8. PROPERTIES OF TRIANGLES

1. The perimeter of Δ ABC is 36cm and its in-radius is 8cm. Then the area of the triangle is **[AP EAMCET 17-09-20_Shift-1]**
   1. 144 cm^2 2. 124 cm^2
   3. 164 cm^2 4. 104 cm^2

2. In a ΔABC, b:c= $\sqrt{3}:\sqrt{2}$ and the angles A, B, C are in A.P, then $\angle A=$ **[AP EAMCET 18-09-20_Shift-1]**
   1. 45^o 2. 60^o
   3. 55^o 4. 75^o

3. For a right-angled triangle having the lengths of two sides as $2\sqrt{2}$ and 5, find the length of the third side. **[AP EAMCET 21-09-20_Shift-1]**
   1. $4\sqrt{2}$ 2. $\sqrt{15}$
   3. $\sqrt{17}$ 4. $\sqrt{13}$

4. If r = inradius, Δ=Area of Δ ABC, s=semi-perimeter then which of the following is true? **[AP EAMCET 21-09-20_Shift-1]**
   1. Δ=r+s 2. $\Delta=\frac{r}{s}$
   3. $\Delta=(rs)^2$ 4. $\Delta=rs$

5. In a $\Delta ABC, a=1, b=\sqrt{3}\ and\ \angle C=\frac{\Pi}{6}$. Then the measure of the third side c= **[AP EAMCET 21-09-20_Shift-2]**
   1. 4 2. 3
   3. 1 4. 2

6. If 2 is the length of a side of a triangle with its opposite angle $\frac{\pi}{3}$, then the circumradius of the triangle is **[AP EAMCET 22-09-20_Shift-1]**
   1. $\frac{2}{\sqrt{3}}$ 2. $\frac{4}{\sqrt{3}}$
   3. 2 4. 4

7. In ΔPQR, find $\sum(q+r)\cos P$, if p, q, r denote its sides and $s=\frac{(p+q+r)}{2}$ **[AP EAMCET 22-09-20_Shift-1]**
   1. S 2. s/2
   3. 2s 4. 4s

8. In a ΔABC, if a,b,c are its sides and $\angle C=60^0$, find the value of $\frac{a}{b+c}+\frac{b}{c+a}$ **[AP EAMCET 22-09-20_Shift-1]**
   1. 1 2. 0
   3. $\frac{\sqrt{3}}{2}$ 4. $\frac{1}{2}$

9. The angles A,B,C of a triangle ABC are in AP. If AB=6, BC=7 then AC= **[AP EAMCET 22-09-20_Shift-2]**
   1. $\sqrt{40}$ 2. $\sqrt{41}$
   3. $\sqrt{43}$ 4. 6

10. If the lengths of the sides of a triangle are 15,20,25 units.Find the circumradius of the triangle ? **[AP EAMCET 23-09-20_Shift-1]**
    1. 30 units 2. 7.5 units
    3. 12.5 units 4. 20 units

11. Let a,b and c be the lengths of the sides of a triangle with its opposite angles A,B and C respectively. If a=3, b=4 and A=$\sin^{-1}(\frac{3}{4})$, then the angle B is **[AP EAMCET 23-09-20_Shift-1]**
    1. 30^0 2. 45^0
    3. 90^0 4. 60^0

12. In ΔABC, $\angle A=30^0+\angle C$ and R- $(\sqrt{3}+1)$r =0 where r is the inradius and R is the circumradius, then ___ **[AP EAMCET 23-09-20_Shift-1]**
    1. ABC is a right-angled triangle
    2. ABC is an equilateral triangle
    3. ABC is acute -angled
    4. $\angle A=75^0, \angle B=60^0, \angle C=45^0$

13. In ΔABC if a=13cm, b=14cm, c=15cm then its circumradius, R=
    1. $\frac{8}{65}CM$ 2. $\frac{7}{65}CM$
    3. $\frac{65}{7}CM$ 4. $\frac{65}{8}CM$

14. In ΔABC, the sides a, b, c are in A.P. if and only if r_1, r_2, r_3 are in
    1. A.P. 2. G.P.
    3. H.P. 4. A.G.P

15. In ΔABC, if $a:b:c=4:5:6$, then the ratio of the circumradius to its inradius is

[TS EAMCET 09-09-20_Shift-1]

1. 16:7
2. 7:16
3. 4:5
4. 5:4

---

16. In a ΔABC, if $b=10, a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=15$, and the area of the triangle is $15\sqrt{3}$ sq. units ,then $\cot\frac{B}{2}=$

[TS EAMCET 09-09-20_Shift-1]

1. $\frac{3}{2}$
2. $\frac{1}{2}$
3. $\frac{2}{\sqrt{3}}$
4. $\frac{5}{\sqrt{3}}$

---

17. If d_1,d_2,d_3 are the diameters of three ex-circles of a ΔABC then $d_1d_2+d_2d_3+d_3d_1=$

[TS EAMCET 09-09-20_Shift-1]

1. $(a+b+c)^2$
2. $ab+bc+ca$
3. $4\Delta^2$
4. $2s^2$

---

18. The perimeter of a ΔABC is 6 times the arithmetic mean of the sine of its angles. if its side BC is of unit length, then $\angle A=$

[TS EAMCET 11-09-20_Shift-1]

1. $\frac{\pi}{6}$
2. $\frac{\pi}{3}$
3. $\frac{\pi}{2}$
4. π

---

19. In a ΔABC, with usual notation, if $r=r_1-r_2-r_3$, then 2R= **[TS EAMCET 11-09-20_Shift-1]**

1. A
2. b+c
3. C
4. c+a

---

20. In a ,let $a,b,c,r,R,I,S,r_1,r_2,r_3$ stand for their usual meaning. Then match the items of List-I with those of the items of List-II

List - I	List - II
$A)\tan\frac{A}{2}=\frac{r}{s-a}$	$I)(AI)\left(\sqrt{\frac{(s-b)(s-c)}{bc}}\right)$
$B)r$	$II)R^2$
$C)(SI)^2+2Rr$	$III)\left(4R+r+\sqrt{2}s\right)\left(4R+r-\sqrt{2}s\right)$
$D)r_1^2+r_2^2+r_3^2$	$IV)\frac{Rr}{S}$
	$V)\frac{(s-b)(s-c)}{\Delta}$

1. A B C D — I V IV III
2. A B C D — V I II III
3. A B C D — V I IV III
4. A B C D — I IV III II

---

21. In a triangle ABC, if $a<b<c$ and $\frac{a^3+b^3+c^3}{\sin^3A+\sin^3B+\sin^3C}=8$, then the maximum value of c is

[TS EAMCET 11-09-20_Shift-2]

1. 3
2. 4
3. 2
4. 6

---

22. In a triangle ABC, if $c=9, s=10$ and $\Delta=10\sqrt{2}$ then $b\left[1+\sqrt{2}\tan\left(\frac{A-B}{2}\right)\right]=$

[TS EAMCET 11-09-20_Shift-2]

1. $a\left[1-\sqrt{2}\tan\left(\frac{A-B}{2}\right)\right]$
2. $c\left[1-\sqrt{2}\tan\left(\frac{A-B}{2}\right)\right]$
3. $a\left[\sqrt{2}\tan\left(\frac{A-B}{2}\right)-1\right]$
4. $c\left[\sqrt{2}\tan\left(\frac{A-B}{2}\right)-1\right]$

---

23. In a triangle ABC, $\cot A+\cot B+\cot C=$

[TS EAMCET 11-09-20_Shift-2]

1. $\frac{a^2+b^2+c^2}{\Delta}$
2. $\frac{a+b+c}{4\Delta}$
3. $\frac{a^2+b^2+c^2}{4\Delta}$
4. $\frac{a^2+b^2+c^2}{2\Delta}$

---

24. In ΔABC, if $a^2-c^2=b(b-c)$, $\sqrt{2}a=2b-c$ and $R=\frac{1}{\sqrt{3}}$ then $b=$

[TS EAMCET 09-09-20_Shift-2]

1. $\frac{\sqrt{2}}{\sqrt{3}}$ 2. $\frac{\sqrt{3}-1}{\sqrt{6}}$
3. $\frac{\sqrt{3}+1}{\sqrt{6}}$ 4. $\frac{\sqrt{3}}{\sqrt{2}}$

25. If ΔABC is a non isosceles triangle and $\angle C=90^0$ then $\frac{a^2+b^2}{a^2-b^2}\sin(A-B)=$

[TS EAMCET 09-09-20_Shift-2]

1. 1 2. 2
3. 0 4. -1

26. In any triangle ABC, $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=$

[TS EAMCET 09-09-20_Shift-2]

1. $1+\frac{2r}{R}$ 2. $2-\frac{1}{2R}$
3. $2\left(1+\frac{r}{4R}\right)$ 4. $2\left(1+\frac{r}{2R}\right)$

27. In a ΔABC if $\angle A=3\angle B, CA=9$ and $BC=16$ then the length of AB is

[TS EAMCET 10-09-20_Shift-1]

1. $\frac{5}{3}$ 2. $\frac{7}{3}$
3. 2 4. $\frac{35}{3}$

28. In $\Delta ABC, \frac{1+\cos C}{r_1+r_2}+\frac{1+\cos A}{r_2+r_3}+\frac{1+\cos B}{r_1+r_3}$

[TS EAMCET 10-09-20_Shift-1]

1. $\frac{2}{3R}$ 2. $\frac{R}{2}$
3. $\frac{3}{2R}$ 4. $\frac{6R}{5}$

29. In a $\Delta ABC, (b^2-c^2)\cot A+(c^2-a^2)\cot B=$

[TS EAMCET 10-09-20_Shift-2]

1. 0 2. $2R^2[\sin 2A-\sin 2B]$
3. $(b^2-a^2)\cot(A+B)$ 4. $2R^2[\tan 2A-\tan 2B]$

30. In a $\Delta ABC, \frac{\Delta^2}{a^2+b^2+c^2}\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^3}+\frac{1}{r^2}\right)=$

[TS EAMCET 10-09-20_Shift-2]

1. 0 2. 1
3. Δ 4. s

31. If $R:r_1:r=5:12:2$, then $r+r_3+r_2-r_1=$

[TS EAMCET 10-09-20_Shift-2]

1. $\cos A$ 2. $\sin A$
3. $2rr_1$ 4. $2r_1^2r$

HEIGHTS AND DISTANCES

32. If the angles of depression of the top and bottom of a short building from the top of a tall building are 30^0 and 60^0 respectively, then the ratio of the heights of short and tall buildings is

[TS_4th MAY_2019_SHIFT-1]

1. 2 : 3 2. 1 : 2
3. 1 : 3 4. 1 : 4

33. For a triangle with the sides of lengths 6,5 and 9, the radius of the in circle is

[AP EAMCET 17-09-20_Shift-1]

1. $\sqrt{3}$ 2. $\sqrt{2}$
3. $\sqrt{5}$ 4. $\frac{\sqrt{3}}{\sqrt{2}}$

34. A man, 2m tall, walks at the rate of $1\frac{2}{3}m/s$ towards a street light which is $5\frac{1}{3}m$ above the ground. The rate at which the length of his shadow is changing when he is $3\frac{1}{3}m$ away from the base of the light is _____

[AP EAMCET 17-09-20_Shift-2]

1. $-1\, m/s$ 2. $2\, m/s$
3. $-2\, m/s$ 4. $1\, m/s$

35. In a $\Delta ABC, \angle C=60^o$ and $\angle A=75^o$. If D is a point on AC such that the area of ΔBAD is $\sqrt{3}$ times of the area of ΔBCD, then the measure of $\angle ABD$

[AP EAMCET 18-09-20_Shift-1]

1. 30^o
2. 45^o
3. 60^o
4. 90^o

36. In a triangle ABC, if $a=3, b=4$ and $\sin A=\frac{3}{4}$ then $\angle CBA=?$

[AP EAMCET 19-08-2021_Shift-1]

1. 60^0
2. 75^0
3. 90^0
4. 45^0

37. In $\Delta ABC, A=75^0, B=45^0$ then the value of $b+c\sqrt{2}=$

[AP EAMCET 19-08-2021_Shift-1]

1. a
2. $3a$
3. $2a$
4. $4a$

38. In ΔABC, suppose the radius of the circle opposite to an angle A is denote by r_1, similarly $r_2 \leftrightarrow$ angle B, $r_3 \leftrightarrow$ angle C. If 'r' is the radius of inscribed circle then, what is the value of $\frac{ab-r_1r_2}{r_3}=$

[AP EAMCET 19-08-2021_Shift-1]

1. $r_1r_2r_3$
2. r
3. $r_1r_2\frac{r_3}{2}$
4. $\frac{r}{2}$

39. In ΔABC, suppose the radius of the circle opposite to an angle A is denoted by r_1, similarly $r_2 \leftrightarrow angle\ B, r_3 \leftrightarrow angle\ C$. If $r_1=2, r_2=3, r_3=6$. what is the value of $r_1+r_2+r_3-r=$
(R- Radius of the circum circle)

[AP EAMCET 19-08-2021_Shift-2]

1. $4R$
2. $3R$
3. $2R$
4. R

40. In triangle ABC, $\frac{a+b+c}{BC+AB}+\frac{a+b+c}{AC+AB}=3$ then $\tan\frac{C}{8}=$ **[AP EAMCET 19-08-2021_Shift-2]**

1. $\sqrt{6}+\sqrt{3}+\sqrt{2}-2$
2. $\sqrt{6}-\sqrt{3}-\sqrt{2}+2$
3. $\sqrt{6}-\sqrt{3}+\sqrt{2}-2$
4. $\sqrt{6}+\sqrt{3}-\sqrt{2}+2$

41. In ΔABC, medians AD and BE are drawn. If $AD=4, \angle DAB=\frac{\pi}{6}$ and $\angle ABE=\frac{\pi}{3}$ then the area of ΔABC is

[AP EAMCET 20-08-2021_Shift-1]

1. $\frac{8}{3}$ sq.units
2. $\frac{16}{3}$ sq.units
3. $\frac{32}{3\sqrt{3}}$ sq.units
4. $\frac{64}{3}$ sq.units

42. In a $\Delta ABC, 2\Delta^2=\frac{a^2b^2c^2}{a^2+b^2+c^2}$ then the triangle is____ **[AP EAMCET 20-08-2021_Shift-1]**

1. Equilateral
2. Isosceles
3. Right angled
4. Acute angled triangle

43. The sides of a triangle inscribed in a given circle subtended angles α, β, γ at the center. The minimum value of the A.M of $\cos\left(\alpha+\frac{\pi}{2}\right), \cos\left(\beta+\frac{\pi}{2}\right)$ and $\cos\left(\gamma+\frac{\pi}{2}\right)$ is equal to **[AP EAMCET 20-08-2021_Shift-1]**

1. $\frac{\sqrt{3}}{2}$
2. $\frac{-\sqrt{3}}{2}$
3. $\frac{-2}{\sqrt{3}}$
4. $\sqrt{2}$

44. What is the value of $(a-b)^2\cos^2\frac{c}{2}+(a+b)^2\sin^2\frac{c}{2}=$

[AP EAMCET 20-08-2021_Shift-2]

1. c^2
2. a^2+b^2
3. $a^2+b^2+c^2$
4. $a^2-b^2+c^2$

45. In ΔABC, suppose the radius of the circle opposite to an angle A is denoted by r_1, similarly $r_2 \leftrightarrow$ angle B, $r_3 \leftrightarrow$ angle C. If $r_1=2, r_2=3, r_3=6$, then what is $(a,b,c)=$

[AP EAMCET 20-08-2021_Shift-2]

1. $(3,4,5)$
2. $(3,5,4)$
3. $(5,4,3)$
4. $(5,3,4)$

46. If in $\Delta ABC, a\tan A+b\tan B=(a+b).\tan\left(\frac{A+B}{2}\right)$, then which of the following holds?

[AP EAMCET 20-08-2021_Shift-2]

1. A =B
2. A = 2 B
3. A =1/2 B
4. A > B

47. If the angles of a triangle are in the ratio $1:2:3$, the corresponding sides are in the ratio **[AP EAMCET 23-08-2021_Shift-1]**

1. $2:\sqrt{3}:1$
2. $1:\sqrt{3}:2$
3. $1:2:3$
4. $\sqrt{3}:2:1$

48. If in a triangle ABC, $s(s-a)=(s-b)(s-c)$, then **[AP EAMCET 23-08-2021_Shift-1]**

1. $\angle A=\frac{\pi}{4}$
2. $\angle B=\frac{\pi}{3}$
3. $\angle A=\frac{\pi}{2}$
4. $\angle B=\frac{\pi}{2}$

