\sqrt{2}}$

$$x\in\left(\frac{-1}{\sqrt{2}},0\right)$$

Interval $=\left(\frac{-1}{\sqrt{2}},0\right)\cup\left(\frac{1}{\sqrt{2}},\infty\right)$

24. $f(x)=2x^3-9ax^2+12a^2x+1$

$$f'(x)=6x^2-18ax+12a^2$$

$$f''(x)=12x-18a$$

$$f'(x)=0\Rightarrow x=a,2a$$

$$f''(a)=-6a<0$$

$f(x)$ has .value at $x=a \quad \therefore p=a$

$$f''(2a)=6a>0$$

$f(x)$ has Min.value at $x=2a \quad \therefore q=2a$

Given that $p^2=q\Rightarrow a^2=2a\Rightarrow a=2$

25. Let $f(x)=x^4-x^2-2x+5$

$$f'(x)=4x^3-2x-2, f''(x)=12x-2$$

$$f'(x)=0\Rightarrow 2x^3-x-1=0$$

If $x=1$, satisfies . $f''(1)=10>0$

$f(x)$ has Min.value at $x=1$

Min. value $=1-1-2+5=3$

26. Let r be the radius of the circular sector

Area of a circular sector of perimeter $=60$

$$2r+r\theta=60\Rightarrow\theta=\frac{60-2r}{r}$$

Area of sector

$$=\frac{1}{2}r^2\theta=\frac{1}{2}r^2\left(\frac{60-2r}{r}\right)=\frac{1}{2}\left(60r-2r^2\right)$$

Let $f(r)=\frac{1}{2}\left(60r-2r^2\right)$

$$f'(r)=\frac{1}{2}(60-4r), f''(r)=-2$$

$$f'(r)=0\Rightarrow r=15, f''(15)=-2<0$$

$f(r)$ has Max. at $r=15$

$\therefore radius(r)=15$

27. $f(x)=kx^3-9x^2+9x+3$

$$f'(x)=3kx^2-18x+9$$

$$f'(x)\geq 0\Rightarrow kx^2-6x+3\geq 0$$

The coefficient of x^2 is positive, in equality is positive.

$\Delta<0\Rightarrow 36-12k<0\Rightarrow k-3>0\Rightarrow k\geq 3$

28. Let $f(x)=x.(\log x)^2$,

$$f'(x)=x.\frac{2\log x}{x}+(\log x)^2$$

$$f''(x)=\frac{2}{x}+\frac{2\log x}{x}$$

$$f'(x)=0\Rightarrow 2\log x(2+\log x)=0$$

$$\Rightarrow x=1, x=e^{-2}$$

$$f''(1)=2+0=2>0$$

$f(x)$ has Min. value at $x=1$

Min. value $=0$.

$$f''\left(e^{-2}\right)=\frac{2}{e^{-2}}<0$$

$f(x)$ has Max. value at $x=e^{-2}$

Max. value $= e^{-2}\left(\log e^{-2}\right)^2 = 4e^{-2}$

29. $f(x)=x^3-6x^2-12x-3$

$f'(x)=3x^2-12x-12, f''(x)=6x-12$

At $x=2, f''(2)=0$

$\therefore$ Point of inflection.

30. $f(x)=x^{40}-x^{20}$

$\Rightarrow f'(x)=40x^{39}-20x^{19}$

$\Rightarrow f''(x)=1560x^{38}-380x^{18}$

$f'(x)=0\Rightarrow x=0, x=\left(\frac{1}{2}\right)^{\frac{1}{20}}$

Absolute Max. value

$= \max\{f(0), f(1), f\left(\frac{1}{2}\right)^{\frac{1}{20}}\}$

$= \max\{0, 0, -1/4\} = 0$

31 Let x and y are length and breadth of rectangle.

$2(x+y)=20\Rightarrow y=10-x$

Area $(A)=xy=10x-x^2$

$f(x)=10x-x^2, f'(x)=10-2x, f''(x)=-2$

$\therefore f'(x)=0\Rightarrow x=5$

$f''(5)=-2<0$

$f(x)$ has Max. value at $x=5 \therefore y=5$

Max. area $= 5\times 5 = 25\, sq.units$

32. $f(x)=x^5-5x^4+5x^3-10$

$f'(x)=5x^4-20x^3+15x^2$

$f''(x)=20x^3-60x^2+30x$

$f'(x)=0\Rightarrow x=0,1,3$

$f''(0)=0$ is point of inflection.

