2. MATHEMATICAL INDUCTION

1. Let $P(n): 2+2^2+2^3+....+2^n=2^{n+1}$, $n \in N$. Then **[AP EAMCET 17-09-20_Shift-2]**
   1. $p(m)$ is true $\Rightarrow P(m+1)$ is true
   2. $P(n)$ is true for all $n \in N$
   3. $P(n)$ is true for all $n \geq 20$
   4. $P(n)$ is true for all $n \leq 10$

2. a^n+b^n is divisible by, if n is any odd positive integer **[AP EAMCET 18-09-20_Shift-1]**
   1. $a-b$
   2. a^2-b^2
   3. a^2+b^2
   4. $a+b$

3. Let P (n):
   $1^2+2^2+3^2+.......n^2 = \dfrac{6(n-1)(n-2).....(n-2020)+2n^3+3n^2+n}{6}$, for all $n \in N$. Then which of the following is correct? **[AP EAMCET 21-09-20_Shift-1]**
   1. P(n) is true for all $n \in N$
   2. P(n) is true for all n > 2020
   3. P(n) is true for all n $\leq$ 2020
   4. P(n) is not true for any $n \in N$

4. The sum of the cubes of three consecutive natural numbers is divisible by **[AP EAMCET 21-09-20_Shift-2]**
   1. 26
   2. 25
   3. 9
   4. 7

5. How many multiples of 5 are there from 10 to 95 including both 10 and 95? **[AP EAMCET 22-09-20_Shift-1]**
   1. 17
   2. 18
   3. 16
   4. 19

6. The ten's digit in 1!+4!+7!+10!+12!+13!+15!+16!+17!is divisible by**[AP EAMCET 22-09-20_Shift-2]**
   1. 4!
   2. 3!
   3. 5
   4. 7

7. $If\ k \in N, then\ 3^{3k}-26^k-1\ divisible\ by$
   1. 676
   2. 8
   3. 64
   4. 26

8. For n $\in$ N, what is the value of 1+4+7+ .... +(3n-2)
   1. $\dfrac{n(3n+1)}{2}$
   2. $\dfrac{(3n-1)}{2}$
   3. $\dfrac{n(3n-1)}{2}$
   4. $\dfrac{(3n+1)}{2}$

9. Let $m=(9n^2+54n+80)(9n^2+45n+54)(9n^2+36n+35)$. The greatest positive integer which divides m, for all positive integers n, is **[TS EAMCET 09-09-20_Shift-1]**
   1. 720
   2. 724
   3. 696
   4. 842

10. The expression for a_n which satisfies $a_0=0, a_1=1\ and\ a_n=a_{n-1}+a_{n-2}\ \forall n \in N-\{0,1\}$ from the following is **[TS EAMCET 09-09-20_Shift-2]**
    1. $\dfrac{1}{\sqrt{5}}\left(\dfrac{1+\sqrt{5}}{2}\right)^n-\dfrac{1}{\sqrt{5}}\left(\dfrac{1-\sqrt{5}}{2}\right)^n$
    2. $\dfrac{1}{\sqrt{7}}\left(\dfrac{1+\sqrt{7}}{2}\right)^n-\dfrac{1}{\sqrt{7}}\left(\dfrac{1-\sqrt{7}}{2}\right)^n$
    3. $\dfrac{1}{\sqrt{2}}\left(\dfrac{1+\sqrt{2}}{2}\right)^n-\dfrac{1}{\sqrt{2}}\left(\dfrac{1-\sqrt{2}}{2}\right)^n$
    4. $\dfrac{1}{\sqrt{3}}\left(\dfrac{1+\sqrt{3}}{2}\right)^n-\dfrac{1}{\sqrt{3}}\left(\dfrac{1-\sqrt{3}}{2}\right)^n$

