27. QUADRATIC EQUATIONS

1. If α and β are the roots of $x^2+7x+3=0$ and $\frac{2\alpha}{3-4\alpha},\frac{2\beta}{3-4\beta}$ are the roots of $ax^2+bx+c=0$ and GCD of a, b, c is 1 then $a+b+c=$ **(2020)**
   1. 11 2. 0
   3. 243 4. 81

2. If α, β are the roots of $x^2+bx+c=0, \gamma, \delta$ are the roots of $x^2+b_1x+c_1=0$ and $\gamma<\alpha<\delta<\beta$, then $(c-c_1)^2=$ **(2020)**
   1. $(b_1-b)(bc_1-b_1c)$ 2. 1
   3. $(b-b_1)^2$ 4. $(c-c_1)(b_1c-b_1c_1)$

3. If α_1, α_2 are the roots of $x^2+ax+1=0$ and α_3, α_4 are the roots of $x^2+bx+1=0$, then $(\alpha_1+\alpha_3)(\alpha_2+\alpha_3)(\alpha_1+\alpha_4)(\alpha_2+\alpha_4)=$ **(2020)**
   1. $3a^2-b^2$ 2. a^2-3b^2
   3. $(a-b)^2$ 4. $(b+a)^2$

4. The roots of the equation $|x^2-x-6|=x+2$ are **[AP EAMCET 17-09-20_Shift-1]**
   1. -2, 1, 4 2. 0, 2, 4
   3. 0, 1, 4 4. -2, 2, 4

5. If x is complex, the expression $\frac{x^2+34x-71}{x^2+2x-7}$ takes all values which lie in the interval (a,b), find the values of a and b **[AP EAMCET 17-09-20_Shift-2]**
   1. $a=-1, b=1$ 2. $a=1, b=-1$
   3. $a=5, b=9$ 4. $a=9, b=5$

6. If the roots of the equation $ax^2+ax+c=0$ are in the ratio $p:q$, then $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}=$ **[AP EAMCET 17-09-20_Shift-2]**
   1. $\sqrt{\frac{a^2}{c}}$ 2. $\sqrt{\frac{a}{2c}}$
   3. $\sqrt{\frac{a}{c}}$ 4. $\sqrt{\frac{a^2}{2c}}$

7. If the sum of the roots of the quadratic equations is 1 and sum of the square of the roots is 13, then find that equation **[AP EAMCET 18-09-20_Shift-1]**
   1. $x^2+x-6=0$ 2. $x^2-x+6=0$
   3. $x^2-x-6=0$ 4. $x^2+x+6=0$

8. If the roots of the given equation $(\cos p-1)x^2+(\cos p)x+\sin p=0$ are real, then **[AP EAMCET 18-09-20_Shift-2]**
   1. $p\in(-\pi,0)$ 2. $p\in(-\frac{\pi}{2},\frac{\pi}{2})$
   3. $p\in(0,\pi)$ 4. $p\in(0,2\pi)$

9. For how many values $a\in C$, the equations $x^2-8x+7=0$ and $x^2-2ax+49=0$ have a common root? **[AP EAMCET 18-09-20_Shift-2]**
   1. 1 2. 3
   3. 2 4. 0

10. If a,b,c are in arithmetic progression (A.P), then the roots of the equation $ax^2-2bx+c=0$ are **[AP EAMCET 18-09-20_Shift-2]**
    1. $1,\frac{c}{a}$ 2. $\frac{-1}{a},-c$
    3. $-1,\frac{-c}{a}$ 4. $-2,\frac{-c}{2a}$

11. Solve the equation $3^{x^2-x}=25-4^{x^2-x}$ **[AP EAMCET 21-09-20_Shift-1]**
    1. -1 2. 2
    3. Both -1 and 2 4. No solution

12. If 2 + 4i is one of the roots of $x^2+bx+c=0$ with b, c∈R, then (b, c) = **[AP EAMCET 21-09-20_Shift-1]**
    1. (4,-20) 2. (4,20)
    3. (-4,-20) 4. (-4,20)