$f''(1)=1-5+5-10=-9<0$

$f(x)$ has Max. value at $x=1,\ a=1$

$f''(3)=90>0$

$f(x)$ has Min. value at $x=3 \therefore b=3$

$\therefore 2a+b=5$

33 $f(x)=a\sin x+\frac{1}{3}\sin 3x$

$f'(x)=a\cos x+\frac{1}{3}\cos 3x(3)$

$f'(x)=0\Rightarrow a\cos x+\cos 3x=0$

At $x=\frac{\pi}{3}, a\cos\frac{\pi}{3}+\cos\left(3\frac{\pi}{3}\right)=0\Rightarrow a=2$.

34. Given $f(x)=2x^3-9x^2+12x+1, (0,2)$

$f'(x)=6x^2-18x+12$

Now, $f'(x)=0\Rightarrow x=1,2$

$f(0)=1, f(1)=6, f(2)=5$

Now absolute maximum is ‘6’.

35. Given $f(x)=\tan x-x$

$f'(x)=\sec^2 x-1\geq 0$ is an increasing function

36. $f'(x)=3ax^2+2bx-\frac{18}{5}$

$f'(-3)=0$

$27a-6b-\frac{18}{5}=0....(1)$

$f'(2)=12a+4b-\frac{18}{5}=0....(2)$

From (1) & (2), $a=1/5, b=3/10$

$f(x)=\frac{1}{5}x^3+\frac{3}{10}x^2-\frac{18}{5}x+\frac{19}{10}$

$f(1)=\frac{-6}{5}$

37 The maximum of $f(x)=\frac{x}{1+4x+x^2}$

$f'(x)=1-x^2\Rightarrow f'(x)=0$

$x=\pm 1 \quad f''(1)<0$

max of $f(x)=\frac{1}{6}$

38 IPE

39. $f'(x) = 3ax^2 + 2bx + c$

For no extreme values
of f should have zero turning points
i.e., above Q.E no real roots

$b^2 - 4ac < 0 \Rightarrow (2b)^2 - 4 \times 3ac < 0$

$\Rightarrow b^2 < 3ac$

40. $PQ.\sqrt{(t+1-t)^2 + (t^3 - 6t - 6 - t^3 + 16t + 3)^3}$

$PQ^2 = 1 + (10t - 3)^2 \underline{\geq 1}$

The minimum distance is 1.

41. $\frac{dy}{dx} = 0$

y min at $x = \frac{a^2}{a+b}$

$$y\min = \frac{b^2}{\left(a - \frac{a^2}{a+b}\right)} + \frac{a^2}{\left(\frac{a^2}{a+b}\right)} = \frac{(a+b)^2}{a}$$

42 Let (x,y) be any point on parabola which is closest to the line y =3x+2

43. Hypotenuse of rectangle triangle 'h' and base b

$A = \frac{1}{2} b\sqrt{h^2 - b^2}$

$\frac{dA}{dx} = 0 \Rightarrow h^2 = 2b^2 \Rightarrow b = \frac{h}{\sqrt{2}}$

$A = \frac{1}{2} \times \frac{h}{\sqrt{2}} \times \frac{h}{\sqrt{2}} = \frac{h^2}{4}$

44. $f^1(x) = a\,cos\,x + \frac{1}{3}\cos 3x$

$f^1\left(\pi/3\right) = 0 \Rightarrow a\left(\frac{1}{2}\right) - 1 = 0$

$a = 2$

45. $A = \frac{1}{2}\,ab\sin c \Rightarrow \frac{dA}{dC} = \frac{1}{2}ab\,\cos c$