49. If in a triangle $\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right)=2$, then the triangle is [AP EAMCET 23-08-2021_Shift-1]

1. Right angled triangle
2. Equilateral triangle
3. $\angle B=60^0$
4. $\angle C=45^0$

50. In triangle ABC, $\frac{16Rs\Delta\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}}{s-c}$ is equal to **[AP EAMCET 23-08-2021_Shift-1]**

1. $\frac{r_1r_2}{r_3}$
2. $r_1.r_2.r_3$
3. $r_1+r_2+r_3$
4. $4r_1r_2r_3$

51. Let A, B, C be three points on a circle of radius R. If O is the centre of the circle and $AOB=45°, BOC=45°$ then the resultant of $\overline{OA}, \overline{OB}$ and $\overline{OC}$ has magnitude **[AP EAMCET 23-08-2021_Shift-1]**

1. $2R$
2. $\left(\sqrt{2}+1\right)R$
3. $2\sqrt{2}R$
4. $4\sqrt{2}R$

52. In a triangle ABC, $2ac\sin\frac{1}{2}(A-B+C)=$ **[AP EAMCET 24-08-2021_Shift-1]**

1. $a^2+b^2+c^2$
2. $a^2+b^2-c^2$
3. $a^2+c^2-b^2$
4. $b^2+c^2-a^2$

53. In triangle ABC, $a=6cm, b=10cm$ and $c=14cm$. Then the sum of the acute angles of the triangle is ______ **[AP EAMCET 24-08-2021_Shift-1]**

1. 180°
2. 120°
3. 90°
4. 60°

54. If 'P' is a point on the altitude $\overline{AD}$ of the triangle ABC, and $\angle ABP=\frac{2B}{3}$, then AP is equal to **[AP EAMCET 24-08-2021_Shift-1]**

1. $C\sin\frac{B}{3}$
2. $2C\sin\frac{B}{3}$
3. $C\sin\frac{2B}{3}$
4. $2C\sin\frac{2B}{3}$

55. If the angles A, B, C of ΔABC are in arithmetic progression, then **[AP EAMCET 24-08-2021_Shift-2]**

1. $b^2=a^2+c^2-ac$
2. $c^2=b^2+a^2-ab$
3. $a^2=b^2+c^2-bc$
4. $c^2=a^2+b^2$

56. In a ΔABC, $\operatorname{cosec}A(\sin B.\cos C+\cos B.\sin C)$ equals to **[AP EAMCET 24-08-2021_Shift-2]**

1. $\frac{c}{a}$
2. $\frac{a}{c}$
3. 1
4. $\frac{a}{b}$

57. What is the greatest angle of the triangle whose sides are $x^2+x+1, 2x+1, x^2-1$? **[AP EAMCET 24-08-2021_Shift-2]**

1. $A=120°$
2. $A=90°$
3. $A=135°$
4. $A=60°$

58. In any ΔABC, $b^2\sin 2c+c^2\sin 2B=$ **[AP EAMCET 25-08-2021_Shift-1]**

1. Δ
2. 2Δ
3. 3Δ
4. 4Δ

59. Number of triangles in which $\tan A+\tan B+\tan C=\cot A+\cot B+\cot C$ is ______ **[AP EAMCET 25-08-2021_Shift-1]**

1. 1
2. Infinite
3. 0
4. 2

60. Let a, b, c denote the lengths of sides of BC, CA, AB of ΔABC. In ΔABC, $\angle BAC = 30°, \angle ABC = 60°$. Then $a:b:c$ is ____

[AP EAMCET 25-08-2021_Shift-2]

1. $2:\sqrt{3}:1$
2. $1:\sqrt{3}:2$
3. $1:2:\sqrt{3}$
4. $2:1:\sqrt{3}$

61. In a triangle ABC, if $b=2, c=3$ and $\angle B = \frac{\pi}{6}$, then 'a' satisfies the equation

[AP EAMCET 25-08-2021_Shift-2]

1. $a^2 + 3\sqrt{3}a + 5 = 0$
2. $a^2 + 3\sqrt{3}a - 5 = 0$
3. $a^2 - 3\sqrt{3}a + 5 = 0$
4. $\sqrt{3}a^2 + 3a + 5 = 0$

62. In ΔABC, if $a:b:c = 4:5:6$ then the ratio between the circumradius and the inradius is ____ **[AP EAMCET 25-08-2021_Shift-2]**

1. $\frac{16}{7}$
2. $\frac{16}{9}$
3. $\frac{7}{16}$
4. $\frac{11}{7}$

63. In a triangle, if the length of the sides a, b and c are three consecutive natural numbers and $a<b<c$, the $(\cos A + \cos B + \cos C)2abc =$

[TS EAMCET 04-08-2021_Shift-2]

1. $3b(b^2-2)$
2. $3b^3 + 6b^2 + 3b$
3. $(3b+2)(3b-2)b$
4. $(b-1)b(b+1)$

64. In a ΔABC, if $(a-b)(s-c) = (b-c)(s-a)$ then r_1, r_2 and r_3 are

[TS EAMCET 04-08-2021_Shift-2]

1. In A.P
2. In G.P
3. In H.P
4. equal

65. In an equilateral triangle ABC, $r:R:r_1$ is

[TS EAMCET 04-08-2021_Shift-2]

1. $1:3:2$
2. $1:3:1$
3. $1:2:3$
4. $2:1:3$

66. In $\Delta ABC, \underline{|B} = \frac{\pi}{4}, \underline{|C} = \frac{\pi}{3}$. If the area of the triangle is $54 + 18\sqrt{3}$ sq.units, then a=

[TS EAMCET 04-08-2021_Shift-1]

1. $\sqrt{3}+1$
2. $2(\sqrt{3}+1)$
3. $4(\sqrt{3}+1)$
4. $6(\sqrt{3}+1)$

67. If a, b, c are the sides of a ΔABC and exradii r_1, r_2, r_3 are respectively 12, 6, 4 then $a+2b+3c =$

[TS EAMCET 04-08-2021_Shift-1]

1. 24
2. 44
3. 30
4. 54

68. In a triangle ABC, if $a:b:c = 4:5:6$, then $\frac{1}{4R}[r_1 + r_2 + r_3] =$

[TS EAMCET 04-08-2021_Shift-1]

1. $\frac{71}{64}$
2. $\frac{4}{5}$
3. $\frac{81}{84}$
4. $\frac{7}{9}$

69. In a triangle ABC, with usual notation, if $a=12, b=16, c=20$, then the ratio of the ex radii of the triangle opposite to the angles in the order $\angle C, \angle B, \angle A$ is

[TS EAMCET 04-08-2021_Shift-1]

1. 3:4:5
2. 6:3:2
3. 12:7:5
4. 2:3:5

70. In a ΔABC, if $4a = b + c$, then $\tan\frac{B}{2}\tan\frac{C}{2} =$

[TS EAMCET 05-08-2021_Shift-1]

1. $\frac{1}{3}$
2. $\frac{3}{5}$
3. $\frac{2}{3}$
4. $\frac{1}{2}$

71. If 4 times area of a ΔABC is $c^2-(a-b)^2$, then $\sin C$ **[TS EAMCET 05-08-2021_Shift-1]**

1. $\frac{\sqrt{3}}{2}$
2. $\frac{1}{\sqrt{2}}$
3. $\frac{1}{2}$
4. 1

72. In a ΔABC, if $\cot\frac{A}{2}\cot\frac{B}{2}=K$, then all the possible values of K lies in

[TS EAMCET 05-08-2021_Shift-1]

1. $(0,1]$
2. $[1,\infty)$
3. $(1,\infty)$
4. $(0,1)$

73. In a $\Delta ABC, b^2\sin 2C+c^2\sin 2B=$

[TS EAMCET 05-08-2021_Shift-1]

1. 0
2. 4Δ
3. 2Δ
4. Δ

74. In ΔABC, if $\angle C=90^0, \frac{a^2+b^2}{a^2-b^2}\sin(A-B)=1$ and $0<B<45^0$, then

[TS EAMCET 05-08-2021_Shift-2]

1. $a>b>c$
2. $c>a>b$
3. $c>b>a$
4. $a<b<c$

75. In a triangle ABC, if $rr_2=r_1r_3$, then $\cos 2B=$

[TS EAMCET 05-08-2021_Shift-2]

1. -1
2. 1
3. 0
4. $\frac{1}{2}$

76. If the side of a triangle are 3, 4 and 5 then the circum radius of the triangle is

[TS EAMCET 05-08-2021_Shift-2]

1. 2
2. $\frac{3}{2}$
3. $\frac{5}{2}$
4. $\frac{7}{2}$

77. In a $\Delta ABC, \frac{\cos^2\left(\frac{B-C}{2}\right)}{(b+c)^2}+\frac{\sin^2\left(\frac{B-C}{2}\right)}{(b-c)^2}=$

[TS EAMCET 05-08-2021_Shift-2]

1. $\frac{1}{a^2}$
2. $\frac{2}{a^2}$
3. $\frac{3}{a^2}$
4. $\frac{4}{a^2}$

78. In ΔABC, if $\frac{1}{a+b}+\frac{1}{c+a}=\frac{3}{a+b+c}$, then $\sin A=$ **[TS EAMCET 06-08-2021_Shift-2]**

1. 1
2. $\frac{1}{2}$
3. $\frac{\sqrt{3}}{2}$
4. $\frac{4}{5}$

79. If the angles of a triangle ABC are in A.P, then

[TS EAMCET 06-08-2021_Shift-2]

1. $c^2=a^2+b^2-ab$
2. $a^2=b^2+c^2-ac$
3. $b^2=a^2+c^2-ac$
4. $b^2=a^2+c^2$

80. In any ΔABC, $\frac{1+\cos(A-B).\cos C}{1+\cos(A-C).\cos B}=$

[TS EAMCET 06-08-2021_Shift-2]

1. $\frac{a^2+c^2}{b^2+c^2}$
2. $\frac{b^2+c^2}{b^2+a^2}$
3. $\frac{a^2+c^2}{a^2+b^2}$
4. $\frac{a^2+b^2}{a^2+c^2}$

81. In any ΔABC, $\frac{b-c\cos A}{c-b\cos A}=$

[TS EAMCET 06-08-2021_Shift-2]

1. $\frac{\sin B}{\sin C}$
2. $\frac{\cos C}{\cos B}$
3. $\frac{\cos B}{\cos C}$
4. $\frac{\sin C}{\sin B}$

82. In a triangle ABC, if $a+c=5b$ then

$\cot\frac{A}{2}\cot\frac{C}{2}=$

[TS EAMCET 06-08-2021_Shift-1]

1. 2
2. $\frac{1}{2}$
3. $\frac{3}{2}$
4. $\frac{2}{3}$

83. In a ΔABC, if $a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=\frac{3b}{2}$, then

[TS EAMCET 06-08-2021_Shift-1]

1. $2b=a+c$
2. $b^2=ac$
3. $b^2=\frac{2ac}{a+c}$
4. $a+b+c=1$

84. In ΔABC, if $B+C=72^0$, then

$\left(1+\frac{a}{c}+\frac{b}{c}\right)\left(1+\frac{c}{a}-\frac{a}{b}\right)=$

[TS EAMCET 06-08-2021_Shift-1]

1. $2+\sqrt{5}$
2. $\frac{\sqrt{5}+1}{2\sqrt{2}}$
3. $\frac{\sqrt{5}-2}{4}$
4. $\frac{5-\sqrt{5}}{2}$

85. In a $\Delta ABC, r_1, r_2, r_3$ respectively denote the radius of ex circles opposite to the vertices A,B,C and r denote the radius of the in circle. If p_1, p_2, p_3 respectively are the altitudes of the triangle from the vertices A,B,C then

$\left(\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}\right)^2=$

[TS EAMCET 06-08-2021_Shift-1]

1. $\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)^2 r^2$
2. $\frac{1}{r}\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)$
3. $\left(\frac{r}{r_1}+\frac{r}{r_2}+\frac{r}{r_3}\right)^2$
4. $rr_1+rr_2+rr_3$

86. In a triangle ABC, if

$a\neq b, \frac{a\cos A-b\cos B}{a\cos B-b\cos A}+\cos C=$

[AP EAMCET 04-07-2022_Shift-1]

1. 0
2. 1
3. 2
4. -1

87. If in a triangle ABC, a =2, b = 3 and c = 4, then tan $\left(\frac{A}{2}\right)=$

[AP EAMCET 04–07–2022_Shift–1]

1. $\sqrt{\frac{3}{15}}$
2. $\sqrt{\frac{4}{15}}$
3. $\sqrt{\frac{2}{15}}$
4. $\sqrt{\frac{1}{15}}$

88. If the angles of a triangle ABC are in the ratio 1:2:3, then the corresponding sides are in the ratio **[AP EAMCET 04–07–2022_Shift–1]**

1. $\sqrt{3}:2:1$
2. $\sqrt{3}:1:2$
3. $1:\sqrt{3}:2$
4. $1:2:\sqrt{3}$

89. In a triangle ABC,

$r_1\cot\frac{A}{2}+r_2\cot\frac{B}{2}+r_3\cot\frac{C}{2}=$

[AP EAMCET 04–07–2022_Shift–1]

1. s
2. 2s
3. 3s
4. $s/2$

90. In a triangle ABC,

$2(bc\cos A+ac\cos B+ab\cos C)=$

[AP EAMCET 04–07–2022_Shift–2]

1. a+b+c
2. 2(a+b+c)
3. $a^2+b^2+c^2$
4. $2(a^2+b^2+c^2)$

91. In a triangle ABC, $\frac{a}{b}=2+\sqrt{3}$ and $\angle C=60^\circ$

Then the measure of $\angle A$ is

[AP EAMCET 04–07–2022_Shift–2]

1. 95^0
2. 65^0
3. 105^0
4. 115^0

92. If a=2,b=3,c=4 in a triangle ABC, then cos C=

[AP EAMCET 04–07–2022_Shift–2]

1. $\frac{1}{4}$ 2. $\frac{-1}{4}$
3. $\frac{1}{2}$ 4. $\frac{-1}{2}$

93. In a triangle ABC,(b+c) cos A+(c+a) cos B+(a+b)cos C=

[AP EAMCET 04–07–2022_Shift–2]

1. 2 abc 2. abc
3. a+b+c 4. (a+b+c)/2abc

94. If any triangles ABC, $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=$

[AP EAMCET 05–07–2022_Shift–1]

1. $a^2+b^2+c^2$ 2. $\frac{a^2+b^2+c^2}{2abc}$
3. $\frac{2abc}{a^2+b^2+c^2}$ 4. a+b+c

95. In a triangle ABC, if r_1=36, r_2=18 and r_3 = 12, then s =

[AP EAMCET 05–07–2022_Shift–1]

1. 6 2. 8
3. 16 4. 36

96. In a triangle ABC, a = 6, b = 5 and c = 4, then cos 2A =

[AP EAMCET 05–07–2022_Shift–1]

1. $-31/32$ 2. $-15/16$
3. $31/32$ 4. $15/16$

97. In a triangle, if $b=5, c=6, \tan\frac{A}{2}=\frac{1}{\sqrt{2}}$, then $a=$

[AP EAMCET 05–07–2022_Shift–2]

1. $\sqrt{41}$ 2. $\sqrt{21}$
3. $\sqrt{14}$ 4. $\sqrt{22}$

98. In a triangle ABC $s\left[\frac{r_1-r}{a}+\frac{r_2-r}{b}+\frac{r_3-r}{c}\right]=$

[AP EAMCET 05–07–2022_Shift–2]

1. $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$ 2. $r_1+r_2+r_3$
3. $r_1r_2r_3$ 4. $\frac{1}{r}-\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$

99. A farmer owns a triangular plot in Guntur. He measures the lengths of the sides of his property as 4cm, 5cm and 7cm. Then the area of land of the farmer in sq. cm is

[AP EAMCET 05–07–2022_Shift–2]

1. $2\sqrt{6}$ 2. $4\sqrt{6}$
3. $\sqrt{6}$ 4. $8\sqrt{6}$

100. In the triangle ABC, if $a=7, b=6$ *and* $A=120^0$, then the approximate value of B is

[AP EAMCET 06–07–2022_Shift–1]