13. Let α and β be the roots of the equation $px^2+qx+r=0, p\neq 0$. if p,q,r in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=4$ then the value of $|\alpha-\beta|$ is **[AP EAMCET 21-09-20_Shift-2]**
    1. $\frac{\sqrt{61}}{9}$ 2. $\frac{2\sqrt{17}}{9}$
    3. $\frac{\sqrt{34}}{9}$ 4. $\frac{2\sqrt{13}}{9}$

14. Solve $(8-t)^2 < (t^2-3t-10)$

[AP EAMCET 21-09-20_Shift-2]

1. $\left(\frac{74}{13},8\right]$ 2. $\left(\frac{74}{13},\infty\right)$
3. $(8,\infty)$ 4. $[8,\infty)$

15. If α,β are the roots of $x^2+px+q=0$, then the values of $\alpha^3+\beta^3$ and $\alpha^4+\alpha^2\beta^2+\beta^4$ are respectively _____ and ______

[AP EAMCET 22-09-20_Shift-1]

1. $(3pq-p^3),(p^4-3p^2q+3q^2)$
2. $-p(3q-p^2),(p^2-q)(p^2+3q)$
3. $(pq-4),(p^4-q^4)$
4. $(3pq-p^3),(p^2-q)(p^2-3q)$

16. The number of solutions for the equation $x^2-5|x|+6=0$ is_______

[AP EAMCET 22-09-20_Shift-1]

1. 4 2. 3
3. 2 4. 1

17. For which value of 'k', the roots of equation $2x^2+5x+k=0$ are rational?

[AP EAMCET 22-09-20_Shift-2]

1. $\frac{5}{8}$ 2. $\frac{25}{8}$
3. $\frac{25}{4}$ 4. $\frac{5}{4}$

18. The polynomial $x^2-6x+12\in Q[x]$ is

1. Irreducible over Q 2. reducible over Q
3. Irreducible over C 4. Zero polynomial

19. If the equations $2ax^2-3bx+4c=0$ and $3x^2-4x+5=0$ have a common root, then $\frac{a+b}{b+c}$ is equal to (a, b, c $\in$R)

1. $\frac{1}{2}$ 2. $\frac{3}{35}$
3. $\frac{34}{31}$ 4. $\frac{29}{23}$

20. **Assertion(A):** $3x^2-16x+4>-16$ is satisfied for some values of real x in $\left(0,\frac{10}{3}\right)$

Reason(R): ax^2+bx+c and a will have the same sign for some values of $x\in R$ when $b^2-4ac>0$ **[TS EAMCET 09-09-20_Shift-1]**

1. (A) is true, (R) is true and (R) is the correct explanation for (A)
2. (A) is true, (R)is true but (R) is not the correct explanation for (A)
3. (A)is true, but(R) is false
4. (A) is false ,but(R)is true

21. If the roots of the quadratic equation $ax^2+bx+c=0$ are imaginary, then for all real values of x, the minimum value of the expression $3a^2x^2+6abx+2b^2$ is

[TS EAMCET 09-09-20_Shift-1]

1. $<4ab$ 2. $>4ac$
3. $>-4ac$ 4. $<-4ab$

22. The equation $\sin^4 x-(k+3)\sin^2 x-k-4=0$ has a solution if **[TS EAMCET 09-09-20_Shift-1]**

1. $k>4$
2. $-4\le k\le -3$
3. k is any positive integer
4. $k=0$

23. The curves $y=x^2+9x+20$ and $y=x^2+bx+c$ intersect the X-axis at the points $(\alpha_i,0),(i=1,2,3,4)$. If $\alpha_1<\alpha_2<\alpha_3<\alpha_4$ be such that $|\alpha_1-\alpha_3|=|\alpha_2-\alpha_4|=8,$ then the sum of all possible values of b and c is

[TS EAMCET 09-09-20_Shift-2]

1. 186 2. 159
3. 216 4. 214

24. If $\frac{x^2+ax+3}{x^2+x+1}$ takes real values for all real values of x then a lies in the interval

[TS EAMCET 09-09-20_Shift-2]

1. $(-2-\sqrt{11},\sqrt{11}-2)$ 2. (4,3)
3. $(-2+\sqrt{2},2+\sqrt{2})$ 4. (-1,0)

25. Let S be the set of all possible integral values of λ in the interval (-3,7) for which the roots of the quadratic equation $\lambda x^2+13x+7=0$ are all rational numbers. Then the sum of the elements in S is