$\frac{dA}{dC} = 0 \Rightarrow \frac{ab}{2}\cos c = 0$

$\cos c = \cos 90^o \Rightarrow c = 90^o$

46. Let

$a + b + c + d = x$

$e + f + g + h = y$

$x + y = 8$

$x^2 + y^2 + 2xy = 64$

$x^2 + y^2 = 64 - 2xy$

$= 64 - 2x(8 - x)$

$= 64 - 16x + 2x^2$

$f^1(x) = 4x - 16, f^1(x) = 0 \Rightarrow x = 4$

$f''(4) = 0$ f $has \min imum = 32$

47 $P = l + 2r =$ constant=k

$2r + r\theta = k$

$r = \frac{k}{2+\theta} \Rightarrow A = \frac{1}{2}r^2\theta$

$= \frac{k^2}{2}\frac{\theta}{(\theta+2)^2}$

$\frac{dA}{d\theta} = 0 \Rightarrow \theta = 2^C \quad \frac{d^2A}{d\theta^2} = \frac{-3k^2}{256} < 0$

max at $\theta = 2^C$

48. Let $f(x) = \sin - x$ such that $\sin x < 0$

$f(x)$ is an increasing function

$f'(x) = \cos x - 1 \geq 0$

$f'(x)$ is an increasing for $\left(0, \pi/2\right)$

49. $f'(x)=6x^2-18ax+12a^2$

$f'(x)=0$

$x=a, x=2a$

$f''(2a)>0, \quad f''(a)<0$

$\therefore P=a \quad q=2a$

$a^2-2a=0$

$a(a-2)=0,\ a>0$

$\Rightarrow a=2$

50. Any point on the curve $y=x^2-4$ is $(x,\ x^2-4)$

$d=\sqrt{x^2+(x^2-4)^2}$

$d^2=x^2+(x^2-4)^2$

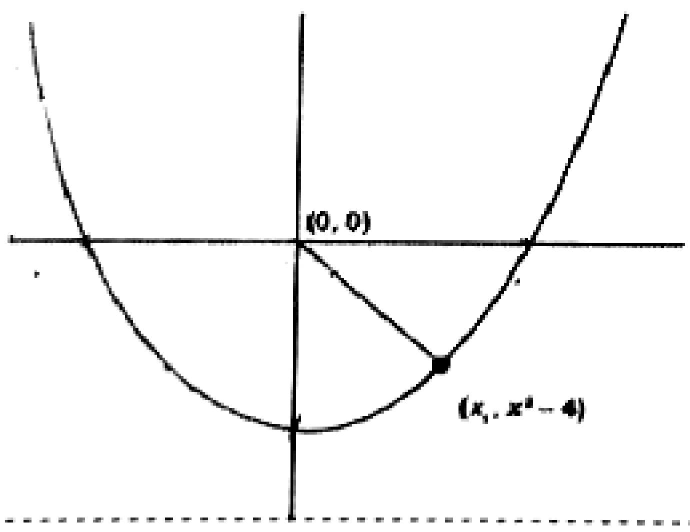
let $f(x)=x^2+(x^2-4)$

$f'(x)=0$

$x=\pm\sqrt{\frac{7}{2}} \Rightarrow d=\sqrt{\frac{7}{2}+\left(\frac{7}{2}-4\right)^2}=$

$d=\sqrt{\frac{7}{2}+\frac{1}{4}}$

$d=\sqrt{\frac{14+1}{4}}=\frac{\sqrt{15}}{2}$



51 $S=2\pi rh+\pi r^2$

$V=\pi r^2 h$

$V=\frac{\pi r^2}{2\pi r}[s-\pi r^2]$

$V=\frac{r}{2}(s-\pi r^2)$

$V'=\frac{s-\pi r^2}{2}-\frac{2\pi r^2}{2}$

$V'=0$

$\frac{S}{3}=\pi r^2$

$\frac{2\pi rh+\pi r^2}{3}=\pi r^2 \Rightarrow \boxed{r=h}$

52. $x+y=k \Rightarrow y=k-x$

$=x^2+y^2$

$f(x)=x^2+(k-x)^2$

$f'(x)=0 \Rightarrow x=\frac{k}{2}$ and $y=\frac{k}{2}$

$x=y$

53. Maximum area of the rectangle inscribed in a circle of radium is $=2r^2$
$=2\times100=200$

54. $f(x)=\cos x-1+\frac{x^2}{2}-\frac{x^3}{3}$

$f'(x)=-\sin x+x-x^2$

$f'(0)=-\sin 0^\circ+0-0=0$

$f''(0)=0$

$\therefore f$ has no extremum value.