1. 47.9^0 2. 44.9^0
3. 59.9^0 4. 61.9^0

101. In any triangle ABC, $a(b\cos C-c\cos B)=$

[AP EAMCET 06–07–2022_Shift–1]

1. $b-c$ 2. $b+c$
3. b^2-c^2 4. b^2+c^2

102. The length of the side of a triangle are 13, 14 & 15. If R and r respectively denote circumradius and inradius of this triangle, then 8R −r =

[AP EAMCET 06-07-2022_Shift-1]

1. 41 2. 51
3. 61 4. 71

103. The sides of a triangle ABC are given by a=3,b=5 and c=3. Then cos A=

[AP EAMCET 07–07–2022_Shift–1]

1. 2/6 2. 1/6
3. 2/3 4. 5/6

104. The triangle ABC, if a=2,b=3 and sin A=2/3 then $\angle B=$

[AP EAMCET 07–07–2022_Shift–1]

1. $\frac{\pi}{2}$ 2. $\frac{\pi}{6}$
3. $\frac{\pi}{3}$ 4. $\frac{\pi}{4}$

105. In a ΔABC, a:b:c=4:5:6. The ratio of radius of the circumcircle to that of the in circle is

[AP EAMCET 07–07–2022_Shift–1]

1. 7:16 2. 17:16
3. 16:17 4. 16:7

106. Let ABC be an acute–angled triangle with area R Then

$\sqrt{a^2b^2-4R^2}+\sqrt{b^2c^2-4R^2}+\sqrt{c^2a^2-4R^2}=$

[AP EAMCET 07–07–2022_Shift–2]

1. a+b+c
2. $a^2+b^2+c^2$
3. $\frac{a^2+b^2+c^2}{2}$
4. $2(a^2+b^2+c^2)$

107. In a triangle ABC ,if a=4,b=5 and c=7 then $\sin\left(\frac{A}{2}\right)=$

[AP EAMCET 07–07–2022_Shift–2]

1. $\sqrt{\frac{3}{35}}$
2. $\sqrt{\frac{35}{3}}$
3. $\sqrt{\frac{2}{35}}$
4. $\sqrt{\frac{1}{35}}$

108. In any triangle ABC, $\frac{\cos 2A}{a^2}-\frac{\cos 2B}{b^2}=$

[AP EAMCET 07–07–2022_Shift–2]

1. a^2-b^2
2. $\frac{1}{a^2}-\frac{1}{b^2}$
3. a^2+b^2
4. $\frac{1}{a^2}+\frac{1}{b^2}$

109. In a triangle ABC, let $\angle C=\frac{\pi}{2}$. If r and R are respectively inradius and circumradius of ABC, then R+r=

[AP EAMCET 07–07–2022_Shift–2]

1. $\frac{a-b}{2}$
2. $\frac{a+b}{2}$
3. $a+b$
4. a–b

110. In any triangle ABC, $\sin\frac{A}{2}\leq$

[AP EAMCET 08–07–2022_Shift–1]

1. $\frac{2a}{\sqrt{bc}}$
2. $\frac{a}{2\sqrt{bc}}$
3. $\frac{3a}{\sqrt{bc}}$
4. $\frac{\sqrt{bc}}{2a}$

111. In a triangle ABC, $(b+c)\sin\frac{A}{2}=$

[AP EAMCET 08–07–2022_Shift–1]

1. $a\cos A$
2. $a\cos\left(\frac{B-C}{2}\right)$
3. $a\sin\left(\frac{B+C}{2}\right)$
4. $a\sin\left(\frac{B-C}{2}\right)$

112. In a triangle ABC, if a–2b+c = 0, then $\cot\left(\frac{A}{2}\right).\cot\left(\frac{C}{2}\right)$

[AP EAMCET 08–07–2022_Shift–1]

1. 1
2. 2
3. 3
4. 4

113. In a ΔABC, if a = 13, b = 14 and c = 15, then $\sin\left(\frac{A}{2}\right)$

[AP EAMCET 08–07–2022_Shift–1]

1. $\frac{1}{\sqrt{5}}$
2. $\frac{1}{\sqrt{7}}$
3. $\frac{3}{4}$
4. $\frac{4}{5}$

114. In a triangle ABC, if $\angle A=60^0$, then $(a+b+c)(b+c-a)=$

[AP EAMCET 08–07–2022_Shift–2]

1. 3bc
2. 2abc
3. abc
4. a+b+c

115. In a triangle ABC, if $r_1=12, r_2=18$ and $r_3=36$ Then b=

[AP EAMCET 08–07–2022_Shift–2]

1. 12
2. 6
3. 24
4. 18

116. If r_1, r_2 and r_3 of a triangle ABC are in Harmonic progress, then a, b and c will be in

[AP EAMCET 08–07–2022_Shift–2]

1. Arithmetic progression
2. Geometric progression
3. Harmonic progression
4. Arithmetic–Geometric progression

117. In a triangle ABC, if $r_1 > r_2 > r_3$, then which of the following is true?

[AP EAMCET 08–07–2022_Shift–2]

1. a>b>c
2. a>b,b<c
3. a<b<c
4. a<b,b>c

118. In a triangle ABC, if

$(b+c)^2\sin^2\frac{A}{2}+(b-c)^2\cos^2\frac{A}{2}=K(1-\cos 2A)$,

then K =

[TS EAMCET 18–07–2022_Shift–1]

1. R^2
2. $2R^2$
3. R
4. 2R

119. In a triangle ABC, if b = 7, $c=4\sqrt{3}$ and

$A=\frac{\pi}{6}$ then a sin B sin C =

[TS EAMCET 18–07–2022_Shift–1]

1. $\frac{\sqrt{13}}{12}$
2. $\frac{\sqrt{13}}{7\sqrt{3}}$
3. $\frac{12}{\sqrt{13}}$
4. $\frac{7\sqrt{3}}{\sqrt{13}}$

120. In triangle ABC, if BC is the hypotenuse, then

$r_2+r_3=$

[TS EAMCET 18–07–2022_Shift–1]

1. r_1+r
2. a
3. $r-r_1$
4. $2(R+r)$

121. In triangle ABC, if A is acute. C is obtuse,

$\sin A=\frac{3\sqrt{3}}{14}, a=3 \ and \ b=5$, then c =

[TS EAMCET 18–07–2022_Shift–2]

1. $\frac{16}{7}$
2. 7
3. $\frac{14}{3}$
4. 6

122. If Δ denotes the area of triangle ABC, then (bsinC+csinB) (bcosC+ccosB) =

[TS EAMCET 18–07–2022_Shift–2]

1. ab cosC
2. 2Δ
3. bccosA
4. 4Δ

123. Let A be the area of in–circle and A_1, A_2, A_3 be the areas of ex–circles of a triangle. If $A_1=4, A_2=9, A_3=16$, then A =

[TS EAMCET 18–07–2022_Shift–2]

1. 81
2. $\frac{61}{169}$
3. $\frac{144}{61}$
4. $\frac{144}{169}$

124. If the sides of a triangle ABC whose perimeter is 42 are in arithmetic progression, its

circum–radius is $\frac{65}{8}$ and $B<A<C$ then

$\sin A=$

[TS EAMCET 19–07–2022_Shift–I]

1. $\frac{4}{13}$
2. $\frac{28}{65}$
3. $\frac{56}{65}$
4. $\frac{14}{65}$

125. In a triangle ABC, if

$a=7, c=11, \cos A=\frac{17}{22}, \cos C=\frac{1}{14}$ then

$b\tan\frac{B}{2}\tan\frac{C-A}{2}=$

[TS EAMCET 19–07–2022_Shift–I]

1. 18
2. 14
3. 2
4. 9

126. If any triangle ABC, $r^2\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}=$

[TS EAMCET 19–07–2022_Shift–I]

1. Δ
2. 2Δ
3. Δ^2
4. 5Δ

127. In a triangle ABC. AD and BE are medians.

If AD=4, $\lfloor DAB=\frac{\pi}{6} \ and \lfloor ABE=\frac{\pi}{3}$ then

The area of ΔABC is

[TS EAMCET 19–07–2022_Shift–2]

1. $\frac{14}{3\sqrt{3}}$
2. $\frac{28}{3\sqrt{3}}$
3. $\frac{11}{3\sqrt{3}}$
4. $\frac{32}{3\sqrt{3}}$

128. If S is the circumcentre of a triangle ABC, a=5,b=6, c=9 and SB=$\frac{27}{4\sqrt{2}}$ then sin 2C

[TS EAMCET 19–07–2022_Shift–2]

1. $\frac{4\sqrt{2}}{9}$ 2. $\frac{4\sqrt{2}}{27}$
3. $\frac{-4\sqrt{2}}{27}$ 4. $\frac{-4\sqrt{2}}{9}$

129. In a triangle ABC, if $\frac{r}{r_1}=\frac{1}{2}$, then 4 tan $\frac{A}{2}$ $\left(\tan\frac{B}{2}+\tan\frac{C}{2}\right)$

[TS EAMCET 19–07–2022_Shift–2]

1. 1 2. 2
3. 3 4. 4

130. If a, b, c are the sides of a triangle ABC and $\begin{vmatrix} b & 1 & a \\ a & 1 & c \\ c & 1 & b \end{vmatrix}=0$,then 2(cosA+cosB+cosC)=

[TS EAMCET 20–07–2022_Shift–1]

1. 1 2. 2
3. 3 4. 4

131. In a triangle ABC, if $A=\frac{\pi}{3}$ and B=$\frac{\pi}{4}$ then $\frac{a^2-b^2}{c^2}=$

[TS EAMCET 20–07–2022_Shift–1]

1. $2-\sqrt{3}$ 2. $2+\sqrt{3}$
3. $\sqrt{2}-1$ 4. $\sqrt{2}+1$

132. In a triangle ABC, if a=3,b=7,c=8 then $\sin\frac{B}{2}\tan\frac{C-A}{2}=$

[TS EAMCET 20–07–2022_Shift–1]

1. $\frac{15\sqrt{3}}{22\sqrt{7}}$ 2. $\frac{5\sqrt{2}}{11\sqrt{7}}$
3. $\frac{5\sqrt{3}}{11}$ 4. $\frac{5\sqrt{3}}{22}$

133. In triangle ABC, if $a=7$, $b=8$, tan C = $\frac{3\sqrt{5}}{2}$ and C is an acute angle, then $c=$

[TS EAMCET 20–07–2022_Shift–2]

1. $\sqrt{145}$ 2. 5
3. 11 4. 9

134. In a triangle ABC, if $\frac{a}{\tan A}=\frac{b}{\tan B}=\frac{c}{\tan C}$ then $\cos^2 A+\cos^2 B+\cos^2 C=$

[TS EAMCET 20–07–2022_Shift–2]

1. $\sqrt{2}$ 2. $\frac{3}{4}$
3. $\frac{\sqrt{3}+1}{2}$ 4. $\frac{2\sqrt{3}-1}{2}$

135. In triangle ABC, if a = 7, b = 10, c = 11, then $\frac{R}{r}=$

[TS EAMCET 20–07–2022_Shift–2]

1. 14 2. 77
3. $\frac{24}{11}$ 4. $\frac{55}{24}$

136. In ΔABC, if $\cot\frac{A}{2}:\cot\frac{B}{2}:\cot\frac{C}{2}=3:7:9$, then a:b:c= **[15th May 2023 Shift 1]**

1. 8:6:5 2. 5:6:8
3. 10:8:5 4. 5:8:10

137. In ΔABC, if $\frac{1}{r_1},\frac{1}{r_2}$ and $\frac{1}{r_3}$ are in arithmetic progression, then r_2:r =

[15th May 2023 Shift 1]

1. 3:2 2. 2:1
3. 1:3 4. 3:1

138. If P_1, P_2 and P_3 are the lengths of the altitudes drawn from the vertices A, B and C of ΔABC respectively, then $\frac{\cos A}{P_1}+\frac{\cos B}{P_2}+\frac{\cos C}{P_3}=$

[15th May 2023 Shift 1]

1. $\frac{1}{R}$ 2. R
3. $\frac{\Delta}{R}$ 4. $\frac{r}{R}$

139. In ΔABC,if $\sin^2 B = \sin C$ and $3\cos^2 B = 2\cos^2 C$, then ΔABC is

[15th May 2023 Shift 2]

1. A right angled triangle
2. An isosceles triangle
3. An equilateral triangle
4. A scalene triangle

140.If ΔABC is a right angled isosceles triangle and $\angle C = 90^0$, then $r : r_3 =$

[15th May 2023 Shift 2]

1. $\sqrt{2}+1 : \sqrt{2}-1$ 2. $\sqrt{2}-1 : \sqrt{2}+1$
3. $\sqrt{2} : 1$ 4. $1 : \sqrt{2}$

141.In ΔABC, if $a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2} = \frac{3b}{2}$, then $a+c : b =$

[15th May 2023 Shift 2]

1. 1 : 1 2. 3 : 2
3. 2 : 1 4. 4 : 3

142. *In* ΔABC, *if* $r_1 = 2r_2 = 3r_3$, *then*

[16th May 2023 Shift 1]

1. b +c=2a 2. a+b=2c
3. a+c=2b 4. $\Delta = abc$

143. *In* ΔABC, *if* $\tan\frac{A}{2} + \tan\frac{C}{2} = \frac{b}{s}$, then $\sin\left(\frac{A+C}{3}\right) =$

[16th May 2023 Shift 1]

1. 1 2. $\frac{\sqrt{3}}{2}$
3. $\frac{1}{\sqrt{2}}$ 4. $\frac{1}{2}$

144. *In* $\Delta ABC, \angle B = 60^o$ *and* $\angle A = 75^o$. if a point D divides BC in the ratio 2:3. Then $\sin\angle BAD : \sin\angle CAD =$

[16th May 2023 Shift 1]

1. $\sqrt{2} : \sqrt{3}$ 2. $\sqrt{3} : 2$
3. $\sqrt{3} : \sqrt{2}$ 4. $3 : \sqrt{2}$

145. In ΔABC, *if* $\cos A + \cos C = 4\sin^2\frac{B}{2}$, then the ratio between the perimeter of the triangle and (a+c) is

[16th May 2023 Shift 2]

1. 1:2 2. 2:3
3. 3:2 4. 1:3

146.In ΔABC, if $\angle A = 90^\circ$, *then* $(r_3 - r_1) =$

[16th May 2023 Shift 2]

1. $r_2 r_3$ 2. $2r_2 r_3$
3. $4r_2 r_3$ 4. 2

147.In ΔABC, if a,b,c are in arithmetic progression and C=2A, then a:c =

[16th May 2023 Shift 2]

1. 4:5 2. 2:3
3. 5:6 4. $\sqrt{3:2}$

148. In $\Delta ABC, \left(\tan\frac{A}{2} + \tan\frac{B}{2}\right)\tan\frac{C}{2} =$

[17th May 2023 Shift 1]

1. $\frac{2c}{a+b+c}$ 2. $\frac{2c}{a+b-c}$
3. $\frac{2c^2}{a^2+b^2+c^2}$ 4. $\frac{c}{a+b+c}$

149. In ΔABC, if $\angle$C=90^0, then $\left(\frac{r_1 - r_3}{r_1}\right)\left(\frac{r_2 - r_3}{r_2}\right) =$

[17th May 2023 Shift 1]

1. 1 2. 3
3. 4 4. 2

150.In ΔABC, A, B and C are in arithmetic progression and $a : c = 1 : 2$, If $b = 4\sqrt{3}\,cm$, then the area of ΔABC (in Sq.cm) is

[17th May 2023 Shift 1]

1. $16\sqrt{3}$ 2. $12\sqrt{3}$
3. $8\sqrt{3}$ 4. $6\sqrt{3}$

151.In Δ ABC, if a, b, c are 5,12 and 13 respectively, then $b^2\sin 2c + c^2\sin 2B =$

[17th May 2023 Shift 2]

1. 60 2. 120
3. 180 4. 90

152. In Δ ABC, $\frac{r_1-r}{a}+\frac{r_2-r}{b}=$

[17th May 2023 Shift 2]

1. $\frac{a}{r_3}$
2. $\frac{b}{r_3}$
3. $\frac{c}{r^3}$
4. 1

153. In Δ ABC, if (a–b)(s–c)=(b-c)(s–a), then r_1, r_2, r_3 are in **[17th May 2023 Shift 2]**

1. AP
2. GP
3. HP
4. AGP

154. $In\, \Delta ABC, if\; a\sin^2\frac{C}{2}+c\sin^2\frac{A}{2}=\frac{b}{2}, then\; a+c:b=$