[TS EAMCET 10-09-20_Shift-1]

1. 4
2. 2
3. 3
4. 1

26. α is the maximum value of $1-2x-5x^2$ and β is minimum values of x^2-2x+r. If $5\alpha x^2+\beta x+6>0$ for all real values x, then the internal in which r lies is

[TS EAMCET 10-09-20_Shift-1]

1. (0,5)
2. $(-5,\infty)$
3. $(-\infty,7)$
4. $(-11,13)$

27. The minimum value of $\dfrac{9.3^{2x}+6.3^x+4}{9.3^{2x}-6.3^x+4}$ is

[TS EAMCET 10-09-20_Shift-2]

1. -1
2. $\frac{1}{2}$
3. $\frac{1}{4}$
4. $\frac{1}{3}$

28. P and q are the roots of the equation $x^2+7x+3=0$. if $\dfrac{3p}{1-2p},\dfrac{3q}{1-2q}$ are the roots of $lx^2+mx+n=0$ and the greatest common divisor of l,m,n is 1, then $l-m+n=$

[TS EAMCET 11-09-20_Shift-1]

1. 11
2. -3
3. -1
4. 12

29. If the quadratic equation $3x^2-7x+2=0$ and $kx^2+7x-3=0$ have a common root then the positive value of k is

[TS EAMCET 11-09-20_Shift-1]

1. 6
2. $\frac{11}{4}$
3. 4
4. $\frac{7}{2}$

30. If α,β are the roots of $ax^2+bx+c=0$ then $\left(\dfrac{\alpha}{a\beta+b}\right)^{3}-\left(\dfrac{\beta}{a\alpha+b}\right)^{3}=$

[TS EAMCET 11-09-20_Shift-2]

1. 0
2. 1
3. $(a+b)^2$
4. $(a-b)^2$

31. The maximum of $\left\{x\in R/\sqrt{x+2}>\sqrt{8-x^2}\right\}=$

[TS EAMCET 11-09-20_Shift-2]

1. 2
2. $\sqrt{2}+1$
3. 3
4. $2\sqrt{2}$

32. If x is real, then the maximum and minimum values of $\dfrac{x^2+14x+9}{x^2+2x+3}$ are respectively

[TS EAMCET 14-09-20_Shift-2]

1. 4,-5
2. 5,-4
3. 9,3
4. 24,6

33. When R is the set of all real numbers, $\left\{x\in R,\dfrac{\sqrt{12-x-x^2}}{x+10}\le\dfrac{\sqrt{12-x-x^2}}{2x+9}\right\}=$

[TS EAMCET 14-09-20_Shift-2]

1. $(-4,1]\cup\{3\}$
2. $[-4,1]$
3. $[-4,1]\cup\{3\}$
4. ϕ, the empty set

34. If $\left(x^2+5x+5\right)^{x+5}=1$ then the number of integers satisfying this equation is

[AP EAMCET 19-08-2021_Shift-1]

1. 2
2. 3
3. 4
4. 5

35. If $1+x^2=\sqrt{3}x$, then $\sum_{n=1}^{24}\left(x^n-\dfrac{1}{x^n}\right)^2$ is equal to

[AP EAMCET 19-08-2021_Shift-2]

1. 48
2. -48
3. -24
4. 24

36. If α,β are the roots of $11x^2+12x-13=0$, then $\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}=$? (approximately close to)

[AP EAMCET 19-08-2021_Shift-2]

1. 4.54
2. 3.54
3. 2.54
4. 1.54

37. If 'a' is a positive integer such that roots of the equation $7x^2-13x+a=0$ are rational numbers, then the smallest possible value of 'a' is___ **[AP EAMCET 19-08-2021_Shift-2]**

1. 5
2. 6
3. 7
4. 8

38. If one root of the equation $ix^2-2(i+1)x+(2-i)=0$ is $(2-i)$, then the other root is **[AP EAMCET 20-08-2021_Shift-1]**

1. $-i$
2. $2+i$
3. i
4. $2-i$

39. If α and β are the roots of the quadratic equation $x^2+x+1=0$, then the equation whose roots are $\alpha^{2021}, \beta^{2021}$ is given by___ **[AP EAMCET 20-08-2021_Shift-1]**