11. Let the greatest common divisor of m, n be 1. $If\ \dfrac{1}{1.7}+\dfrac{1}{7.13}+\dfrac{1}{13.19}+.....up\ to\ 20\ terms = \dfrac{m}{n}, then\ 5m+2n=$ **[TS EAMCET 10-09-20_Shift-1]**
    1. 325
    2. 330
    3. 342
    4. 337

12. If S_n is the sum of the first n terms of the series $1^2+2\times2^2+3^2+2\times4^2+5^2+2\times6^2+....\infty,$ Then, when n is even $S_n=$ **[TS EAMCET 10-09-20_Shift-2]**

1. $\frac{n(n+1)}{2}$
2. $\frac{n^2(n+1)}{2}$
3. $\frac{n(n+1)^2}{2}$
4. $\frac{n^2(n+2)}{2}$

13. n^5-5n^3+4n is divisible by 120 is true for

[TS EAMCET 11-09-20_Shift-1]

1. all positive integers n
2. all positive integers for $n \ge 3$ only
3. all positive integers for $n \ge 1$ only
4. all positive integers for $n \ge 5$ only

14. For $n>2$ and $n \in N$, the product of the roots of $(x-n)\left((x^2-2nx)^2+(2n^2-5)(x^2-2nx)+(n^4-5n^2+4)\right)=0$ is divisible by

[TS EAMCET 11-09-20_Shift-2]

1. 625
2. 25
3. 120
4. 80

15. If $f(1)=3$, and $f(n+1)-f(n)=3(4^n-1)$, then $\forall n \in N$, $f(n)=$

[TS EAMCET 14-09-20_Shift-1]

1. 4^n-1
2. 4^n-5n+4
3. 4^n-3n+2
4. 4^n+4n-5

16. $n \in N$ then the statement $8n+16 \le 2^n$ is true for

[AP EAMCET 19-08-2021_Shift-1]

1. n=2
2. n=3
3. n=6
4. n=5

17. For what natural numbers $n \in N$, the inequality $2^n > n+1$ is valid?

[AP EAMCET 19-08-2021_Shift-2]

1. $\forall n \in N$
2. $\forall n \ge 2$
3. $\forall 1 \le n \le 3$
4. $\forall n \in N-\{2,3\}$

18. Using mathematical induction,the numbers a_n's are defined by $a_0=1, a_{n+1}=3n^2+n+a_n \, (n \ge 0)$, then a_n= **[AP EAMCET 20-08-2021_Shift-2]**

1. n^3+n^2+1
2. n^3-n^2+1
3. n^3-n^2
4. n^3+n^2

19. If 'n' is a positive integer, then $2.4^{2n+1}+3^{3n+1}$ is divisible by

[AP EAMCET 23-08-2021_Shift-1]

1. 2
2. 9
3. 11
4. 27

20. For what values of $m \in N$, the following divisibility $x+y / x^m+y^m$ holds?

[AP EAMCET 24-08-2021_Shift-1]

1. Even numbers
2. Odd numbers
3. All natural numbers
4. Only when $m=1$

21. If $1^4+2^4+3^4+....+n^4=f(n)(1^2+2^2+.....+n^2), \forall n \in N$, then $f(4)=$

[AP EAMCET 24-08-2021_Shift-2]

1. $\frac{58}{5}$
2. $\frac{57}{5}$
3. $\frac{59}{5}$
4. $\frac{56}{5}$

22. For all $n \in N, (n+24)(n+25)(n+26)(n+27)$ is divisible by

[AP EAMCET 25-08-2021_Shift-1]

1. 27
2. 26
3. 29
4. 24

23. For all $n \in N$, $2^{2n+1}+3^{2n+1}$ is divisible by

[TS EAMCET 04-08-2021_Shift-2]

1. 7
2. 5
3. 11
4. 8

24. For any $n \in N$,

$$\frac{1}{2.5}+\frac{1}{5.8}+....+\frac{1}{(3n-1)(3n+2)}=$$

[TS EAMCET 04-08-2021_Shift-1]