[18th May2023 shift -1]

1. 2:1
2. 1:2
3. 3:2
4. 4:3

155. $In\, \Delta ABC, r_1+r_2+r_3-r=$

[18th May2023 shift -1]

1. 4R
2. 2R
3. 4R sinA
4. 4R cosA

156. $In\, \Delta ABC$, if s– a : s– b : s– c = 2 : 3 : 4, then cotA : cotC =

[18th May2023 shift -1]

1. 6:7
2. 19:30
3. 6:19
4. 1:5

157. In ΔABC, if $r_1 : r_2 = 7:8$ and $r_1 : r_3 = 3:4$, then a:b:c= **[18th May 2023 Shift 2]**

1. 24:21:28
2. 8:7:6
3. 13:14:15
4. 7:8:6

158. In ΔABC, if(sinA+sinB)(sinA-SinB)=sinC(sinB+sinC), then$\angle$A=

[18th May 2023 Shift 2]

1. 60^o
2. 30^o
3. 150^o
4. 120^o

159. In ΔABC, if A,B,C are in arithmetic progression, then $\frac{c}{a}\sin 2A+\frac{a}{c}\sin 2C=$

[18th May 2023 Shift 2]

1. $\frac{\sqrt{3}}{2}$
2. $\sqrt{3}$
3. 1
4. $\frac{1}{2}$

160. In $\Delta ABC, if\; r=1, R=4\; and\; \Delta=8, then\; \frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=$

[19th May 2023 Shift 1]

1. 8
2. $\frac{1}{4}$
3. $\frac{1}{8}$
4. $\frac{1}{16}$

161. In ΔABC, if r is the in radius and r_1, r_2, r_3 are the ex–radii, then $\frac{1}{4}\left[b^2\sin 2C+C^2\sin 2B\right]=$

[19th May 2023 Shift 1]

1. $rr_1\tan\frac{A}{2}$
2. $bc\cos A$
3. $rr_1r_2r_3$
4. $rr_1\cot\frac{A}{2}$

162. $In\; \Delta ABC, \frac{a}{s-a}+\frac{b}{s-b}+\frac{c}{s-c}=$

[19th May 2023 Shift 1]

1. $\frac{4R}{r}-1$
2. $\frac{R}{r}-3$
3. $\frac{2R}{r}-1$
4. $\frac{4R}{r}-2$

163. In an isosceles right angled triangle, a straight line is drawn from the mid point of one of the equal sides to the opposite vertex. Then a pair of possible values of the cotangents of the two angles so formed at that vertex are

[12TH MAY 2023 SHIFT-1]

1. 1 and 2
2. 2 and 3
3. 3 and 4
4. 4 and 5

164. In a triangle ABC, if $r_1=2r_2=3r_3, then\; \frac{a}{b}+\frac{b}{c}+\frac{c}{a}=$

[12TH MAY 2023 SHIFT-1]

1. $\frac{75}{60}$
2. $\frac{155}{60}$
3. $\frac{176}{60}$
4. $\frac{191}{60}$

165. PQR is an isosceles triangle with $PQ=PR$. If the radius of the circum circle of ΔPQR is equal to the length of PQ, then $\underline{|P}=$

[12TH MAY 2023 SHIFT -2]

1. 30^o
2. 60^o
3. 45^o
4. 120^o

166. If ΔABC, if $\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c}$ and side $a=2$, then area of the ΔABC (in sq. units) is **[12TH MAY 2023 SHIFT -2]**

1. $8\sqrt{2}$
2. $4\sqrt{3}$
3. $\frac{\sqrt{3}}{2}$
4. $\sqrt{3}$

167. In ΔABC, if A is an acute angle , b=6, c=9 and $\sin A=\frac{2\sqrt{14}}{9}$, then $3a(\cos B+\cos C)=$ **[13TH MAY 2023 SHIFT-1]**

1. 14
2. 20
3. 17
4. 23

168. If the roots of the equation $x^3-11x^2+36x-36=0$ are the ex-radii of a triangle ABC then the perimeter of the triangle ABC is **[13TH MAY 2023 SHIFT-1]**

1. 24
2. 18
3. 12
4. 9

169. In ΔABC, if $a:b:c=4:5:6$, then the ratio of the circum radius to its inradius is **[EAPCET 14-05-23 SHIFT-1]**

1. 16:7
2. 25:11
3. 5:4
4. 9:5

170. The perimeter of a ΔABC is 6times the arithmetic mean of the values of the sine of its angles. If its side BC is of unit length, then $\angle A=$ **[EAPCET 14-05-23 SHIFT-1]**

1. $\frac{\pi}{6}$
2. $\frac{\pi}{3}$
3. $\frac{\pi}{2}$
4. π

171. In triangle ABC, if b = 6, c = 7 and $\tan\frac{A}{2}=\frac{1}{\sqrt{6}}$, *then the inradius of* ΔABC *is* **[EAPCET 13-05-23 SHIFT-2]**

1. $\sqrt{\frac{2}{3}}$
2. $\frac{2\sqrt{6}}{9}$
3. $\frac{\sqrt{2}}{9}$
4. $\frac{2\sqrt{6}}{3}$

172. In ΔABC, if a = 7, b = 8 and c = 9 then $\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=$ **[EAPCET 13-05-23 SHIFT-2]**

1. $\frac{97}{360}$
2. $\frac{5}{72}$
3. $\frac{169}{360}$
4. $\frac{67}{72}$

KEY

1)	1	2)	4	3)	3	4)	4	5)	3
6)	1	7)	3	8)	1	9)	3	10)	3
11)	3	12)	1	13)	4	14)	3	15)	1
16)	4	17)	1	18)	1	19)	1	20)	2
21)	3	22)	4	23)	3	24)	3	25)	1
26)	3	27)	4	28)	3	29)	2	30)	2
31)	1	32)	1	33)	2	34)	4	35)	1
36)	3	37)	3	38)	2	39)	1	40)	3
41)	3	42)	3	43)	2	44)	1	45)	1
46)	1	47)	2	48)	3	49)	1	50)	4
51)	2	52)	3	53)	4	54)	2	55)	1
56)	3	57)	1	58)	4	59)	3	60)	2
61)	3	62)	1	63)	1	64)	1	65)	3
66)	4	67)	2	68)	1	69)	2	70)	2
71)	4	72)	3	73)	2	74)	2	75)	3
76)	3	77)	1	78)	3	79)	3	80)	4
81)	2	82)	3	83)	1	84)	4	85)	2
86)	1	87)	4	88)	3	89)	3	90)	3
91)	3	92)	2	93)	3	94)	2	95)	4
96)	1	97)	1	98)	2	99)	2	100)	1
101)	3	102)	3	103)	4	104)	1	105)	4
106)	3	107)	1	108)	2	109)	2	110)	2
111)	2	112)	3	113)	1	114)	1	115)	3
116)	1	117)	1	118)	2	119)	4	120)	2
121)	2	122)	4	123)	4	124)	3	125)	3
126)	1	127)	4	128)	4	129)	2	130)	3
131)	1	132)	4	133)	4	134)	2	135)	4
136)	1	137)	4	138)	1	139)	4	140)	2
141)	3	142)	3	143)	4	144)	1	145)	3
146)	2	147)	2	148)	1	149)	4	150)	3
151)	2	152)	3	153)	1	154)	1	155)	1
156)	4	157)	3	158)	4	159)	2	160)	3
161)	4	162)	4	163)	2	164)	4	165)	4

166) 4 **167)** 2 **168)** 3 **169)** 1 **170)** 1
171) 4 **172)** 2

SOLUTIONS

1. 2s=36, s=18
Use $\Delta = r.s$

2. In a ΔABC,
$A+B+C=180^0 \rightarrow (1)$
Angles are in A.P
$2B = A+C$
$3B = 180^0 \Rightarrow B = 60^0$
$\frac{b}{c}=\frac{\sqrt{3}}{\sqrt{2}}$
$\Rightarrow \frac{2R\sin B}{2R\sin C}=\frac{\sqrt{3}}{\sqrt{2}}$
$\Rightarrow \sqrt{2}\sin B = \sqrt{3}\sin C$
$\Rightarrow \sqrt{2}\frac{\sqrt{3}}{2}=\sqrt{3}\sin C$
$\Rightarrow C = 45^0 \Rightarrow A = 75^0$

3. using cosine rule

4. $r=\frac{\Delta}{s}$
$\Delta = rs$

5. Given: $a=1, b=\sqrt{3}$ *and* $\angle C = \frac{\Pi}{6}$
w.k.t cosine rule ,
$\cos C = \frac{a^2+b^2-c^2}{2ab}$
$\frac{\sqrt{3}}{2}=\frac{4-c^2}{2\sqrt{3}}$
$c^2 = 4-3$
$\therefore c = 1$

6. Using sine rule
$\frac{a}{\sin A}=2R$
$\Rightarrow \frac{2}{\left(\frac{\sqrt{3}}{2}\right)}=2R \Rightarrow R=\frac{2}{\sqrt{3}}$

7. Conceptual

8. $\frac{ac+a^2+b^2+bc}{(b+c)(c+a)}$
$=\frac{ac+c^2+ab+bc}{(b+c)(c+a)}$
$=\frac{(b+c)(c+a)}{(b+c)(c+a)}=1$

9. $A, B, C \rightarrow A.P \quad \angle B = 60^0$
By cosine rule
$AC^2 = 6^2+7^2-2.6.7\cos 60^0$
$=36+49-42=43$
$\Rightarrow AC=\sqrt{43}$

10. $a=15, b=20, c=25, s=30$
$\Delta = \sqrt{30\times15\times10\times5}=15\times10=150$
$R=\frac{abc}{4\Delta}=\frac{15\times20\times25}{150\times4}=12.5$

11. It is right angled triangle, right angled at 'B'

12. GIVEN:
$\angle A = 30^0 + \angle C$
$\angle A - \angle C = 30^0$
GIVEN:
$R=\left(\sqrt{3}+1\right)r$
$R=\left(\sqrt{3}+1\right)4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
$\frac{\sqrt{3}-1}{2}=2\sin\frac{B}{2}\left(\cos\frac{A-C}{2}-\cos\frac{A+C}{2}\right)$
$\frac{\sqrt{3}-1}{4}=\sin\frac{B}{2}\left(\cos 15^0-\sin\frac{B}{2}\right)$
$\frac{\sqrt{3}-1}{4}=\frac{\sqrt{3}+1}{2\sqrt{2}}\sin\frac{B}{2}-\left(\sin\frac{B}{2}\right)^2$
Clearly , $\sin\frac{B}{2}=\frac{1}{\sqrt{2}}$ satisfy above equation
$\Rightarrow \frac{B}{2}=45^0 \Rightarrow B=90^0$

13. $a=13, b=14, c=15$
Circumradius $R=\frac{abc}{4\Delta}$

14. $a, b, c \rightarrow A.P$

$s-\frac{\Delta}{r_1}, s-\frac{\Delta}{r_2}, s-\frac{\Delta}{r_3} \to A.P$

$\frac{\Delta}{r_1}, \frac{\Delta}{r_2}, \frac{\Delta}{r_3} \to A.P$

$\frac{1}{r_1}, \frac{1}{r_2}, \frac{1}{r_3} \to A.P$

$r_1, r_2, r_3 \to H.P$

15. $\frac{a}{4}=\frac{b}{5}=\frac{c}{6}=k$

$a=4k \; b=5k \; c=6k$

$S=\frac{15k}{2}$

$\Delta=\sqrt{\left(\frac{15k}{2}\right)\left(\frac{7k}{2}\right)\left(\frac{5k}{2}\right)\left(\frac{3k}{2}\right)}$

$=\frac{15k^2\sqrt{7}}{4}$

$\frac{R}{r}=\left(\frac{abc}{4\Delta}\right)\left(\frac{S}{\Delta}\right)=\frac{16}{7}$

16. $b=10, a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=15$

$a.\frac{s(s-c)}{ab}+c.\frac{s(s-a)}{bc}=15$

$\frac{s}{b}(2s-c-a)=15$

$\frac{s}{b}(b)=15 \Rightarrow s=15$

$\Delta=15\sqrt{3}$

$\cot\frac{B}{2}=\frac{s(s-b)}{\Delta}=\frac{15\times 5}{15\sqrt{3}}=\frac{5}{\sqrt{3}}$

17. $d_1=2r_1, d_2=2r_2, d_3=2r_3$

$d_1d_2+d_2d_3+d_3d_1=4(r_1r_2+r_2r_3+r_3r_1)$

$=4S^2$

$=(a+b+c)^2$ S

18. Given perimeter of ΔABC is 6 times of A.M of sine of its angles

$i.e.\ a+b+c=6\left[\frac{\sin A+\sin B+\sin C}{3}\right]$

$\Rightarrow 2R[\sin A+\sin B+\sin C]=2[\sin A+\sin B+\sin C]$

$\Rightarrow 2R=2$

$\Rightarrow R=1$

Now,

$a=2R\sin A \Rightarrow 1=2\sin A \quad \begin{pmatrix} given \\ BC=a=1 \end{pmatrix}$

$\Rightarrow \sin A=\frac{1}{2}$

$\angle A=\frac{\pi}{6}$

19. Given

$r=r_1-r_2-r_3$

$\Rightarrow r+r_2=r_1-r_3$

On simplifying

$\Rightarrow \tan\frac{B}{2}=\tan\left(\frac{A-C}{2}\right)$

$\Rightarrow \frac{B}{2}=\frac{A-C}{2} \Rightarrow B+C=A$

w.k.t

$A+B+C=180^0 \Rightarrow \angle A=90^0$

$now,\ a=2R\sin A$

$\therefore 2R=a$

20. Conceptual

21. $\frac{8R^3(\sin^3 A+\sin^3 B+\sin^3 C)}{\sin^3 A+\sin^3 B+\sin^3 C}=8$

$\Rightarrow R=1$

$c=2R\sin A=2\sin A \Rightarrow \max .value=2$

22. $C=9, S=10, \Delta=10\sqrt{2}, 2S=20 \Rightarrow a+b=11$

$\Delta=10\sqrt{2} \Rightarrow S(S-a)(S-b)(S-c)=200$

$a=5, b=6, c=9$

$\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\frac{C}{2}=\frac{-1}{11\sqrt{2}}$