1. $x^2-x+1=0$
2. $x^2+x-1=0$
3. $x^2-x-1=0$
4. $x^2+x+1=0$

40. If $f(10-x)=3x^2+4x-5$ & $f(x)=px^2+qx+r$ then $p+q+r=$ **[AP EAMCET 20-08-2021_Shift-2]**

1. 272
2. 274
3. 275
4. 273

41. For $a \neq b$, if the equation $x^2+ax+b=0$ & $x^2+bx+a=0$ have a common root, then the value of $a+b=$ **[AP EAMCET 20-08-2021_Shift-2]**

1. -1
2. 0
3. 1
4. 2

42. Let a, b, c be positive real numbers. If $\frac{x^2-bx}{ax-c}=\frac{m-1}{m+1}$ has two roots which are numerically equal but opposite in sign, then the value of 'm' is **[AP EAMCET 23-08-2021_Shift-1]**

1. c
2. $\frac{1}{c}$
3. $\frac{a+b}{a-b}$
4. $\frac{a-b}{a+b}$

43. For the equation $x^2-5|x|-14=0$ **[AP EAMCET 23-08-2021_Shift-1]**

1. All roots are real
2. All the roots are imaginary
3. Two roots are real
4. No real roots

44. The number of real roots of the equation $\frac{(x^2+1)^3}{x^3}+\frac{x^2+1}{3x}=0\ (x \neq 0)$ is **[AP EAMCET 23-08-2021_Shift-2]**

1. 1
2. 0
3. 2
4. 3

45. If one of the roots of the equation $x^2+px+q=0$ is equal to the square of the other then ____ **[AP EAMCET 23-08-2021_Shift-2]**

1. $p(q^2-3p)=q(p-1)$
2. $p(3p-q^2)=p(p+1)$
3. $p(3q-p^2)=q(q-1)$
4. $p(3q-p^2)=q(q+1)$

46. The equations $x^2-ax+b=0$ and $x^2+bx-a=0$ have a common root, then **[AP EAMCET 24-08-2021_Shift-1]**

1. $a=b$
2. $a+b=1$
3. $a+b=0$ or $a-b=1$
4. $a-b=2$

47. If α and β are the roots of the equation $x^2-x+1=0$ then $\alpha^{2009}+\beta^{2009}=$ **[AP EAMCET 24-08-2021_Shift-1]**

1. -2
2. -1
3. 1
4. 2

48. Which of the following condition imply that roots of the equation $\left(\frac{1}{4}\right)x^2+bx+c=0$ are integers? **[AP EAMCET 24-08-2021_Shift-2]**

1. $b^2-c>0$
2. b & c are even integers
3. b^2-c is the square of an integer and b is an integer
4. b & c are integers

49. Let m and n be two integers such that $0 \leq m \leq 10$ and $0 \leq n \leq 10$. Then the number of ordered pairs (m, n) such that $x^2+mx+n=0$ has real roots is ____ **[AP EAMCET 25-08-2021_Shift-1]**

1. 71
2. 73
3. 75
4. 72

50. If x^2+px+1 is a factor of ax^3+bx+c, then **[AP EAMCET 25-08-2021_Shift-1]**

1. $a^2+c^2=-ab$
2. $a^2-c^2=-ab$
3. $a^2-c^2=ab$
4. $a^2+c^2=ab$

51. Let S be the set of all quadratic equations of the form $x^2+bx+c=0$ where $b,c\in\{1,2,3,4,5,6\}$. If an equation is selected at random from S, then the probability that the equation has real roots is ____ **[AP EAMCET 25-08-2021_Shift-2]**

1. $\frac{9}{12}$
2. $\frac{9}{36}$
3. $\frac{19}{36}$
4. $\frac{7}{36}$

52. The smallest negative integer satisfying both the quadratic inequalities $x^2<4x+77$ & $x^2>4$ is **[TS EAMCET 04-08-2021_Shift-2]**

1. -3
2. -6
3. -2
4. -7

53. If the roots of equation $x^2-2cx+ab=0$ are real and unequal, then the roots of $x^2-2(a+b)x+a^2+b^2+2c^2=0$ are **[TS EAMCET 04-08-2021_Shift-2]**