1. $\frac{n}{6n+4}$
2. $\frac{n^2}{6n+4}$
3. $\frac{1}{2}.\frac{n^2}{6n+4}$
4. $\frac{n}{6n^2+4}$

25. If $f(1)=0$ and $f(n+1)-f(n)=5n\ \forall n\in N$

then $f(n)=$

[TS EAMCET 05-08-2021_Shift-1]

1. $\frac{5}{2}(n^2+n)$ 2. $\frac{5}{2}(n^2-n)$

3. $\frac{5}{3}(3n^2-n)$ 4. $\frac{5}{4}(4n^2-1)(n-1)$

26. If $\frac{1}{1\cdot5}+\frac{1}{5\cdot9}+\frac{1}{9\cdot13}+\ldots to\ n\ terms=\frac{27}{109}$ then $n=$ **[TS EAMCET 05-08-2021_Shift-2]**

1. 21 2. 27
3. 63 4. 189

27. For all natural numbers $n, 3(5^{2n+1})+2^{3n+1}$ is divisible by

[TS EAMCET 06-08-2021_Shift-2]

1. 559 2. 17
3. 19 4. 23

28. For any $n\in N, 4^n+15n-1$ is divisible by

[TS EAMCET 06-08-2021_Shift-1]

1. 2 2. 9
3. 5 4. 6

KEY

1)	1	2)	4	3)	3	4)	3	5)	2
6)	2	7)	4	8)	3	9)	1	10)	1
11)	3	12)	3	13)	1	14)	3	15)	3
16)	3	17)	2	18)	2	19)	3	20)	4
21)	3	22)	4	23)	2	24)	1	25)	2
26)	2	27)	2	28)	2	29)		30)	

SOLUTIONS

1. Given

$p(n)=2+2^2+2^3+\ldots\ldots..+2^n=2^{n+1}, n\in N$

$P(m)$ is true

Now we that P.T $p(m+1)$ is true

$(2+2^2+2^3+\ldots\ldots..2^m)+2^{m+1}=2^{m+1}+2^{m+1}$

$=2(2^{m+1})$

$=2^{(m+1)+1}$

$\therefore$ p(m+1) is true

2. a^n+b^n is divisible by a+b if n is odd positive integer

3. $\frac{n(n+1)(2n+1)}{6}=$

$\frac{6(n-1)(n-2)\ldots\ldots..(n-2020)+2n^3+3n^2+n}{6}$

Since

$2n^3+3n^2+n=n(n+1)(2n+1)$

$6(n-1)(n-2)\ldots\ldots.(n-2020)=0$

$\therefore n=1,2,3,\ldots\ldots\ldots.2020$

$n\le 2020$

4. Let $(n-1), n, (n+1)$ are 3 consecutive natural no's

Given condition is

$(n-1)^3+n^3+(n+1)^3=6n+3n^3$

$=3n(n^2+2)$

$=3n(n^2-1+3)$

$=3(n(n-1)(n+1)+3n)$

$\therefore n(n-1)(n+1)$ & $3n$ are both divisible by 3.

Hence their sum is also divisible by 3

$\therefore$ the whole no is divisible by 9

5. 10,15,…,95

95=10+(n-1)5

85=(n-1)5

n=18

6. conceptual

7. $3^{3k}-26^k-1$ is divisible by

$n=1\Rightarrow$ G.E =0

$n=2\Rightarrow$ G.E =52

$\therefore$ G.E is divisible by 26.

8. $1+4+7+\ldots\ldots+(3n-2)$

$\sum(3n-2)=3\sum n-2\sum 1$

$=3\frac{n(n+1)}{2}-2n=\frac{n(3n-1)}{2}.$

9. $m=(9n^2+54n+80)(9n^2+45n+54)(9n^2+36n+35)$

For n =1

$m=143\times108\times80=1235520$

By verification 720 divides m

10. $a_0=0, a_1=1$

$a_n=a_{n-1}+a_{n-2}$

$a_2=a_1+a_0=1$

$a_3=a_2+a_1=2$

$a_4=a_3+a_2=3$

By verification n=3

$$a_3=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^3-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^3$$