$b\left(1+\sqrt{2}\tan\frac{A-B}{2}\right)=\frac{60}{11}$

Option A

$a\left[1-\sqrt{2}\tan\frac{A-B}{2}\right]=\frac{60}{11}$

23. $\cot A+\cot B+\cot C$

$$=\frac{b^2+c^2-a^2}{4\Delta}+\frac{c^2+a^2-b^2}{4\Delta}+\frac{a^2+b^2-c^2}{4\Delta}$$

$$=\frac{a^2+b^2+c^2}{4\Delta}$$

24. $a^2-c^2=b^2-bc$

$$a^2=b^2+c^2-2\cos 60^0 bc$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\angle A=60^0$$

$$a=2R\sin A=2\left(\frac{1}{\sqrt{3}}\right)\frac{\sqrt{3}}{2}=1$$

$$\sqrt{2}a=2b-c$$

$$2a^2=4b^2+c^2-4bc$$

$$2a^2=4a^2-4c^2+c^2$$

$$c=\sqrt{\frac{2}{3}},\Rightarrow\sqrt{2}a=2b-c$$

$$\Rightarrow\sqrt{2}=2b-\sqrt{\frac{2}{3}}$$

$$\Rightarrow b=\frac{\sqrt{3}+1}{\sqrt{6}}$$

25. $\angle C=\frac{\pi}{2},A+B=\frac{\pi}{2}$

$$\frac{a^2+b^2}{a^2-b^2}\sin(A-B)=\frac{\sin^2 A+\sin^2 B}{\sin^2 A-\sin^2 B}\sin(A-B)$$

$$=\sin^2 A+\sin^2 B$$

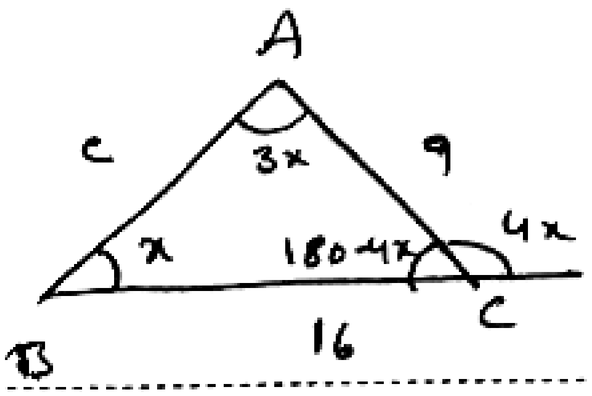
$$=1-\left(\cos^2 A-\sin^2 B\right)$$

$$=1-\cos(A+B)\cos(A-B)=1$$

26. Conceptual

27. $\frac{16}{\sin 3x}=\frac{9}{\sin x}$

$$\frac{16}{9}=3-4\sin^2 x$$



$$\sin x=\frac{\sqrt{11}}{6}$$

$$\cos x=\frac{5}{6}$$

$$\cos(180-4x)=\frac{81+256-c^2}{2(9)(16)}$$

$$-\left(2\cos^2 2x-1\right)=\frac{337-c^2}{288}$$

$$-2\left(2\cos^2 x-1\right)^2=\frac{337-c^2}{288}-1$$

$$-2\left(2\left(\frac{25}{36}\right)-1\right)^2==\frac{49-c^2}{288}$$

$$c^2=\frac{1225}{9}\Rightarrow c=\frac{35}{3}$$

28. $$\frac{2\cos^2\frac{C}{2}}{4R\cos^2\frac{C}{2}}+\frac{2\cos^2\frac{A}{2}}{4R\cos^2\frac{A}{2}}+\frac{2\cos^2\frac{B}{2}}{4R\cos^2\frac{B}{2}}$$

$$=\frac{1}{2R}+\frac{1}{2R}+\frac{1}{2R}=\frac{3}{2R}$$

29. $\left(b^2-c^2\right)\cot A=4R^2\left(\sin^2 B-\sin^2 C\right)\cot A$

$$=4R^2\sin A\sin(B-C)\frac{\cos A}{\sin A}$$

$$=-4R^2\sin(B-C)\cos(B+C)$$

$$=-2R^2\left(2\sin(B-C)\cos(B+C)\right)$$

$$=-2R^2\left(\sin 2B-\sin 2C\right)........(1)$$

Similarly

$$\left(c^2-a^2\right)\cot B=-2R^2\left(\sin 2C-\sin 2A\right)..........(2)$$

$$(1)+(2)\Rightarrow 2R^2\left(\sin 2A-\sin 2B\right)$$

30. Consider $\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^3}+\frac{1}{r^2}\right)=$

$$=\frac{s^2}{\Delta^2}+\frac{(s-a)^2}{\Delta^2}+\frac{(s-b)^2}{\Delta^2}+\frac{(s-c)^2}{\Delta^2}$$

$$=\frac{1}{\Delta^2}\left[s^2+s^2+a^2-2as+s^2+b^2-2sb+s^2+c^2-2sc\right]$$

$$=\frac{1}{\Delta^2}\left[4s^2-2s(a+b+c)+a^2+b^2+c^2\right]$$

$$=\frac{1}{\Delta^2}\left[4s^2-4s^2+a^2+b^2+c^2\right]$$

$$=\frac{a^2+b^2+c^2}{\Delta^2}$$

$$\therefore \frac{\Delta^2}{a^2+b^2+c^2}\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^3}+\frac{1}{r^2}\right)=1$$

31. $R:r_1:r=5:12:2$

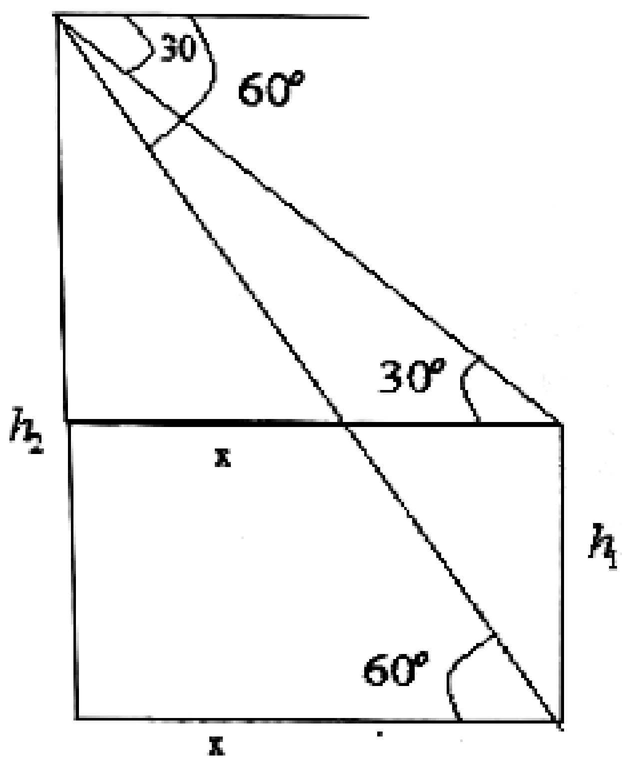
It forms a right angled triangle

$A=90^0$

$r+r_3+r_2-r_1=4R\cos A=0=\cos A$

HEIGHTS AND DISTANCE SOLUTIONS

32.



$$\tan 30^o=\frac{h_2-h_1}{x}\Rightarrow\frac{1}{\sqrt{3}}=\frac{h_2-h_1}{x}\ldots(1)$$

$$\tan 60^o=\frac{h_2}{x}\Rightarrow\sqrt{3}=\frac{h_2}{x}\ldots.(2)$$

$$\frac{(1)}{(2)}\Rightarrow\frac{1}{3}=\frac{h_2-h_1}{h_2}$$

$h_1:h_2=2:3$

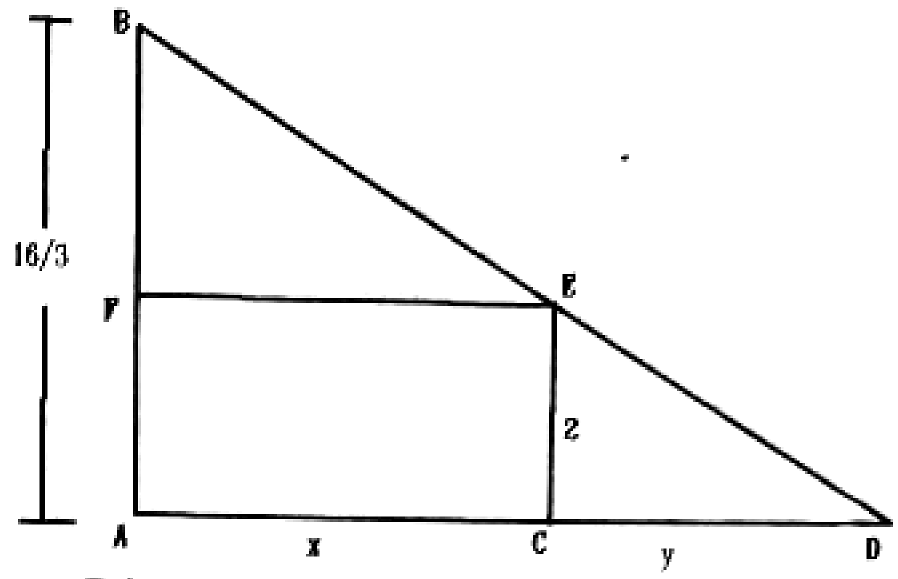
33. $a=6,b=5,c=9$

$\Rightarrow s=10$

$$\Delta=\sqrt{s(s-a)(s-b)(s-c)}=10\sqrt{2}$$

$use\quad r=\frac{\Delta}{s}$

34. $CE=2, AB=\frac{16}{3}$



From Diagram

$let\ AC=x,\ CD=y$

$$\frac{dx}{dt}=\frac{5}{3}$$

$\Delta ABD\cong\Delta ECD$

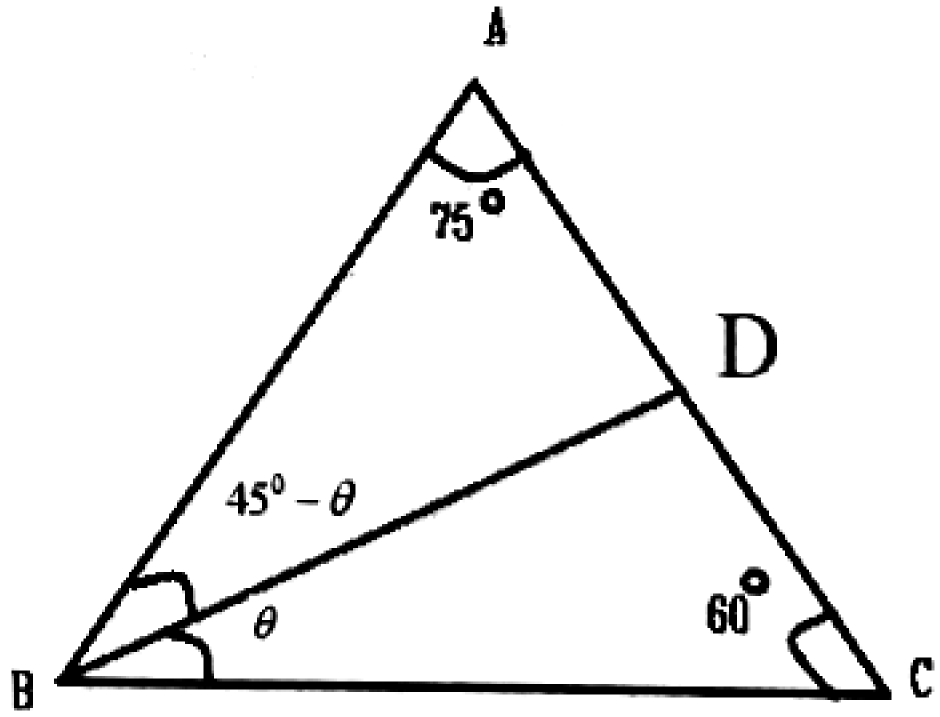
$$\frac{AB}{CE}=\frac{AD}{CD}\Rightarrow\frac{\frac{16}{3}}{2}=\frac{x+y}{y}$$

$$\Rightarrow\frac{16}{6}=\frac{x+y}{y}\Rightarrow 10y=6x$$

$$\Rightarrow 10\frac{dy}{dt}=6\frac{dx}{dt}$$

$$\Rightarrow\frac{dy}{dt}=\frac{6}{10}\times\frac{5}{3}=1$$

35.



Given Area of ΔBAD =(Area of ΔBCD) $\sqrt{3}$

$$\frac{1}{2}.AD.h=\frac{1}{2}.CD.h\sqrt{3}$$

$AD=\sqrt{3}CD$

$In\Delta ABD$, sine rule

$$\frac{AD}{\sin(45^0-\theta)}=\frac{BD}{\sin 75^0}$$

$$BD=\frac{\sin 75^0 AD}{\sin(45^0-\theta)}$$

$In\Delta BCD$, sine rule

$$\frac{CD}{\sin\theta}=\frac{BD}{\sin 60^0}$$

$$BD=\frac{\sin 60^0 CD}{\sin\theta}$$

$$\frac{AD\sin 75^0}{\sin(45^0-\theta)}=\frac{\sin 60^0 CD}{\sin\theta}$$

$$\frac{\sqrt{3}+1}{\sqrt{2}}\sin\theta=\frac{1}{\sqrt{2}}(\cos\theta-\sin\theta)$$

$$(\sqrt{3}+2)\sin\theta=\cos\theta$$

$$\Rightarrow \tan\theta=\frac{1}{\sqrt{3}+2}\times\frac{\sqrt{3}-2}{\sqrt{3}-2}$$

$$\tan\theta=2-\sqrt{3}\Rightarrow\theta=15^0$$

$$\angle ABD=45^0-\theta=30^0$$

36. $use\dfrac{a}{\sin A}=\dfrac{b}{SinB}$

37. $We\,have\,A+B+C=180^o$

$$\Rightarrow C=60^o$$

$Use\,a:b:c=SinA:SinB:SinC$

38. $Use\,a=2RSinA, b=2R\sin B$

$$r_1=4R\ Sin A/2CosB/2Cos C/2$$

$$r_2=4R\cos A/2SinB/2CosC/2$$

$$r_3=4R\cos A/2\cos B/2SinC/2$$

39. $we\,have\,r_1+r_2+r_3-r=4R$

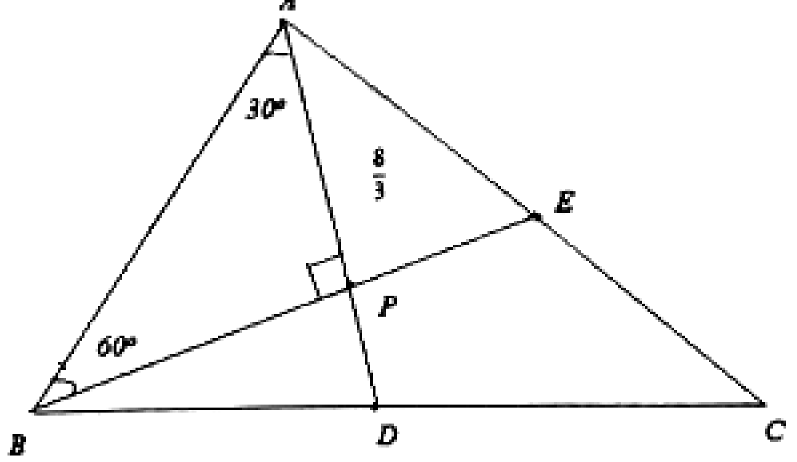
40. $\dfrac{a+b+c}{a+c}+\dfrac{a+b+c}{b+c}=3$

$$\Rightarrow c^2=a^2+b^2-ab$$

$$C=60^o$$

$Now\,find\,\tan C/8=\tan 7\dfrac{1}{2}^o$

41.



$$AD=4\Rightarrow AP=\frac{8}{3}$$

$$In\Delta ABP,\frac{BP}{\sin 30^o}=\frac{\frac{8}{3}}{\sin 60^o}$$

$$BP=\frac{8}{3\sqrt{3}}$$

$$Ar(ABC)=2(Ar(\Delta ABD)$$

$$=2\left[\frac{1}{2}4\times\frac{8}{3\sqrt{3}}\right]$$

$$=\frac{32}{3\sqrt{3}}$$

42. $2\left(\dfrac{abc}{4R}\right)^2=\dfrac{a^2b^2c^2}{a^2+b^2+c^2}$

$$\Rightarrow a^2+b^2+c^2=8R^2$$

$$\Rightarrow \sin^2 A+Sin^2B+Sin^2C=2$$

$Now\,take\ A=30^o, B=60^o, C=90^o$ and varify.

43. $\alpha+\beta+\gamma=360^o$......(1)

$A.M=G.M$

$$\Rightarrow\cos\left(\alpha+\frac{\pi}{2}\right)=\cos\left(\beta+\frac{\pi}{2}\right)=\cos\left(\gamma+\frac{\pi}{2}\right)$$

$$\Rightarrow\alpha=\beta=\gamma...(2)$$

$(1)\,of\,(2)\ \gamma=120^o=\beta=\gamma$

$$Min\,value=\frac{\cos(120^o+90^o)+\cos(120^o+90^o)+\cos(120^o+90^o)}{3}$$

44. $(a^2+b^2)\left[\cos^2\dfrac{C}{2}+Sin^2\dfrac{C}{2}\right]-2ab\left[\cos^2\dfrac{C}{2}-\sin^2\dfrac{C}{2}\right]$

$$a^2+b^2-2ab\cos C$$

$$=c^2$$

45. $\dfrac{1}{r}=\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}$

$$\Rightarrow r=1$$

$$\Delta=\sqrt{rr_1r_2r_3}=6$$

$$use\,r_1=\frac{\Delta}{s-a}, r_2=\frac{\Delta}{s-b}$$

46. Take A=60°, B=60° and then check

47. $k+2k+3k=180^{o}$

$\Rightarrow k=30^{o}$

$\Rightarrow A=30^{o}, B=60^{o}, C=90^{o}$

$use\ a:b:c=\sin A:SinB:SinC$

48. $1=\dfrac{(S-b)(S-c)}{S(S-a)}$

$\tan A/2=1$

$\Rightarrow A=\dfrac{\pi}{2}$

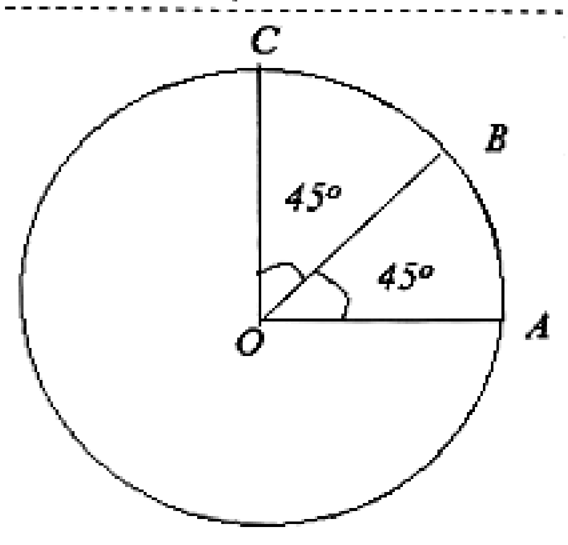
49. $Use\ r_1=\dfrac{\Delta}{S-a}, r_2=\dfrac{\Delta}{s-b}, r_3=\dfrac{\Delta}{s-C}$

50. $Use\,\Delta=\dfrac{abc}{4R}, SinA/2=\sqrt{\dfrac{(s-b)(s-c)}{bc}}$

$Sin\,B/2=\sqrt{\dfrac{(s-a)(s-c)}{ac}}$

$Cos\,C/2=\sqrt{\dfrac{s(s-c)}{ab}}$

51.