1. Real and unequal
2. Imaginary
3. Irrational & unequal
4. Real and equal

54. If $\frac{\alpha}{\alpha+1}$ and $\frac{\beta}{\beta+1}$ are the roots of the quadratic equation $x^2+7x+3=0$, then the equation having roots α and β is **[TS EAMCET 04-08-2021_Shift-1]**

1. $3x^2-x-3=0$
2. $11x^2+13x+3=0$
3. $13x^2+11x+13=0$
4. $11x^2+3x+13=0$

55. If $y=\frac{x^2+14x+9}{x^2+2x+3}\ \forall x\in R$, then the interval of maximum length in which y lies is **[TS EAMCET 04-08-2021_Shift-1]**

1. $[-5,4]$
2. $[-4,5]$
3. $\left[\frac{1}{3},3\right]$
4. $\left[\frac{-1}{3},3\right]$

56. If $x^2-5x-14>0 \Rightarrow x$ lie outside $[\alpha,\beta]$, then $\frac{\alpha}{\beta}=$ **[TS EAMCET 05-08-2021_Shift-1]**

1. $\frac{-2}{7}$
2. $\frac{-7}{2}$
3. $\frac{2}{7}$
4. $\frac{7}{2}$

57. For $x\in R\setminus\{-6\}$, the value of $\frac{(x+2)(x+5)}{(x+6)}$ does not lie in the interval **[TS EAMCET 05-08-2021_Shift-1]**

1. $[-9,-1]$
2. $[-5,-2]$
3. $(-5,-2)$
4. $(-9,-1)$

58. If $x=2+2^{2/3}+2^{1/3}$, then $x^3-6x^2+6x=$ **[TS EAMCET 05-08-2021_Shift-1]**

1. 3
2. 2
3. 1
4. 0

59. $f(x)=ax^2-bx-a$ is a quadratic expression. If K is the least real number such that $f(x)\le K\ \forall x\in R$, then **[TS EAMCET 05-08-2021_Shift-2]**

1. $K=0$
2. $K<-2$
3. $K>0$
4. $-1<K<0$

60. **Assertion(A):** The maximum value of $-x^2+3x+1$ is $\frac{11}{4}$

Reason(R): If $a<0$, the maximum value of ax^2+bx+c exist at $x=\frac{-b}{2a}$

The correct option among the following is **[TS EAMCET 05-08-2021_Shift-2]**

1. (A) is true, (R) is true and (R) is the correct explanation for (A)
2. (A) is true, (R) is true but (R) is not the correct explanation for (A)
3. (A) is true but (R) is false
4. (A) is false but (R) is true

61. If $f(x)\equiv x^2+ax+2=0$ and $g(x)\equiv x^2+2x+a=0$ have only one real common root, then sum of the roots of $f(x)+g(x)=0$ is **[TS EAMCET 05-08-2021_Shift-2]**

1. $\frac{-1}{2}$
2. 0
3. $\frac{1}{2}$
4. 1

62. Suppose α is minimum value of x^2+bx+5 and β is maximum value of $-x^2+ax+5$. If $[\alpha,\beta]$ is the interval of maximum length for x in which $x^2-10x+24\le 0$, then $a^2b^2=$

[TS EAMCET 06-08-2021_Shift-2]

1. 25
2. 16
3. 4
4. 18

63. If the minimum value of the quadratic expression x^2+5x-2 is M and it exists at a then $\frac{M}{a}=$

[TS EAMCET 06-08-2021_Shift-2]

1. 3.5
2. $\frac{33}{5}$
3. 2.5
4. -0.25

64. For $\forall x\in R$ the minimum value $\frac{1}{3}$ and the maximum value 3 of $\frac{x^2+x+1}{x^2-x+1}$ exist at $l\,\&\,m$ respectively, then $l+m=$

[TS EAMCET 06-08-2021_Shift-1]

1. -22
2. 0
3. 17
4. -7

65. If 2 and 3 are the two roots of the equation $2x^3+mx^2-13x+n=0$, then the values of m, n are respectively

[TS EAMCET 06-08-2021_Shift-1]

1. -5,-30
2. -5,30
3. 5,30
4. 5,-30

66. If $f(x)=ax^2+bx+c$ for some a, b, c $\in\mathbb{R}$ with a + b + c =3 and $f(x+y)=f(x)+f(y)+xy\ \forall x,y\in\mathbb{R}$. The $\sum_{n=1}^{10}f(n)=$