55 $f(x)=\left(1+\frac{1}{\sin^n\alpha}\right)\left(1+\frac{1}{\cos^n\alpha}\right)$

$f'(x)=0 \Rightarrow x=\pi/4$

$f(\pi/4)=\left(1+2^{n/2}\right)^2$

56 The least intercept made by a tangent $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $a+b$

57. Conceptual

58. $f(x)=-(x-2)^3(x+2)^2$

$f'(x)=0$ *them* $x=-2/5$

Maximum $\frac{12^3.8^3}{5^5}$

59. AB height of cone

$AC=\sqrt{3}$ *radius of cone*

$\text{Tan } 60^\circ=\frac{AC}{AB}$

$\sqrt{3}=\frac{\sqrt{3}}{AB}\Rightarrow \boxed{AB=1}$

apply similar triangle property

$\boxed{\frac{1}{h}=\frac{\sqrt{3}}{\sqrt{3}-r}}\Rightarrow \sqrt{3}-r=h\sqrt{3}$

$\boxed{\sqrt{3}(1-h)=r}$

$V.cylinder=\pi r^2h$

$=\pi 3(1-h)^2h$

$=3\pi\left(1+h^2-2h\right)h$

Let $f(h)=3\pi\left(h+h^3-2h^2\right)$

$f'(h)=0$

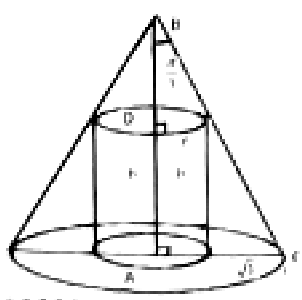
$3\pi\left(1+3h^2-4h\right)=0$

$3h^2-3h-h+1=0$

$3h(h-1)-1(h-1)=0$

$(3h-1)=0 \quad h=1$

$h=1/3 \quad f''(h)<0$



60 $f(x)$ *continous at* $x=1$

$1+6-3=1+\log_2\left(b^2+7\right)$

$8=b^2+7$

$b^2=1 \quad b=\pm 1$

$[-1,1]$

61. $x+2y=10 \quad x^2y^3$

$x=10-2y$

$=(10-2y)^2y^3$

Let $f(y)=\left(100+4y^2-40y\right)y^3$

$=100y^3+4y^5-40y^4$

$f'(y)=0$

$y=3$ *then* $x=4$

$\Rightarrow x^2+2y^3=16+54=70$

62 $f'(x)=0$

$x=-2,3$

$f(-2)=53$

63 Given that $9x\geq x^2+20\Rightarrow x^2-9x+20\leq 0$

$\therefore x=4,5$

For max $f'(x)=0$

$\Rightarrow 6x^2-30x+36=0$

$\Rightarrow x=2,3$

Max of $f(x)=$

Max of $\{f(2),f(3),f(4),f(5)\}$

$=\{-20,-21,-16,7\}$

64. *Let* $y=f(x)=2x^3-15x^2+36x+c$

$\therefore \frac{dy}{dx}=6x^2-30x+36$

for stationary points, $\frac{dy}{dx}=0$

