24. MEAN VALUE THEOREM

1. Find the value of 'p' and 'q' if the function $f(t)=t^3-6t^2+pt+q$ defined on $[1,3]$ satisfies the Rolle's theorem

For c $c=\frac{2\sqrt{3}+1}{\sqrt{3}}$

[AP EAMCET 21-09-20_Shift-2]

1. $p\in R, q=11$
2. $p=11, q\in R$
3. $p\in R, q\in R$
4. P=11,q=11

2. The number of admissible values of C obtained when the LaGrange's mean value theorem is applied for the function

$f(x)=x\ on[2,5]\ is$

[TS EAMCET 09-09-20_Shift-1]

1. 0
2. Only one
3. infinite
4. Finitely many

3. If the Rolles's theorem hlods for the function $f(x)=x^4+ax^3+bx$ in $-1\le x\le 1$, and

$f'\left(\frac{1}{2}\right)=0$, then $ab=$

[TS EAMCET 09-09-20_Shift-2]

1. -4
2. -64
3. -1
4. -8

4. In each of the choices given below, a function and an interval are given. The correct choice having a function and the associated interval for which the lagranges's mean value theorem is not valid is **[TS EAMCET 10-09-20_Shift-1]**

1. $|x|;[1,5]$
2. $\log x;[1,e]$
3. $\frac{2x-1}{3x-4};[1,2]$
4. $(x-2)^2(x-4)^2;[2,4]$

5. Consider the following statements

Statement(I):if $a_0+\frac{a_1}{2}+\frac{a_2}{3}+...+\frac{a_n}{n+1}=0$, where $a_0, a_1....a_n$ are real numbers, then the polynomial $a_0+a_1x+a_2x^2+...+a_nx^n$ has a zero in the interval $(0,1)$

Statement(II):if $f:[a,b]\to R$ is continuous on $[a.b]$ and f is differentiable in $(a.b)$,where $a>0$ and $\frac{f(a)}{a}=\frac{f(b)}{b}$, then there exists $c\in(a,b)$ such that $cf'(c)=f(c)$

[TS EAMCET 11-09-20_Shift-1]

1. Only (I) is true
2. Only (II) is true
3. Neither (I) nor (II) is true.
4. Both (I) and (II) are true.

6. The value of 'c', of the lagrange's mean value theorem, for $f(x)=\sqrt{x^2-x}, x\in[1,4]$ is ____

[AP EAMCET 23-08-2021_Shift-1]

1. $\frac{4}{3}$
2. $\frac{3}{2}$
3. $\frac{5}{4}$
4. 3

7. If the function $f(x)=ax^3+bx^2+26x-24$ satisfies the conditions of Rolle's theorem in $[2,4]$ and $f'\left(3+\frac{1}{\sqrt{3}}\right)=0$, then the value of $ab=$

[AP EAMCET 23-08-2021_Shift-2]

1. -9
2. 9
3. -3
4. 3

8. If $f(x)=x^\alpha \log x$ and $f(0)=0$, then the value of 'α' for which Rolle's theorem can be applied in [0, 1] is ______

[AP EAMCET 24-08-2021_Shift-1]

1. -2
2. -1
3. 0
4. $\frac{1}{2}$

9. If the function $f(x)=x(x+3)e^{\frac{-x}{2}}$ satisfies all the conditions of rolle's theorem in $[-3,0]$, then a root of $f'(x)=0$ is

[TS EAMCET 04-08-2021_Shift-1]

1. 3
2. -1
3. -2
4. -3

10. The Rolle's theorem is not applicable to $f(x)=\begin{cases} x \ , & 0\le x\le 1 \\ 2-x \ , & 1\le x\le 2 \end{cases}$ $on\ [0,2]$ because

[TS EAMCET 05-08-2021_Shift-1]

1. $f(x)$ is not defined everywhere
2. $f(x)$ is not continuous
3. $f(0)\neq f(2)$
4. $f(x)$ is not differentiable

11. $f:[1,3]\to R$ is a function defined as $f(x)=x^3+ax^2+bx$. If $f(1)-f(3)=0$ and $f'\left(\frac{2\sqrt{3}+1}{\sqrt{3}}\right)=0$. Then, $a-b=$

[TS EAMCET 06-08-2021_Shift-2]

1. 5
2. -17
3. $4\sqrt{3}$
4. $-2\sqrt{3}$

12. $f:[2,10]\to R$ is defined as

$$f(x)=\begin{cases} \frac{1}{2}(x-6)^2-3, & x\le 4 \\ x-5, & x>4 \end{cases}$$

Which of the following is true?