$\overline{OA}=R\bar{i}$

$\overline{OB}=R\dfrac{1}{\sqrt{2}}\bar{i}+R\dfrac{1}{\sqrt{2}}\bar{j}$

$\overline{OC}=R\bar{j}$

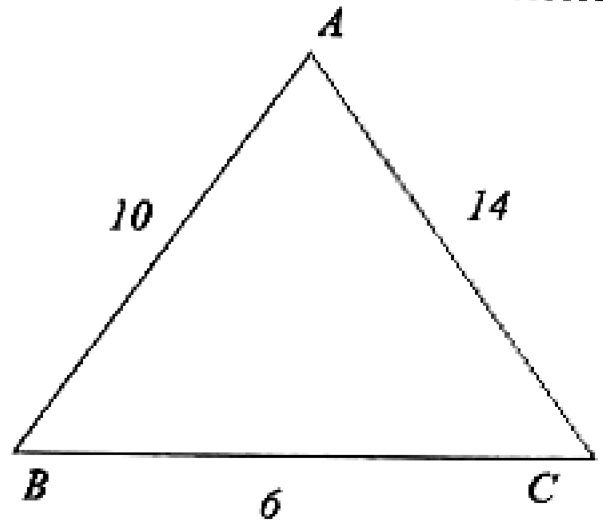
$\left|\overline{OA}+\overline{OB}+\overline{OC}\right|=\sqrt{(R+R/\sqrt{2})^2+(R+R/\sqrt{2})^2}=R(\sqrt{2}+1)$

52. $2ac\,Sin\dfrac{1}{2}(180^{o}-2B)$

$\Rightarrow 2ac\cos B$

$\Rightarrow a^2+c^2-b^2$ (cosine rule)

53.

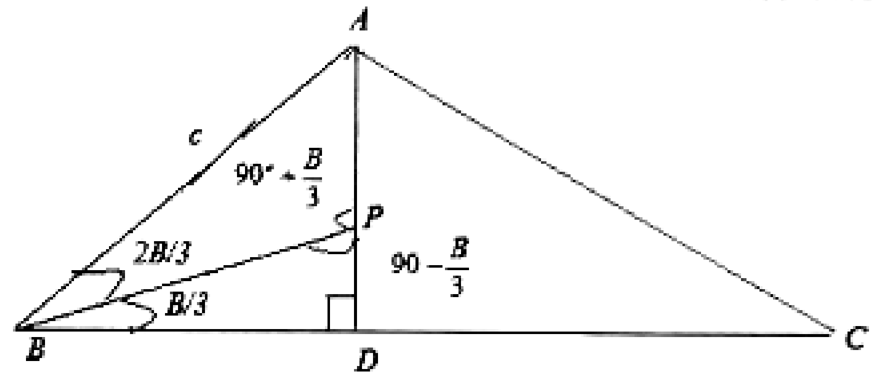


$CosB=\dfrac{a^2+c^2-b^2}{2ac}=$

$\Rightarrow B=120^{o}(obtuse)\Rightarrow Sum\,of\ acute=180^{o}-120^{o}$

$=60^{o}$

54.



$In\,\Delta ABP$

$\dfrac{AP}{Sin\dfrac{2B}{3}}=\dfrac{c}{\sin\left(90^{o}+\dfrac{B}{3}\right)}$

$\dfrac{AP}{2\sin\dfrac{B}{3}}=c$

$AP=2cSin\dfrac{B}{3}$

55. $2B=A+C$

$\Rightarrow B=60^{o}$

$use\,b^2=a^2+c^2-2ac\cos B$

56. $Co\sec A\sin(B+C)$

$use\,A+B+C=180^{o}$

57. Put x=2, then a=7, b=5, c=3

$\Rightarrow CosA=\dfrac{b^2+c^2-a^2}{2bc}$

$=-\dfrac{1}{2}$

58. $b^2 2sinC\cos C + c^2 2\sin B\cos B$

$use \frac{b}{\sin B} = \frac{c}{sinC}$

$\Rightarrow 2bc\, sinB\cos C + 2cbsinC\,cos\,B$

$2bc\sin(B+C)$

$2bc\sin A = 2(2\Delta) = 4\Delta$

59. In triangle A+B+C =180°

$\Rightarrow \tan A + \tan B + \tan C = \tan A.\tan B.\tan C$

60. $\angle A = 30^o, \angle B = 60^o$

$\Rightarrow \angle C = 90^o$

$use\, a:b:c = SinA : SinB : SinC$

61. Use $b^2 = a^2 + c^2 - 2ac\cos B$

62. $\frac{R}{r} = \frac{(abc)S}{4\Delta^2}....(1)$

$a = 4k, b = 5k, c = 6k$

$\Rightarrow S = \frac{15k}{2}, \Delta^2 = S(S-a)(S-b)(S-c)$

$= \frac{15k}{2}.\frac{7k}{2}\frac{5k}{2}\frac{3k}{2}$

$Sub\, above\, value\, in\, eq(1)$

63. $\left(\frac{b^2+c^2-a^2}{2bc} + \frac{a^2+c^2-b^2}{2ac} + \frac{a^2+b^2-c^2}{2ab}\right)2abc$

$\Rightarrow -a^3 - b^3 - c^3 + ab^2 + a^2b + ac^2 + a^2c + c^2b + b^2c$

variety with a $= 1$, $b = 2, c = 3$

64. $[(S-b)-(s-a)](s-c) = ((s-c)-(s-b))(s-a)$

$\Rightarrow \left(\frac{1}{r_2} - \frac{1}{r_1}\right)\frac{1}{r_3} = \left(\frac{1}{r_3} - \frac{1}{r_2}\right)\frac{1}{r_1}$

$\Rightarrow r_1 + r_3 = 2r_2$

65. $r : R : r_1 = 4RSinA/2\,SinB/2\,SinC/2 : R : 4R\,Sin$

$A/2\,Cos\,B/2\,Cos\,C/2$

$Put\, A = B = C = 60^o$

66. Given $\angle B = \frac{\pi}{4}, \angle C = \frac{\pi}{3}$

$\angle A = 75^0$

$W.K.T\ \Delta = 2R^2 \sin A.\sin B.\sin C$

$54 + 18\sqrt{3} = 2R^2.\frac{\sqrt{3}+1}{2\sqrt{2}}.\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2}$

$54 + 18\sqrt{3} = \frac{3+\sqrt{3}}{4}R^2$

$R^2 = \frac{4\left(54+18\sqrt{3}\right)}{3+\sqrt{3}}$

$R = \frac{2\sqrt{18\left(3+\sqrt{3}\right)}}{\sqrt{3+\sqrt{3}}} = \frac{2.3\sqrt{2}.\sqrt{3+\sqrt{3}}}{\sqrt{3+\sqrt{3}}} = 6\left(\sqrt{2}\right)$

$a = 2R\sin A = 6\left(\sqrt{3}+1\right)$

67. $r_1 = 12, r_2 = 6, r_3 = 4$ $\quad \Delta = \sqrt{rr_1r_2r_3}$ $\quad r_1 = \frac{\Delta}{s-a}$

$\frac{1}{r} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4}$ $\quad = \sqrt{2.12.6.4}$ $\quad s-a = \frac{\Delta}{r_1} = \frac{24}{12} = 2$

$\frac{1}{r} = \frac{1+2+3}{12}$ $\quad = \sqrt{576}$ $\quad s-b = \frac{\Delta}{r_1} = \frac{24}{6} = 4$

$\frac{1}{r} = \frac{1}{2}$ $\quad = 24$ $\quad s-c = \frac{\Delta}{r_3} = \frac{24}{4} = 6$

$\therefore r = 2$ $\quad 3s - 2s = 12$

$s = 12$

$\therefore s - a = \frac{\Delta}{r_1}$

$s - \frac{\Delta}{r_1} = a \Rightarrow 12 - \frac{24}{12} = a \Rightarrow a = 10$

$s - \frac{\Delta}{r_2} = b \Rightarrow 12 - \frac{24}{6} = b \Rightarrow b = 8$

$s - \frac{\Delta}{r_3} = c \Rightarrow 12 - \frac{24}{4} = c \Rightarrow c = 6$

$\therefore a + 2b + 3c = 10 + 2(8) + 3(6)$

$= 16 + 18 + 10$

$= 44$

68. $a:b:c = 4:5:6$

$a = 4k, b = 5k, c = 6k$

$2s = 15k$

$\Delta = \frac{\sqrt{1575}.k^2}{4}$

$\frac{1}{4R}\left[r_1 + r_2 + r_3\right] = \frac{4R+r}{4R} = 1 + \frac{r}{4R} = \frac{71}{64}$

69. $a = 12, b = 16, c = 20$

$r_3 : r_2 : r_1 = 6:3:2$

70. $4a = b + c$

$$\sqrt{\frac{(s-a)(s-c)}{s(s-b)} \times \frac{(s-a)(s-b)}{s(s-c)}}$$

$$= \frac{s-a}{s} = \frac{\frac{a+b+c}{2} - a}{\frac{a+b+c}{2}}$$

$$= \frac{b+c-a}{a+b+c} = \frac{4a-a}{a+4a} = \frac{3}{5}$$

71. $4\Delta = c^2 - (a-b)^2$

$$4\Delta = (c+a-b)(c-a+b)$$

$$= (2s-2b)(2s-2a)$$

$$4\Delta = 4(s-b)(s-a)$$

$$1 = \frac{(s-a)(s-b)}{\Delta}$$

$$1 = \tan\frac{C}{2}$$

$$\frac{C}{2} = 45^0 \Rightarrow \boxed{C = 90^0}$$

$$\boxed{\sin C = 1}$$

72. Conceptual

73. $b^2 \sin 2C + c^2 \sin 2B$

$$8R^2 \sin^2 B(\sin C \cos C) + 8R^2 \sin^2 C \sin B \cos B$$

$$8R^2 \sin B \sin C[\sin B \cos C + \cos B \sin C]$$

$$8R^2 \sin B \sin C[\sin(B+C)]$$

$$4(2R^2 \sin A \sin B \sin C) = 4\Delta$$

74. $\frac{4R^2(\sin^2 A + \sin^2 B)}{4R^2(\sin^2 A - \sin^2 B)}.\sin(A-B) = 1$

$$\frac{(\sin^2 A + \sin^2 B)}{\sin(A+B)\sin(A-B)}.\sin(A-B) = 1$$

$$\sin^2 A + \sin^2 B = \sin(A+B)$$

$$\sin^2 A + \sin^2 B = \sin(180^0 - C) = \sin C = 1$$

$$A + B = 90^0$$

Given $0 < B < 45^0 \Rightarrow A > 45^0$

Given $C = 90^0$

$\therefore B < A < C$

$\sin B < \sin A < \sin C$

$$\frac{b}{2R} < \frac{a}{2R} < \frac{c}{2R} \Rightarrow c > a > b$$

75. $rr_2 = r_1 r_3$

$$\frac{\Delta}{s}.\frac{\Delta}{s-b} = \frac{\Delta}{s-a}.\frac{\Delta}{s-c}$$

$$\frac{(s-c)(s-a)}{s(s-b)} = 1$$

$$\tan^2\frac{B}{2} = 1$$

$$B = 90^0$$

$$\cos 2B = \cos 180^0 = -1$$

76. $R = \frac{AC}{2} = \frac{5}{2}$

77. $\frac{\cos^2\left(\frac{B-C}{2}\right)}{4R^2(\sin B + \sin C)^2} + \frac{\sin^2\left(\frac{B-C}{2}\right)}{4R^2(\sin B - \sin C)^2}$

$$\frac{\cos^2\left(\frac{B-C}{2}\right)}{4R^2\left(4\sin^2\left(\frac{B+C}{2}\right)\cos^2\left(\frac{B-C}{2}\right)\right)} + \frac{\sin^2\left(\frac{B-C}{2}\right)}{4R^2\left(4\cos^2\left(\frac{B+C}{2}\right)\sin^2\left(\frac{B-C}{2}\right)\right)}$$

$$\frac{1}{16R^2}\left[\frac{1}{\sin^2\left(\frac{B+C}{2}\right)} + \frac{1}{\cos^2\left(\frac{B+C}{2}\right)}\right]$$

$$\frac{1}{4R^2}\left[\frac{1}{\left(2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B+C}{2}\right)\right)^2}\right]$$

$$\frac{1}{4R^2}\left[\frac{1}{(\sin(B+C))^2}\right] = \frac{1}{(2R\sin A)^2} = \frac{1}{a^2}$$

78. $\frac{1}{a+b}+\frac{1}{c+a}=\frac{3}{a+b+c}$

$(2a+b+c)(a+b+c)=3[(a+b)(a+c)]$

$\frac{b^2+c^2-a^2}{2bc}=\frac{1}{2}$

$\cos A=\frac{1}{2}\Rightarrow A=60^0$

$\therefore \sin A=\frac{\sqrt{3}}{2}$

79. Angles are in A.P.

$A=30^0, B=60^0, C=90^0$

$\cos B=\frac{a^2+c^2-b^2}{2ac}$

$\Rightarrow b^2=a^2+c^2-ac$

80. $\frac{1+\cos(A-B).\cos C}{1+\cos(A-C).\cos B}$

$=\frac{1-\cos(A+B)\cos(A-B)}{1-\cos(A+C)\cos(A-C)}$

$=\frac{1-\cos^2 A+\sin^2 B}{1-\cos^2 A+\sin^2 C}=\frac{\sin^2 A+\sin^2 B}{\sin^2 A+\sin^2 C}$

$=\frac{a^2+b^2}{a^2+c^2}$

81. In $\Delta ABC, \frac{b-c\cos A}{c-b\cos A}$

$=\frac{c\cos A+a\cos C-c\cos A}{a\cos B+b\cos A-b\cos A}=\frac{\cos C}{\cos B}$

82. $\cot\frac{A}{2}\cot\frac{C}{2}=\frac{s(s-a)}{\Delta}\times\frac{\Delta}{(s-a)(s-b)}$

$=\frac{2s}{2s-2b}=\frac{3}{2}$

83. $a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=\frac{3b}{2}$

$a\frac{s(s-c)}{ab}+c.\frac{s(s-a)}{bc}=\frac{3b}{2}$

$\frac{s}{b}[s-c+s-a]=\frac{3b}{2}$

$2b=a+c$

84. In $\Delta ABC, B+C=72^0\Rightarrow A=108^0$

Now, $\left(1+\frac{a}{c}+\frac{b}{c}\right)\left(1+\frac{c}{b}-\frac{a}{b}\right)$

$=\left(\frac{a+b+c}{c}\right)\left(\frac{b+c-a}{b}\right)=\frac{2s}{c}\times\frac{2(s-a)}{b}$

$=4\cos^2\frac{A}{2}=4\cos^2 54^0$

$=\frac{5-\sqrt{5}}{2}$

85. $\Delta=\frac{1}{2}ap_1=\frac{1}{2}bp_2=\frac{1}{2}cp_3$

$\Rightarrow p_1=\frac{2\Delta}{a}, p_2=\frac{2\Delta}{b}, p_3=\frac{2\Delta}{c}$

$\left(\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}\right)^2=\left(\frac{a}{2\Delta}+\frac{b}{2\Delta}+\frac{c}{2\Delta}\right)^2$