$$=\frac{1}{\sqrt{5}}\left(\frac{10\sqrt{5}+6\sqrt{5}}{8}\right)=2$$

11. n^{th} term $=\dfrac{1}{(6n+1)(6n-5)}$

$$\frac{1}{1.7}+\frac{1}{7.13}+\frac{1}{13.19}+.....+\frac{1}{(6n+1)(6n-5)}$$

$$=\frac{1}{6}\left[\frac{7-1}{1.7}+\frac{13-7}{7.13}+\frac{19-13}{13.19}+.....+\frac{(6n+1)-(6n-5)}{(6n+1)(6n-5)}\right]$$

$$=\frac{1}{6}\left[1-\frac{1}{6n+1}\right]=\frac{n}{6n+1}$$

$$put\ n=20\rightarrow\frac{n}{6n+1}=\frac{20}{121}$$

Here m=20,n=121

5m+2n=342

12. $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+........\infty$

$=1^2+2.2^2+3^2+2.4^2+......+(n-1)^2+2n^2$

$=1^2+2.2^2+3^2+2.4^2+......+(2m-1)^2+2(2m)^2$

$[1^2+2^2+3^2+.......+(2m-1)^2+(2m)^2]$

$+\ 2^2[1^1+2^2+....+m^2]$

$$=\frac{2m(2m+1)(4m+1)}{6}+4\left(\frac{m(m+1)(2m+1)}{6}\right)$$

$$=\frac{n(n+1)(2n+1)}{6}+\frac{4\left(\frac{n}{2}\right)\left(\frac{n}{2}+1\right)(n+1)}{6}$$

$$=\frac{n(n+1)^2}{2}$$

Put $n=0,\ 0^5-5.0^3+4.0=0$ is divisible by 120

Put $n=1,\ 1^5-5.1^3+4.1=0$ is divisible by 120

Put $n=2,\ 2^5-5.2^3+4.2=0$ is divisible by 120

Put $n=3,\ 3^5-5.3^3+4.3=120$ is divisible by 120

Clearly, among the options, n^5-5n^3+4n is divisible by 120 $\forall\ n\in Z^+$

13. Put $n=0,\ 0^5-5.0^3+4.0=0$ is divisible by 120

Put $n=1,\ 1^5-5.1^3+4.1=0$ is divisible by 120

Put $n=2,\ 2^5-5.2^3+4.2=0$ is divisible by 120

Put $n=3,\ 3^5-5.3^3+4.3=120$ is divisible by 120

Clearly, among the options, n^5-5n^3+4n is divisible by 120 $\forall\ n\in Z^+$

14. $x\left[(x^2-2nx)^2+(2x^2-5)(x^2-2nx)+(x^4-5x^2+4)\right]$

$-n\left[(x^2-2nx)^2+(2n^2-5)(x^2-2nx)-n(n^4-5n^2+4)\right]=0$

Product of the roots

$=n(n^4-5n^2+4)$

$=n(n^2-4)(n^2-1)$

$=(n-2)(n-1).n(n+1)(n+2)$

It is divisible by $5!=120$

15. $f(n+1)-f(n)=3(4^n-1)$

Put n=1

$f(2)-f(1)=3(3)$

$f(2)=12$ it is true

$f(n)=4^n-3n+2$ (option verification)

16. $n\in N,\ 8n+16\le 2^n$

For $n=6,\ 64\le 2^6\Rightarrow 64\le 64$ which is true.

$n\in N,\ 2^n>n+1$

17. For $n=1,\ 2>2$ not true.

For $n=2,\ 4>3$ which is true.

$\therefore \forall n>1,$ given statement is true.

18. $a_0=1, a_{n+1}=3n^2+n+a_n$

Put $n=0$

$a_1=a_0=1$

Option verify for $n=0$ & $n=1$

19. Let $S(n)=2.4^{2n+1}+3^{3n+1}$

$S(1)=2.(64)+81=209=11\times 19$

Clearly divisible by 11, as per options.