$\Rightarrow 6(x-2)(x-3)=0$

$\Rightarrow x=2,3\Rightarrow a=2,\Rightarrow b=3$

$\therefore a=f(2)=16-60+72+c$

$\Rightarrow a=28+c$

Now $b=3$ & $f(3)=19$

$\Rightarrow 54-135+108+c=19$

$\Rightarrow 27+c=19$

$\Rightarrow c=-8$

Hence $a+b+c=20+3-8$

$=15$

65. $f(x)=px^3+qx^2+rx+t$

$f'(x)=3px^2+2qx+r$

$f'(x)=0$

$\Rightarrow x=-2,2$

$q=0, r=4$

$9x^2-1=0$

$x=\pm\frac{1}{3}$

$p=\frac{-1}{3}$

Then $3p+q+r=3$

66. Given

$2(l+b)=48$

$l+b=24$

diameter $=b$

$2r=b$

$r=\frac{b}{2}$

$v=\pi r^2 h=\pi\left(\frac{b}{2}\right)^2\times(24-b)$

$f'(b)=0 \quad b=4, l=20$

67. $f(x)=\frac{4}{3}x^3-4x \; in[0,2]$

$f^1(x)=\frac{4}{\cancel{3}}\cancel{3}x^2-4=4x^2-4$

$f^1(x)=0 \Rightarrow 4x^2-4=0 \Rightarrow x^2=1 \Rightarrow x=\pm 1$

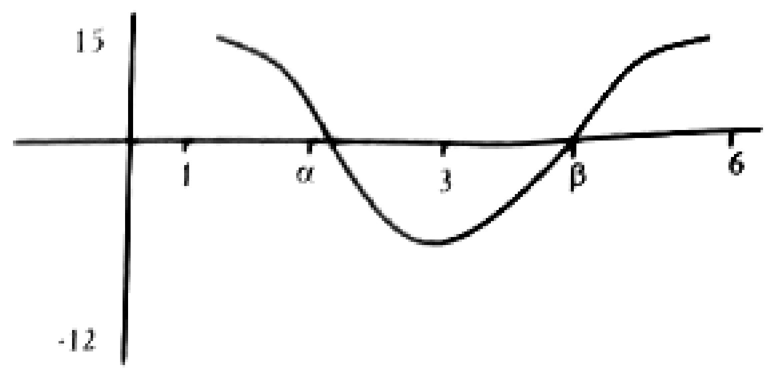
$f(0)=\frac{4}{3}(0)^3-4(0)=0$

$f(1)=\frac{4}{3}-4=\frac{-8}{3}$

$f(2)=\frac{32}{3}-8=\frac{8}{3}$

$sum=0$

68



$1<\alpha<3 \;,\; 3<\beta<6$

$-6<-\beta<-3$

$-5<\alpha-\beta<0$

$0<|\alpha-\beta|<5$

69. $a^2x^4+b^2y^4=c^6$

$b^2y^4=c^6-a^2x^4$

$x^4b^2y^4=x^4c^6-a^2x^8$

$x^4y^4=\frac{x^4c^6-a^2x^8}{b^2}$

Let, $f(x)=\frac{x^4c^6-a^2x^8}{b^2}$

$f^1(x)=0$

$\Rightarrow 4x^3c^6-a^2 8x^7=0$

$x^4=\frac{c^6}{2a^2}$

$x=\left(\frac{c^6}{2a^2}\right)^{1/4}$

Here, $f^{11}(x)<0 \; for \; x=\left(\frac{c^6}{2a^2}\right)^{1/4}$

i.e., f has the maximum value.

maximum value of f(x)

$=\frac{1}{b^2}\left[\frac{c^6}{2a^2}c^6-a^2\frac{c^{12}}{4a^4}\right]$

$=\frac{c^{12}}{b^2 2a^2}\left[1-\frac{1}{2}\right]=\frac{1}{2}\left(\frac{c^{12}}{2a^2b^2}\right)=\frac{c^{12}}{4a^2b^2}$