$=\frac{(a+b+c)^2}{4\Delta^2}=\frac{1}{r^2}=\frac{1}{r}.\left[\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right]$

86. Use

$a=2R\sin A$

$b=2R\sin B$

87. Use $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

88. $k+2k+3k=180^0$

$k=30^0$

$a:b:c=\sin A:\sin B:\sin C$

$=\sin 30^0:\sin 60^0:\sin 90^0$

89. use $r_1=S\tan A/2$

$r_2=S\tan B/2$

$r_3=S\tan C/2$

90. Use cosine formula

91. $\frac{a}{b}=\frac{\sin A}{\sin B}$

$\frac{\frac{a}{b}-1}{\frac{a}{b}+1}=\frac{\frac{\sin A}{\sin B}-1}{\frac{\sin A}{\sin B}+1}$

$$\frac{2+\sqrt{3}-1}{2+\sqrt{3}+1}=\tan\frac{C}{2}\tan\left(\frac{A-B}{2}\right)$$

$\Rightarrow A-B=90^0$

from given data $A+B=120^0$

$\Rightarrow A=105^0$

92. Use $\cos C=\dfrac{a^2+b^2-c^2}{2ab}$

93. Use projection rule

94. Use cosine formula

95. *Use*

$$\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$$

$\Rightarrow r=6$

and $\Delta^2=rr_1r_2r_3$

$\Rightarrow \Delta=6^3$

finally use $S=\dfrac{\Delta}{r}$

96. *use*

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

$\Rightarrow \cos A=\dfrac{1}{8}$

Now use $\cos 2A=2\cos^2 A-1$

97. $\tan\dfrac{A}{2}=\dfrac{1}{\sqrt{2}}$

$$\Rightarrow \cos A=\frac{1-\tan^2 A/2}{1+\tan^2 A/2}$$

$$=\frac{1}{3}$$

Now use $a^2=b^2+c^2-2bc\cos A$

98. *use*

$$r_1=\frac{\Delta}{s-a},r_2=\frac{\Delta}{s-b}$$

$$r_3=\frac{\Delta}{s-c},r=\frac{\Delta}{s}$$

99. $a=4,b=5,c=7$

use $s=\dfrac{a+b+c}{2}$

$$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$$

100. $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}$

$$\frac{7}{\frac{\sqrt{3}}{2}}=\frac{6}{\sin B}$$

$$\sin B=\frac{3\sqrt{3}}{7}=0.742$$

$$\sin 60=\frac{\sqrt{3}}{2}=0.86$$

$$0.742\cong\sin(47.9)$$

101. Use cosine rule

102. *use*

$$s=\frac{a+b+c}{2}$$

$$=21$$

$$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$$

$$=84$$

$$R=\frac{abc}{4\Delta}=\frac{65}{8}$$

$$r=\frac{\Delta}{s}=4$$

103. $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$

104. *use*

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$

105. $a=4k,b=5k,c=6k$

$$S=\frac{15k}{2},\Delta=\sqrt{\frac{15}{2}\frac{7}{2}\frac{5}{2}\frac{3}{2}}$$

$$\frac{R}{r}=\frac{abcs}{4\Delta^2}$$

$$=\frac{4.5.6.\frac{15}{2}}{4.\frac{15}{2}.\frac{7}{2}\frac{5}{2}.\frac{3}{2}}=\frac{16}{7}$$

106. Use

$$R=\frac{1}{2}ab\sin C=\frac{1}{2}bc\sin A=\frac{1}{2}ac\sin B$$

and cosine rule

107. Use

$$\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$$

108. $\frac{1-2\sin^2 A}{a^2}-\left(\frac{1-2\sin^2 B}{b^2}\right)$

$$\frac{1}{a^2}-\frac{1}{b^2}-2\left(\frac{\sin^2 A}{a^2}-\frac{\sin^2 B}{b^2}\right)$$

$$=\frac{1}{a^2}-\frac{1}{b^2}(\textit{from}\text{ sine rule})$$

109. $R=\frac{c}{2}$

$$r=(s-c)\tan\frac{c}{2}$$

$$=s-c$$

$$R+r=\frac{c+2s-2c}{2}$$

$$=\frac{a+b}{2}$$

110. $\sin\frac{A}{2}=\sqrt{\frac{1-\cos A}{2}}$

$$\sin^2\frac{A}{2}=\frac{1-\cos A}{2}$$

$$\sin^2\frac{A}{2}=\frac{1-\frac{b^2-a^2+c^2}{2bc}}{2}$$

$$=\frac{a^2-(b-c)^2}{4bc}\le\frac{a^2}{4bc}$$

$$\sin^2\frac{A}{2}\le\frac{a^2}{4bc}$$

$$\sin\frac{A}{2}\le\frac{a}{2\sqrt{bc}}$$

111. Use mollwiede's rule

112. $2b=a+c$

$$\Rightarrow s=\frac{3b}{2}$$

use

$$\cot\frac{A}{2}=\frac{s(s-a)}{\Delta}$$

$$\cot\frac{C}{2}=\frac{s(s-c)}{\Delta}$$

113. Use $\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$

114. $\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$

$$\Rightarrow\frac{3}{4}=\frac{s(s-a)}{bc}\text{.........(1)}$$

$$\textit{take }(a+b+c)(b+c-a)$$

$$=2s(2s-2a)$$

$$=4\frac{3}{4}bc=3bc\text{......}\textit{from}(1)$$

115. Use

$$b^2=(r_2-r)(r_1+r_2)$$

$$\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$$

116. $\frac{1}{r_1},\frac{1}{r_2},\frac{1}{r_3}\text{........}A.P$

$$\frac{s-a}{\Delta},\frac{s-b}{\Delta},\frac{s-c}{\Delta}\text{..........}A.P$$

$$a,b,c\text{.........}A.P$$

117. Use

$$r_1=\frac{\Delta}{s-a},r_2=\frac{\Delta}{s-b},r_3=\frac{\Delta}{s-c}$$

118. $(b+c)^2\sin^2\frac{A}{2}+(b-c)^2\cos^2\frac{A}{2}=k(1-\cos 2A)$

$$b^2+c^2-2bc\left(\cos^2\frac{A}{2}-\sin^2\frac{A}{2}\right)=k(1-\cos 2A)$$

$$a^2=k2\sin^2 A$$

$$4R^2\sin^2 A=2k\sin^2 A$$

$$k=2R^2$$

119. $a^2=b^2+c^2-2bc\sin\text{A}$

$$\Rightarrow\text{a}=\sqrt{13}$$

$$\frac{a}{\sin A}=\frac{b}{\sin\text{B}}$$

$$\Rightarrow\sin B=\frac{7}{2\sqrt{13}}$$

$$a\sin B\sin C = 2R\sin A\sin B\sin C$$
$$= c\sin A\sin B$$
$$=4\sqrt{3}\frac{1}{2}.\frac{7}{2\sqrt{13}} = \frac{7\sqrt{3}}{\sqrt{13}}$$

120. $r_2 + r_3 = 4R\cos^2\frac{A}{2}$
$$= 2R.1$$
$$= 2R\sin A$$
$$= a$$

121. $\sin A = \frac{3\sqrt{3}}{14}$
$$\Rightarrow \cos A = \frac{13}{14}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$9 = 25 + c^2 - 2(5)c\frac{13}{14}$$
$$7c^2 - 65c + 112 = 0$$
$$c = 7, \frac{16}{7}$$
$$c = 7(obtuse)$$

122. $(b\sin c + c\sin B)a$
$$= 2\Delta + 2\Delta$$
$$= 4\Delta$$

123. Use $\frac{1}{\sqrt{A}} = \frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}}$

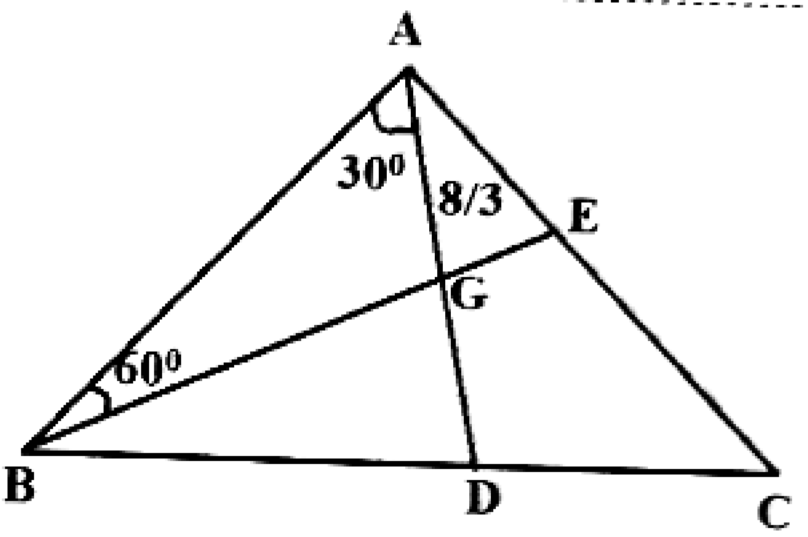
124. $2a = b + c ...(1)$
$$a + b + c = 42$$
from (1) $3a = 42 \Rightarrow a = 14$

Use $a = 2R\sin A$

125. $b\tan\frac{B}{2}\tan\frac{C-A}{2}$
$$= b\left(\frac{c-a}{c+a}\right) ....(1)$$
$$c^2 = a^2 + b^2 - 2ab\cos c$$
$$\Rightarrow b = 9$$
From (1) $9\left(\frac{4}{18}\right) = 2$

126. Use
$$\cot\frac{A}{2} = \frac{s(s-a)}{\Delta}$$
$$\cot\frac{B}{2} = \frac{s(s-b)}{\Delta}$$
$$\cot\frac{C}{2} = \frac{s(s-c)}{\Delta}$$

127.



ΔAGB
$$\frac{\frac{8}{3}}{\sin 60^0} = \frac{BG}{\sin 30^0}$$
$$\frac{\frac{8}{3}}{\frac{\sqrt{3}}{2}} = \frac{BG}{\frac{1}{2}}$$
$$BG = \frac{8}{3\sqrt{3}}$$
Area of $\Delta ABD = \frac{1}{2}4\times\frac{8}{3\sqrt{3}}$
$$= \frac{16}{3\sqrt{3}}$$
Area of $\Delta ABC = \frac{32}{3\sqrt{3}}$

128. $R = \frac{27}{4\sqrt{2}}$
$$\cos C = \frac{b^2 + a^2 - c^2}{2ab} = \frac{-1}{3}$$
$$\sin C = \frac{c}{2R} = \frac{4\sqrt{2}}{3}$$
$$\sin 2C = 2\sin c\cos c$$

129.

$$\frac{r}{r_1}=\frac{1}{2}\Rightarrow\frac{\frac{\Delta}{s}}{\frac{\Delta}{s-a}}=\frac{1}{2}\Rightarrow\frac{s-a}{s}=\frac{1}{2}\Rightarrow s=2a$$

use

$$\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\tan\frac{B}{2}=\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\tan\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

130. $a^2+b^2+c^2-ab-bc-ca=0$

$$(a-b)^2+(b-c)^2+(c-a)^2=0$$

$$a=b=c$$

Δ^{le} *is equilateral*

$$A=B=C=60^0$$

131. $\dfrac{a^2-b^2}{c^2}$

$$=\frac{\sin^2 A-\sin^2 B}{\sin^2 C}$$

$$=\frac{\sin(A+B)\sin(A-B)}{\sin^2 C}$$

take $A=60^0, B=45^0, C=75^0$

132. $\sin\dfrac{B}{2}\tan\dfrac{C-A}{2}$

$$=\sin\frac{B}{2}\left(\frac{c-a}{c+a}\right)\cot\frac{B}{2}$$

$$=\left(\frac{c-a}{c+a}\right)\cos\frac{B}{2}.....(1)$$

$$\cos\frac{B}{2}=\sqrt{\frac{s(s-b)}{ac}}=\frac{\sqrt{3}}{2}$$

from (1) $\sin\dfrac{B}{2}\tan\dfrac{C-A}{2}=\left(\dfrac{8-3}{8+3}\right)\dfrac{\sqrt{3}}{2}$

133.

$$\tan C=\frac{3\sqrt{5}}{2}$$

$$\Rightarrow\cos C=\frac{2}{7}$$

$$c^2=a^2+b^2-2ab\cos c$$

134. $\dfrac{a}{\tan A}=\dfrac{b}{\tan B}=\dfrac{c}{\tan C}$

$$2R\sin A.\frac{\cos A}{\sin A}=2R\sin B.\frac{\cos B}{\sin B}=2R\sin c.\frac{\cos c}{\sin c}$$

$$\cos A=\cos B=\cos C$$

triangle is equilateral

$$A=B=C=60$$

$$\cos^2 A+\cos^2 B+\cos^2 C=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$$

$$=\frac{3}{4}$$

135. Use $R=\dfrac{abc}{4\Delta}\Rightarrow r=\dfrac{\Delta}{s}$

136. $\cot\dfrac{A}{2}:\cot\dfrac{B}{2}:\cot\dfrac{C}{2}=3:7:9$

$$\frac{s(s-a)}{\Delta}:\frac{s(s-b)}{\Delta}:\frac{s(s-c)}{\Delta}=3:7:9$$

$$\frac{s-a}{3}=\frac{s-b}{7}=\frac{s-c}{9}=k$$

$$s-a=3k\ \ s-b=7k\ \ s-c=9k$$

$$s=19k$$

$$a=16k\ \ b=12k\ \ c=10k$$

$$a:b:c=8:6:5$$

137. $\dfrac{2}{r_2}=\dfrac{1}{r_1}+\dfrac{1}{r_3}\Rightarrow 2\left(\dfrac{s-b}{\cancel{\Delta}}\right)=\dfrac{s-a+s-c}{\cancel{\Delta}}$

$$\cancel{2s}-2b=\cancel{2s}-(a+c)\Rightarrow 2b=a+c\Rightarrow s=\frac{3b}{2}$$

$$\frac{r_2}{\Delta}=\frac{s}{s-b}=\frac{\frac{3b}{2}}{\frac{b}{2}}=3:1$$

138. $\Delta=\dfrac{1}{2}aP_1=\dfrac{1}{2}bP_2=\dfrac{1}{2}cP_3$

$$\frac{\cos A}{P_1}+\frac{\cos B}{P_2}+\frac{\cos C}{P_3}$$

$$=\frac{1}{2\Delta}\times R(\sin 2A+\sin 2B+\sin 2C)$$

$$\frac{R(4\sin A\sin B\sin C)}{2(2R^2\sin A\sin B\sin C)}=\frac{1}{R}$$

139. $3-3\sin^2 B = 2-2\sin^2 C$

$\Rightarrow 3-3\sin C = 2-2\sin^2 C$

$\Rightarrow 2\sin^2 C - 3\sin C + 1 = 0$

$\sin C = \frac{3\pm\sqrt{1}}{4} = 1 \text{ (or) } \frac{1}{2}$

$C = 90^0 \text{ (or)} 30^0$

$if\ C = 90^0 \Rightarrow B = 90^0 \Rightarrow Not\ possible$

$if\ C = 30^0 \Rightarrow \sin^2 B = \frac{1}{2} \Rightarrow B = 45^0$

$A = 105^0$

---

140. $r : r_3 = \sin\frac{A}{2}\sin\frac{B}{2} : \cos\frac{A}{2}\cos\frac{B}{2}$

$\Rightarrow \cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right) : \cos\left(\frac{A+B}{2}\right) + \cos\left(\frac{A-B}{2}\right)$

$\Rightarrow 1-\frac{1}{\sqrt{2}} : 1+\frac{1}{\sqrt{2}}$

$\Rightarrow \sqrt{2}-1 : \sqrt{2}+1$

---

141. $a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2} = \frac{3b}{2}$

$\Rightarrow a\left(\frac{\cos C+1}{2}\right) + c\left(\frac{\cos A+1}{2}\right) = \frac{3b}{2}$

$\Rightarrow a\cos C + c\cos A + a + c = 3b$

$\Rightarrow b + a + c = 3b$

$\Rightarrow a + c = 2b \Rightarrow \frac{a+c}{b} = \frac{2}{1}$

---

142. $r_1 = 2r_2 = 3r_3$

$a : b : c = 5 : 4 : 3$

Verify options

---

143. $\tan\frac{A}{2} + \tan\frac{B}{2} = \frac{b}{s}$

$\frac{(s-b)(s-c)}{\Delta} + \frac{(s-a)(s-b)}{\Delta} = \frac{b}{s}$

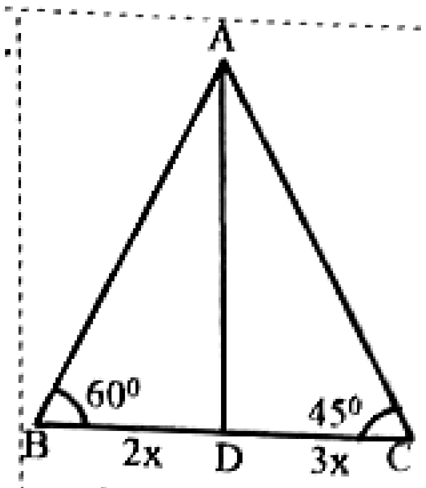
$\frac{s-b}{\Delta}(2s-a-c) = \frac{b}{s}$

$\frac{(s-b)}{\Delta}\times b = \frac{b}{s} \Rightarrow \frac{s(s-b)}{\Delta} = 1$

$\Rightarrow \cot\frac{B}{2} = 1 \Rightarrow B = 90^0 \Rightarrow A + C = 90^0$

$\Rightarrow \sin\left(\frac{A+C}{3}\right) = \frac{1}{2}$

144.