[AP EAMCET 04-07-2022_Shift-1]

1. 330
2. 255
3. 165
4. 190

67. The number of positive real roots of the equation $3^{x+1}+3^{-x+1}=10$ is

[AP EAMCET 04-07-2022_Shift-1]

1. 3
2. 2
3. 1
4. Infinitely many

68. The number of real roots of the equation $\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=\frac{13}{6}$ is

[AP EAMCET 04-07-2022_Shift-1]

1. 1
2. 2
3. 3
4. 4

69. If $4^x-3^{x-1/2}=3^{x+1/2}-2^{2x-1}$ then the value of x is **[AP EAMCET 04-07-2022_Shift-1]**

1. 7/2
2. 5/2
3. 1/2
4. 3/2

70. If f(f(0))=0, where f(x)= x^2+ax+b, $b\neq 0$ then a+b= **[AP EAMCET 04-07-2022_Shift-2]**

1. 2
2. 1
3. -1
4. -2

71. The sum of the real roots of the equation $|x-2|^2+|x-2|-2=0$ is

[AP EAMCET 04-07-2022_Shift-2]

1. 4
2. -4
3. 2
4. -2

72. If the difference between the roots of $x^2+ax+b=0$ and that of the roots of $x^2+bx+a=0$ is same and $a\neq b$, then

[AP EAMCET 04-07-2022_Shift-2]

1. a-b-4=0
2. a-b+4=0
3. a+b+4=0
4. a+b-4=0

73. For what values of $a\in Z$, the quadratic expression (x+a)(x+1991)+1 can be factorised as (x+b)(x+c), where b,c $\in Z$?

[AP EAMCET 04-07-2022_Shift-2]

1. 1990
2. 1989
3. 1991
4. 1992

74. If $S=\left\{\begin{matrix}m\in R: x^2-2(1-3m)x+7(3+2m)\\ =0\text{ has distinct roots}\end{matrix}\right\}$, then the number of elements in S is

[AP EAMCET 05-07-2022_Shift-1]

1. 2
2. 3
3. 4
4. Infinite

75. The sum of the real roots of the equation $x^4 - 2x^3 + x - 380 = 0$ is

[AP EAMCET 05-07-2022_Shift-1]

1. -1
2. 0
3. 1
4. 2

76. If $x = -5 + 2\sqrt{-4}$, then the value of $x^4 + 9x^3 + 35x^2 - x + 4$ is

[AP EAMCET 05-07-2022_Shift-2]

1. 80
2. 160
3. -160
4. -80

77. α, β are the roots of $x^2 - 10x - 8 = 0$ with α > β. If $a_n = \alpha^n - \beta^n$ for $n \in N$, then the value of $\frac{a_{10} - 8a_8}{5a_9}$ is

[AP EAMCET 05-07-2022_Shift-2]

1. -3
2. 3
3. -2
4. 2

78. The number of real values of m so that the equation $x^2 + (2m + 1)x + m = 0$ has equal roots is

[AP EAMCET 05-07-2022_Shift-2]

1. 1
2. 0
3. 2
4. 3

79. If $f(x) = ax^2 + bx + c$ satisfies $f(1) + 2f(2) = 0$ and $2f(1) + f(2) = 0$, then $3a + b =$ **[AP EAMCET 06-07-2022_Shift-1]**

1. 2
2. -1
3. 0
4. 1

80. The sum of squares of roots of the equation $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$ is

[AP EAMCET 06-07-2022_Shift-2]

1. 82
2. 65
3. 50
4. 37

81. If a,b, c,d are real numbers such that a < b < c<d, then the roots of the equation (x-a) (x-c)+2(x-b) (x-d)=0 are

[AP EAMCET 06-07-2022_Shift-2]

1. Real & need not be distinct
2. Real and distinct
3. Non- real and distinct
4. Non- real and need not be distinct

82. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n[th] power of the other, then $\left(ac^n\right)^{1/n+1} + \left(a^n c\right)^{1/n+1} =$

[AP EAMCET 06-07-2022_Shift-2]