70 $f(x)=x^2+\frac{54}{x}$

$f^1(x)=2x-\frac{54}{x^2}$

Here, $f^1(x)<o\ for\ x\in(-\infty,o)$

f is decreasing on $(-\infty,o)$

Take $f^1(x)=0$

$$2x-\frac{54}{x^2}=0$$

$$x=3\notin(-\infty,0)$$

i.e., f has neither maximum nor minimum in $(-\infty,0)$

71. $f^1(x)=\cos x\left[\frac{-4}{\sin^2 x}+\frac{1}{(1-\sin x)^2}\right]$

For max or min $f^1(x)=0$

$$\Rightarrow\frac{1}{(1-\sin x)^2}=\frac{4}{\sin^2 x}$$

$$\Rightarrow\sin^2 x=4(1-\sin x)^2$$

$$\Rightarrow\sin x=2(1-\sin x)$$

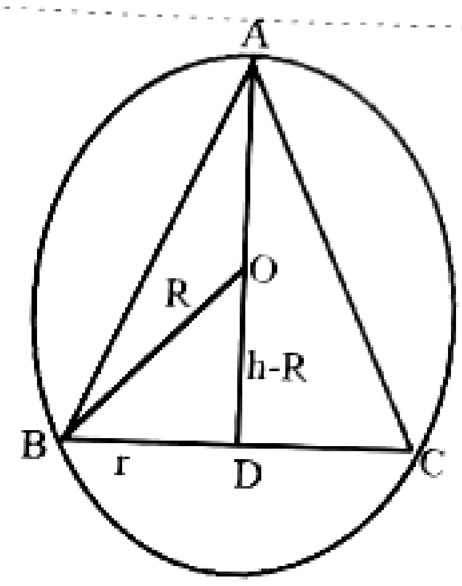
$$\Rightarrow\sin x=\frac{2}{3}$$

$\therefore$ min value $f(x)=\frac{4}{\frac{2}{3}}+\frac{1}{1-\frac{2}{3}}$

$=6+3$

$=9$

72.



i.e., OA = OB = R, AD = h, BD = r

OD = AD – OA = h–R

In

$\Delta^{le} OBD,(OB)^2=(OD)^2+(BD)^2\Rightarrow R^2=(h-R)^2+r^2\Rightarrow\boxed{r^2=2hR-h^2}$

Volume of the cone

$$V=\frac{1}{3}\pi r^2h=\frac{\pi}{3}(2hR-h^2)h=\frac{2\pi h^2R}{3}-\frac{\pi h^3}{3}$$

Now

$$\frac{dv}{dh}=0\Rightarrow\frac{4\pi hR}{3}-\pi h^2=0\Rightarrow\boxed{h=\frac{4R}{3}}$$

$$\therefore V=\frac{2\pi R}{3}.\frac{16R^2}{9}-\frac{64\pi R^3}{81}=\frac{32\pi R^3}{81}$$

$$=\frac{8}{27}\times\frac{4}{3}\pi R^3$$

W.K.T volume of sphere is $V_s=\frac{4}{3}\pi R^3$

73. Given $f(x)=\frac{x}{5}+\frac{5}{x}$

$f^1(x)=\frac{1}{5}\quad\frac{-5}{x^2}\Rightarrow f^1(x)=0\Rightarrow x=\pm 5$

$f^{11}(x)=\frac{10}{x^3}\Rightarrow f^{11}(-5)<0$

$\therefore f(x)\, has \max\ at\ x=-5=\alpha$

$Now\sqrt{\alpha^2+2\alpha-6}=\sqrt{25-10-6}=\sqrt{9}=3$

74. $2x+3y=50$

$2x=50-3y\Rightarrow 4x^2=(50-3y)^2$

$f(x)=(50-3y)^2\, y^3$

$f'(y)=2(50-3y)(-3)y^3+3y^2(50-3y)^2=0$

$\Rightarrow 3y^2(50-3y)(-2y+50-3y)=0$

$5y=50$

$2x=20$

$y=10=\beta$

$\boxed{x=10}$

$10=\alpha$

$\frac{\alpha}{2}+\frac{\beta}{5}=5+2=7$

75. $f(x)\ =x.e^{-x}$

$f^1(x)=x(-e^{-x})+e^{-x}$

$f^1(x)=0$

$-x+1=0$

$x=1$

$f(1)\ =\beta=1.e^{-1}=\frac{1}{e}$

$(\alpha_1\beta)=\ (1,1/e)$

76 $f(x)=x^3-2x^2+x-3$

For maximum or minimum $f^1(x)=0$

$\Rightarrow 3x^2-4x+1=0$

$\Rightarrow 3x^3-x-3x+1=0$

$\Rightarrow x[3x-1]-[3x-1]=0$

$x=1/3, x=1$

$x=0\Rightarrow f(0)=-3=M$

$x=2\Rightarrow f(2)=-1=m$

$M+m=-3-1=-4$