20. Let $S(m)\equiv\dfrac{x+y}{x^m+y^m}, m\in N$

Clearly, $S(1)\equiv\dfrac{x+y}{x+y}=1$

For any $m>1$, $S(m)$ is not true.

21. $1^4+2^4+3^4+....+n^4=f(n)\left(1^2+2^2+.....+n^2\right)$

For $n=4$

$1^4+2^4+3^4+4^4=f(4)\left(1^2+2^2+3^2+4^2\right)$

$\Rightarrow 354=f(4).30$

$\therefore f(4)=\dfrac{59}{5}$

22. $n\in N,(n+24)(n+25)(n+26)(n+27)$

The product any 4 consecutive integers are always divisible by $4!=24$.

23. $S\equiv 2^{2n+1}+3^{2n+1}, n\in N$

Suppose $n=1$

$S\equiv 2^3+3^3=8+27=35$

$n=2$

$S\equiv 2^5+3^5=32+243=275$

So S is divisible by 5

24. $\dfrac{1}{2.5}+\dfrac{1}{5.8}+....+\dfrac{1}{(3n-1)(3n+2)}$

$=\dfrac{1}{3}\left\{\dfrac{1}{2}-\dfrac{1}{5}+\dfrac{1}{5}-\dfrac{1}{8}+...+\dfrac{1}{3n-1}-\dfrac{1}{3n+2}\right\}$

$=\dfrac{1}{3}\left\{\dfrac{1}{2}-\dfrac{1}{2n+1}\right\}$

$=\dfrac{n}{6n+4}$

25. Given $f(n+1)-f(n)=5n \text{ and } f(1)=0$

$n=0\Rightarrow f(1)=0$

$n=1\Rightarrow f(2)=5+f(1)$

$n=2\Rightarrow f(3)=10+f(2)=10+5+f(1)$

$n=3\Rightarrow f(4)=15+f(3)=15+10+5+f(1)$

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$f(n)=5(n-1)+5(n-2)+....+15+10+5+f(1)$

$\Rightarrow f(n)=f(1)+5\left[1+2+3+...+(n-1)\right]$

$\Rightarrow f(n)=0+5\left[\dfrac{(n-1)(n-1+1)}{2}\right]$

$\therefore f(n)=\dfrac{5}{2}\left[n^2-n\right]$

26. 1,5,9…in A.P. then n^{th} term $=a+(n-1)d$

$=1+(n-1)4=(4n-3)$

n^{th} term of the given series $=\dfrac{1}{(4n-3)(4n+1)}$

$\dfrac{1}{4}\left[\dfrac{5-1}{1.5}+\dfrac{9-5}{5.9}+\dfrac{13-9}{9.3}+...+\dfrac{(4n+1)-(4n-3)}{(4n+1)(4n-3)}\right]=\dfrac{27}{109}$

$\dfrac{1}{4}\left[1-\dfrac{1}{5}+\dfrac{1}{5}-\dfrac{1}{9}+...+\dfrac{1}{4n-3}-\dfrac{1}{4n+1}\right]=\dfrac{27}{109}$

$\Rightarrow\dfrac{1}{4}.\dfrac{4n+1-1}{4n+1}=\dfrac{27}{109}$

$\Rightarrow\dfrac{n}{4n+1}=\dfrac{27}{109}$

$\Rightarrow 109n=108n+27\Rightarrow n=27$

27. $3\left(5^{2n+1}\right)+2^{3n+1}$

Put $n=1,\ 3.5^3+2^4=391=23\times 17$

$n=2,\ 3.5^5+2^7=9503=559\times 17$

$\therefore 3.5^{2n+1}+2^{3n+1}$ is divisible by 17.

28. Let $S(n)=4^n+15n-1$

$S(1)=18, S(2)=45$

H.C.F of 18, 45 is 9

$S(n)=4^n+15n-1$ is divisible by 9.