[TS EAMCET 06-08-2021_Shift-1]

1. $f(2)\neq f(10)$
2. $f(x)$ is not continuous on $[2,10]$
3. Rolle's theorem is not applicable for $f(x)$ in $[2,10]$
4. Rolle's theorem is applicable for $f(x)$ in $[2,10]$ and Rolle's point $c=6$

13. $y=x^3-ax^2+48x+7$ is an increasing function for all real values of x, then a lies in the interval

[AP EAMCET 05-07-2022_Shift-1]

1. $(-14, 14)$
2. $(-12, 12)$
3. $(-16, 16)$
4. $(-21, -21)$

14. In the interval $(7,\infty)$, $f(x)=|x-5|+2|x-7|$ is

[AP EAMCET 05-07-2022_Shift-2]

1. increasing function
2. decreasing function
3. constant function
4. attains maximum value

15. *If* $f(x)=\sqrt{3}\sin x-\cos x-2ax+b$ decreases for all values of x, then

[AP EAMCET 08-07-2022_Shift-2]

1. $a\ge 1$
2. a = 1
3. $a\le 1$
4. a<1

16. If $0<x<\frac{\pi}{2}$, then $f(x)=\frac{2}{\pi}-\frac{\sin x}{x}$

[AP EAMCET 08-07-2022_Shift-2]

1. $\frac{2}{\pi}>\frac{\sin x}{x}$
2. $\frac{2}{\pi}<\frac{\sin x}{x}$
3. $\frac{\sin x}{x}>1$
4. $2<\frac{\sin x}{x}$

17. The curve represented by $X=t^5+5t^3+20t+7$ and $y=4t^3-3t^2-18t+3$ is decreasing in the interval

[18th May2023 shift -1]

1. (-2, -1)
2. (3/2, 2)
3. (-1, 3/2)
4. (-2, 2)

18. In the interval $\left(\frac{1}{e},e\right)$, a decreasing function among the following functions is

[19th May 2023 Shift 1]

1. $f(x)=\frac{\log x}{x}$
2. $f(x)=x^2\log x$
3. $f(x)=x\log x$
4. $f(x)=x^{-x}$

KEY

1)	2	2)	3	3)	1	4)	3	5)	4
6)	2	7)	1	8)	4	9)	3	10)	4
11)	2	12)	3	13)	2	14)	1	15)	1
16)	2	17)	3	18)	4				

SOLUTIONS

1. Given $f(t)=t^3-6t^2+pt+q$ in $[1,3]$

w.k.t rolle's theorem $i.e., f'(c)=3c^2-12c+p$

$$c=\frac{12\pm\sqrt{144-12p}}{6}\ \frac{2\sqrt{3}+1}{\sqrt{3}}=\frac{12}{6}\pm\frac{\sqrt{144-12p}}{6}$$

$$\frac{1}{\sqrt{3}}=\frac{\sqrt{144-12p}}{6}$$

$1=12-p$

$p=11, q\in R$

2. $f(x)=x,\ f'(x)=1$

Lagranges theorem is applied at infinite points.

3. $f(x)=x^4+ax^3+bx\ x\in[-1,1]$

$f'(\frac{1}{2})=0$

$f'(x)=4x^3+3ax^2+b$

$f(-1)=f(1)$

$a=-b$

$f'(\frac{1}{2})=0$

$2+3a+4b=0$

$a=2, b=-2$

$ab=-4$

4. $f(x)=\dfrac{2x-1}{3x-4}$

$$f'(x)=\frac{-5}{(3x-4)^2}$$

$f(1)=-1$

$$f(2)=\frac{3}{2}$$

By L.M.V.T,

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$\frac{-5}{(3c-4)^2}=\frac{\frac{3}{2}+1}{2-1}$$

$9c^2+16-24c=-2$

$3c^2-8c+6=0$

$$c=\frac{8\pm2\sqrt{2}i}{6}$$

$\therefore c\notin[1,2]$

L.M.V.T fails

5. **Statement-I**

$$Let\ f(x)=a_0x+a_1\frac{x^2}{2}+a_2\frac{x^3}{3}+...+a_n\frac{x^{n+1}}{n+1}$$

$f(0)=0$

$$f(1)=a_0+\frac{a_1}{2}+\frac{a_3}{3}+...+\frac{a_n}{n+1}=0$$

$i.e., f(0)=f(1)$

By using Rolle's theorem, $f'(c)=0$ where $c\in(0,1)$

Now $f'(x)=a_0+a_1x+a_2x^2+...+a_nx^n$

$f'(c)=0$ where $c\in(0,1)$

Statement-II

Given $\dfrac{f(a)}{a}=\dfrac{f(b)}{b}\Rightarrow\dfrac{f(a)}{f(b)}=\dfrac{a}{b}$

By using L.M.V.T ,

$$f'(c)=\frac{f(b)-f(a)}{b-a}=\frac{kb-ka}{b-a}=k$$

$$f(c)=\int f'(c).dc=\int k.dc=k.\int dc=kc$$

$$\Rightarrow f(c)=f'(c).c\left[\because k=f'(c)\right]$$

6. $$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$\Rightarrow 4c^2-4c-3=0$

$$\Rightarrow c=\frac{3}{2}\in[1,4]$$

7. $f(2)=f(4)\Rightarrow 28a+6b+26=0.\to(1)$

$$f'\left(3+\frac{1}{\sqrt{13}}\right)=0\Rightarrow$$

$$28a+6\sqrt{3}a+6b+\frac{2}{\sqrt{3}}b+26=0\to(2)$$

$$(2)-(1) \Rightarrow 6\sqrt{3}a = -\frac{2}{\sqrt{3}}b$$

$$\Rightarrow b = -9a$$

$$from(1)\, a = 1$$

$$\Rightarrow b = -9$$

$$\therefore ab = -9$$

8. $f(x)$ is continuous at $x = 0$.