$\frac{\sin\underline{|BAD}}{2x} = \frac{\sin 60^0}{AD} \text{ ---(1)}$

$\frac{\sin\underline{|CAD}}{3x} = \frac{\sin 45^0}{AD} \text{ ---(2)}$

simplify

145.

CosA+CosC= $4\sin^2\frac{B}{2}$

$2\cos\frac{A+C}{2}\cos\frac{A-C}{2} = 4\sin^2\frac{B}{2}$

$\cancel{2}\sin \cancel{B}/2 \cos\frac{A-C}{2} = \cancel{4}^2 \sin^2 B/2$

$\cos\frac{A-C}{2} = 2\sin B/2$ [By Mollwied's Rule]

$\sin \cancel{B/2}\ \frac{a+c}{b} = 2\sin \cancel{B/2}$

2b=a+c

Now $\frac{a+b+c}{a+c} = \frac{3b}{2b} = 3/2$

= 3:2

---

146. Given $A = 90^\circ$

We have

$r_1 = s\,Tan\,A/2$

$r_1 = s$

$and\ r_1r_2 + r_2r_3 + r_3r_1 = s^2$

$r_2r_3 - s^2 = -r_1r_2 - r_3r_1$

Now ,

$(r_2 - r_1)(r_3 - r_1) = r_2r_3 - r_1r_2 - r_1r_3 + r_1^2$

$= r_2r_3 + r_1r_3 - s^2 + r_1^2 = 2r_2r_3$

147. $A+B+C=\pi$ $\quad 2b=a+c$

$3A+B=\pi$ $\quad 2\sin B=\sin A+\sin C$

$B=\pi-3A$ $\quad 2\sin(\pi-3A)=\sin A+\sin 2A$

$2\sin 3A=\sin A[1+2\cos A]$

$6-8\sin^2 A=1+2\cos A$

$\Rightarrow 8\cos^2 A-2\cos A-3=0$

$\cos A=\frac{-1}{2},\ 3/4$

Now $a:c=\sin A:\sin C$

$=\sin A:2\sin A\cos A$

$=1:2\cos A=1:2\left(\frac{3}{4}\right)$

$=2:3$

148. Consider $\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)\tan\frac{C}{2}$

$=\left(\frac{\Delta}{s(s-a)}+\frac{\Delta}{s(s-b)}\right)\frac{\Delta}{s(s-c)}$

$=\left(\frac{s-b+s-a}{(s-a)(s-b)}\right)\frac{\Delta^2}{s^2(s-c)}$

$=\frac{(2s-a-b)\Delta^2}{s^2(s-a)(s-b)(s-c)}$

$=\frac{c(\Delta^2)}{s\Delta^2}$

$=\frac{c}{\left(\frac{a+b+c}{2}\right)}$

$=\frac{2c}{a+b+c}$

149. Given $\angle C=90^o$

$c^2=a^2+b^2\rightarrow 1$

Consider $\left(1-\frac{r_3}{r_1}\right)\left(1-\frac{r_3}{r_2}\right)$

$=\left(1-\frac{s-a}{s-c}\right)\left(1-\frac{s-b}{s-c}\right)$

$=\frac{(a-c)(b-c)}{(s-c)^2}$

$=\frac{4\left(ab-ac-bc+c^2\right)}{(a+b-c)^2}$

$=\frac{4\left(ab-ac-bc+c^2\right)}{2\left(a^2+b^2+ab-bc-ca\right)}$

$=\frac{2\left(ab-ac-bc+c^2\right)}{c^2+ab-bc-ca}$ $(\because from 1)$

$=2$

150. Given $2B=A+C$

$2B=180-B$

$B=60^o$

Also given $b=4\sqrt{3}$

And $a=k,\ c=2k$

$\cos B=\frac{a^2+c^2-b^2}{2ac}$

$\frac{1}{2}=\frac{k^2+4k^2-48}{2\times 2k^2}$

$k^2=16$

$k=4$

Area of

$ABC=\frac{1}{2}ac\sin B$

$=\frac{1}{2}(k)(2k)\left(\frac{\sqrt{3}}{2}\right)$

$=\frac{\sqrt{3}k^2}{2}$

$=\frac{\sqrt{3}\times 16}{2}$

$=8\sqrt{3}$ Sq units

151. $a=5,\ b=12,\ c=13$

$\therefore 2S=a+b+c=30$

$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{15(10)(3)(2)}$

$=30$

$b^2\sin 2c+c^2\sin 2B=4\Delta=120$

152. $\frac{r_1-r}{a}+\frac{r_2-r}{b}$

$=\frac{4R\sin^2\frac{A}{2}}{2R\sin A}+\frac{4R\sin^2\frac{B}{2}}{2R\sin B}$

$=\frac{4R\sin^2\frac{A}{2}}{4R\sin\frac{A}{2}\cos\frac{A}{2}}+\frac{4R\sin^2\frac{B}{2}}{4R\sin\frac{B}{2}\cos\frac{B}{2}}$

$=Tan\frac{A}{2}+Tan\frac{B}{2}$

$=\frac{(s-b)(s-c)}{\Delta}+\frac{(s-c)(s-a)}{\Delta}$

$=\frac{s-c}{\Delta}(s-b+s-a)$

$=\frac{s-c}{\Delta}(c)=\frac{c}{r_3}$

153.

$(a-b)(s-c)=(b-c)(s-a)$

$[(s-b)-(s-a)](s-c)=[(s-c)-(s-b)](s-a$

$\Rightarrow (s-b)(s-c)+(s-a)(s-b)=2(s-a)(s-c)$

$\Rightarrow \frac{1}{s-a}+\frac{1}{s-c}=\frac{2}{s-b}$

$\Rightarrow \frac{\Delta}{s-a}+\frac{\Delta}{s-c}=2\frac{\Delta}{s-b}$

$\Rightarrow r_1+r_3=2r_2$

$\therefore r_1, r_2, r_3$ *are in A.p.*

154. $a\frac{(s-a)(s-b)}{ab}+c\frac{(s-b)(s-c)}{bc}=\frac{b}{2}$

$\frac{(s-b)}{b}[s-a+s-c]=\frac{b}{2}$

$\frac{a+c}{b}=\frac{2}{1}\Rightarrow a+c:b=2:1$

155. CONCEPTUAL

156. $\frac{(s-a)}{2}=\frac{(s-b)}{3}=\frac{(s-c)}{4}=k$

$k=\frac{3s-2s}{9}=\frac{s}{9}$

$\Rightarrow \frac{a}{7}=\frac{b}{6}=\frac{c}{5}$

$\cot A=\frac{b^2+c^2-a^2}{4\Delta}$ $\cot C=\frac{a^2+b^2-c^2}{4\Delta}$

$\frac{\cot A}{\cot C}=\frac{b^2+c^2-a^2}{a^2+b^2-c^2}=\frac{36+25-49}{49+36-25}=\frac{12}{60}=\frac{1}{5}$

$\cot A:\cot C=1:5$

157.

$\frac{r_1}{7}=\frac{r_2}{8}\Rightarrow\frac{r_1}{21}=\frac{r_2}{24}$

$\frac{r_1}{3}=\frac{r_3}{4}\Rightarrow\frac{r_1}{21}=\frac{r_3}{28}$

r1 : r2 : r3 = 21 : 24 : 28

$\frac{\Delta}{s-a}:\frac{\Delta}{s-b}:\frac{\Delta}{s-c}=21:24:28$

$\Rightarrow s-a=\frac{1}{21k},\ s-b=\frac{1}{24k},\ s-c=\frac{1}{28k}$

...... (1) .... (2) .... (3)

$(1)+(2)+(3)\rightarrow 3s-2s=\frac{1}{k}\left(\frac{21}{168}\right)$

$S=\frac{1}{8k}$

(1) $\Rightarrow\frac{1}{8k}-\frac{1}{21k}=a\Rightarrow a=\frac{1}{k}\left[\frac{13}{8\times 21}\right]$

$= b=\frac{1}{k}\left[\frac{2}{24}\right], c=\frac{1}{k}\left[\frac{5}{56}\right]$

a : b : c = $\frac{13}{168}:\frac{14}{168}:\frac{15}{168}=13:14:15$

158. sin2 A – sin2 B = sin C (sin A + sin C)

$\cancel{\sin(A+B)}.\sin(A-B)=\cancel{\sin C}(\sin B+\sin C)$

$\Rightarrow \sin(A-B)-\sin C=\sin B$

$\Rightarrow \sin(A-B)-\sin(A+B)=\sin B$

$\Rightarrow$ - 2 cos A. sin B = sin B

$\Rightarrow \cos A=\frac{-1}{2}\Rightarrow$ A = 1200

159. 2B = A + C $\Rightarrow$ B = 600

$$\frac{c}{a}\sin 2A+\frac{a}{c}\sin 2C=\frac{c}{a}2\sin A\cos A$$

$$+\frac{a}{c}2\sin c\cos c$$

$$=\frac{c}{R}\cos A+\frac{a}{R}\cos C$$

$$=\frac{1}{R}\left[c\cos A+a\cos C\right]=\frac{1}{R}(b)$$

$$=\frac{1}{\cancel{R}}(2\cancel{R}\sin B)=2\times\frac{\sqrt{3}}{2}=\sqrt{3}$$

160. $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=\frac{a+b+c}{abc}=\frac{2s}{4R\Delta}=\frac{1}{2Rr}=\frac{1}{8}$

161. $\frac{1}{4}\left[4R^2\sin^2 B.2\sin C\cos C+4R^2\sin^2 C.2\sin B\cos B\right]$

$$2R^2\sin B.\sin C\sin(B+C)$$

$$2R^2.\frac{b}{2R}.\frac{c}{2R}.\frac{a}{2R}=\frac{abc}{4R}=\Delta$$

$$r r_1\cot\frac{A}{2}$$

$$=\frac{\Delta}{s}.\frac{\Delta}{s-a}.\frac{s(s-a)}{\Delta}=\Delta$$

162. $r_1=\frac{\Delta}{s-a},\Rightarrow s-a=\frac{\Delta}{r_1}\Rightarrow\frac{1}{s-a}=\frac{r_1}{\Delta}$

$$=\frac{ar_1}{\Delta}+\frac{br_2}{\Delta}+\frac{cr_3}{\Delta}=\frac{1}{\Delta}\sum ar_1$$

$$=\frac{1}{\Delta}\sum 2R\sin A.\ s.\mathrm{Tan}\frac{A}{2}$$

$$=\frac{4RS}{\Delta}\sum\sin^2\frac{A}{2}$$

$$=\frac{4R}{2}\left(1-2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$$

$$=\frac{4R}{r}\left(1-\frac{r}{2R}\right)$$

$$=\frac{4R}{r}-2$$

163. $\cot\theta_1=\frac{a}{\frac{a}{2}}=2$

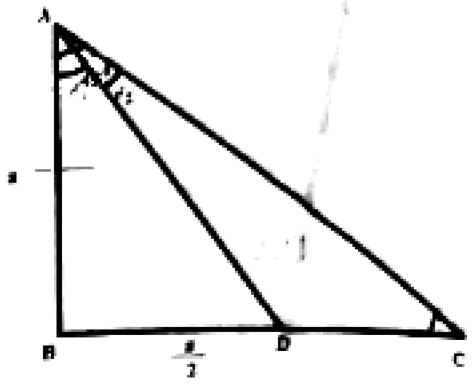
$$\theta_1+\theta_2=45^0$$

$$\cot(\theta_1+\theta_2)=1$$

$$\frac{\cot\theta_1\cot\theta_2-1}{\cot\theta_1+\cot\theta_2}=1$$

$$2\cot\theta_2-1=2+\cot\theta_2$$

$$\cot\theta_2=3$$



164. $Given\ r_1=2r_2=3r_2$

$we\ know\ that\ r_1=\frac{\Delta}{s-a},r_2=\frac{\Delta}{s-b},r_3=\frac{\Delta}{s-c}$

$Take\ a=5,b=4,c=3$

$Now,\frac{a}{b}+\frac{b}{c}+\frac{c}{a}=\frac{5}{4}+\frac{4}{3}+\frac{3}{5}=\frac{191}{60}$

165. Conceptual

166. $\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c}\ and\ a=2$

$$\Delta=\frac{\sqrt{3}}{4}\times a^2=\sqrt{3}$$

167. $b=6,c=9,\ \sin A=\frac{2\sqrt{14}}{9}$

$\cos A=5/9$ (By Right angle triangle)

$$a^2=b^2+c^2-2bc\cos A$$

$$a^2=57$$

Given

$$3a[\cos B+\cos C]=3a\left[\frac{c^2+a^2-b^2}{2ca}+\frac{a^2+b^2-c^2}{2ab}\right]$$

$$=\frac{3a\left[c^2b+a^2b-b^3+a^2c+b^2c-c^3\right]}{2abc}=20$$

168. $f(x)=x^3-11x^2+36x-36$

$a+b+c=11$..........(1)

$ab+bc+ca=36$.....(2)

$abc=36$........(3)

By solving (1),(2),(3) we get

$a=2(or)6,\ b=3,\ c=2(or)6$

$r_1 = 2$, $r_2 = 3$, $r_3 = 6$, ; $r_1 = 6$, $r_2 = 3$, $r_3 = 2$

$s^2 = r_1r_2 + r_2r_3 + r_3r_1$

$s = 6$

$\boxed{2s = 12}$

$s = 6$

$\boxed{2s = 12}$

169. Given a:b:c = 4:5:6

$$S = \frac{a+b+c}{2} = \frac{4+5+6}{2} = \frac{15}{2}$$

$$\Delta = \sqrt{\frac{15}{2} \times \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2}} =$$

Circum Radius : In radius = R: r

$$= \left(\frac{abc}{4\Delta}\right) : \frac{\Delta}{s}$$

$$= \frac{(abc)(s)}{4\Delta^2} = 16.7$$

170. In ΔABC,

$$\text{Given } (2s) = 6\left(\frac{\sin A + \sin B + \sin C}{3}\right)$$

$$(a+b+c) = 6\left(\frac{\sin A + \sin B + \sin C}{3}\right)$$

$$(2R)\ (\sin A + \sin B + \sin C) = 2(\sin A + \sin B + \sin C)$$

R=1

$$\sin A = \frac{a}{2R}$$

Given BC=a=1 unit

$$\text{SinA} = \frac{1}{2}$$

$$\text{angleA} = 300 = \frac{\pi}{6}$$

171. $b = 6,\ c = 7,\ Tan\frac{A}{2} = \frac{1}{\sqrt{6}}$

$$\Rightarrow \cos A = \frac{1 - Tan^2\frac{A}{2}}{1 + Tan^2\frac{A}{2}} = \frac{5}{7}$$

$$\Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow a = 5$$

$\Rightarrow s = 9$ *and* $\Delta = 6\sqrt{6}$

$$r = \frac{\Delta}{s} = \frac{6\sqrt{6}}{9} = \frac{2\sqrt{6}}{3}$$

172. $a = 7, b = 8, c = 9$

$s = 12,\ \Delta = 12\sqrt{5}$

$$\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{25}{144(5)} + \frac{16}{144(5)} + \frac{9}{144(5)}$$

$$= \frac{50}{144 \times 5} = \frac{5}{72}$$