$$\Rightarrow \lim_{x \to 0^+} f(x) = f(0).$$

$$\Rightarrow \lim_{x \to 0^+} \frac{\log x}{x^{-\alpha}} = 0.$$

$$\Rightarrow \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\alpha . x^{-\alpha-1}} = 0$$

$$\Rightarrow \lim_{x \to 0^+} \frac{x^{\alpha}}{-\alpha} = 0.$$

It is true when $\alpha > 0$.

$$\alpha = \frac{1}{2}$$

9. $f(x) = x(x+3)e^{-x/2}$

Clearly $f(-3) = f(0) = 0$

$$f'(x) = x(x+3)e^{-x/2}\left(-\frac{1}{2}\right) + (x+3)e^{-x/2} + xe^{-x/2}$$

$$= \frac{e^{-x/2}}{2}\left(-x^2 + x + 6\right)$$

now, $f'(x) = 0$

$$\Rightarrow -x^2 + x + 6 = 0 \left[\because e^{-x/2} \neq 0\right]$$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x-3)(x+2) = 0$$

$$x = 3, -2$$

$$\therefore x = -2 \ \left[\because x \in [-3, 0]\right]$$

10. $$f(x) = \begin{cases} x \ , & 0 \le x \le 1 \\ 2 - x \ , & 1 \le x \le 2 \end{cases}$$

$$LHL = \lim_{x \to 1^-} f(x) = \lim_{x \to 1} x = 1$$

$$RHL = \lim_{x \to 1^+} f(x) = \lim_{x \to 1} 2 - x = 1$$

$$\Rightarrow \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$\Rightarrow$ *continuous at* $x = 1$

$$LHD = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{x - 1}{x - 1} = 1$$

$$RHD = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{2 - x - 1}{x - 1} = -1$$

$$LHD \neq RHD$$

$\Rightarrow f(x)$ is not differentiable at $x = 1$

Hence Rolle's can't be applied.

11. $f(x) = x^3 + ax^2 + bx$

$$f(1) = f(2)$$

$$4a + b = -13 \ldots\ldots (1)$$

$$f'\left(2 + \frac{1}{\sqrt{3}}\right) = 0 \in 2 + \frac{1}{\sqrt{3}} \in [1, 3]$$

$$f'(x) = 0$$

$$f'\left(2 + \frac{1}{\sqrt{3}}\right) = \left(4 + \frac{2}{\sqrt{3}}\right)a + b + 4\sqrt{3} + 13 = 0$$

$$(1) - (2) \Rightarrow \frac{2a}{\sqrt{3}} + 4\sqrt{3} = 0 \Rightarrow \boxed{a = -6}$$

From (1), $\boxed{b = 1}$

$$\therefore a - b = -6 - 11 = -17$$

12. Conceptual

13. $y = x^3 - ax^2 + 48x + 7$

$$\frac{dy}{dx} = 3x^2 - 2ax + 48 > 0, \forall x \in R$$

$$\Delta < 0$$

$$(-2a)^2 - 4(3)(48) < 0$$

$$a^2 - 144 < 0$$

14. $f(x)=|x-5|+2|x-7|, \quad x\in(7,\infty)$

$=(x-5)+2(x-7)$

$=3x-19$

$f'(x)=3>0$

15. $f(x)=\sqrt{3}\sin x-\cos x-2ax+b$

$f'(x)=\sqrt{3}\cos x+\sin x-2a<0$

16. $0<x<\frac{\pi}{2}$

$f(x)=\frac{2}{\pi}-\frac{\sin x}{x}$

Find $f'(x)$ and check if increasing or decreasing.

17. $\frac{dx}{dt}=5t^4+15t^2+20=5(t^4+3t^2+4)$

$\frac{dy}{dt}=12t^2-6t-18=6(2t^2-t-3)$

$\frac{dy}{dx}=\frac{6(2t^2-t-3)}{5(t^4+3t^2+4)}<0(\text{decreasing})$

$2t^2-t-3<0$

$2t^2-3t+2t-3<0$

$(t+1)(2t-3)<0$

$t\in\left(-1,\frac{3}{2}\right)$

$\therefore t^4+3t^2+4 \text{ is +ve } \forall t\in R$

18. Take $f(x) = x^{-n}$
Now $f^{l}(x)<0$
i.e., $f^{l}(x) = -x^{-n}[1+\log x]$
Now $-x^{-x}(1+\log x)<0$ (decreasing function)

$\therefore \boxed{x\in\left(\frac{1}{e},e\right)}$