1. -2b
2. -b
3. b-1
4. b+1

83. The range of the function $f(x) = \frac{x^2 + x + 1}{x^2 - x + 1}$ is

[AP EAMCET 07-07-2022_Shift-1]

1. $\left[\frac{1}{3}, 3\right]$
2. $\left[\frac{1}{2}, 2\right]$
3. $\left[-\frac{1}{2}, -\frac{1}{4}\right]$
4. $\left[-\frac{1}{2}, 2\right]$

84. Which of the following quadratic equations whose real roots x_1, x_2 satisfy the conditions $x_1^2 + x_2^2 = 5, 3\left(x_1^5 + x_2^5\right) = 11\left(x_1^3 + x_2^3\right)$

[AP EAMCET 07-07-2022_Shift-1]

1. $x^2 \pm 3x + 2 = 0$
2. $x^2 \pm 3x + 11 = 0$
3. $x^2 \pm 5x + 2 = 0$
4. $x^2 \pm 5x + 11 = 0$

85. If α, β are the roots of $ax^2 + bx + c = 0$, then the quadratic equation whose roots are $\sqrt{5}\alpha$, $\sqrt{5}\beta$ is

[AP EAMCET 07-07-2022_Shift-2]

1. $ax^2 + \sqrt{5}bx + 5c = 0$
2. $ax^2 + \sqrt{5}bx + \sqrt{5}c = 0$
3. $ax^2 + 5bx + \sqrt{5}c = 0$
4. $ax^2 + 5bx + 5c = 0$

86. If $a^2 + b^2 + c^2 = 1$, $a, b, c \in \mathbb{R}$ then the set of extreme values of ab+bc+ca is

[AP EAMCET 07-07-2022_Shift-2]

1. $\left\{\frac{1}{2}, 2\right\}$
2. $\{-1, 2\}$
3. $\left\{-1, \frac{1}{2}\right\}$
4. $\left\{\frac{-1}{2}, 1\right\}$

87. If $x^2 + px + 1$ is a factor of $ax^3 + bx + c$, then

[AP EAMCET 08-07-2022_Shift-1]

1. $a^2 + c^2 = ab + 3$
2. $a^2 - c^2 = ab$
3. $a^2 - c^2 = -ab$
4. $a^2 + c^2 = ab$

88. The quadratic equation whose sum of the roots is 11 and sum of squares of the roots is 61 is

[AP EAMCET 08-07-2022_Shift-1]

1. $x^2+11x-30=0$
2. $x^2+11x+30=0$
3. $x^2-11x-30=0$
4. $x^2-11x+30=0$

89. The number of pairs of consecutive positive even integers such that the sum of their squares is 290 is

[AP EAMCET 08-07-2022_Shift-2]

1. 0
2. 1
3. 2
4. 3

90. The range of the function $f(x)=\frac{x}{x^2-5x+9}$ is

[AP EAMCET 08-07-2022_Shift-2]

1. $\left[\frac{1}{11},1\right]$
2. $\left[\frac{-1}{11},1\right]$
3. $\left[-1,\frac{-1}{11}\right]$
4. $\left[-1,\frac{1}{11}\right]$

91. If α,β are the roots of the equation $2x^2+6x+k=0$, then the maximum value of $\left[\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right]$ **[AP EAMCET 08-07-2022_Shift-2]**

1. 0
2. 1
3. -1
4. -2

92. If $A=\left\{x\in R/\sqrt{x^2-8x+15}\in R\right\}$ and $B=\left\{x\in R/\frac{x-3}{2x-5}<\frac{x-6}{2x-11}\right\}$, then $A\cap B$ =

[TS EAMCET 18-07-2022_Shift-1]

1. ϕ
2. $\left(\frac{5}{2},3\right]\cup\left[5,\frac{11}{2}\right)$
3. $\left(\frac{5}{2},\frac{21}{4}\right)$
4. $\left(\frac{5}{2},\frac{11}{2}\right)$

93. If the extreme value of $3x-2x^2+1$ is k then the set of all real values of x for which $kx^2+2x+1>0$ is

[TS EAMCET 18-07-2022_Shift-1]

1. $\left(\frac{1}{2},1\right)$
2. $\left(-\infty,\frac{1}{2}\right)\cup(1,\infty)$
3. $(-\infty,\infty)$
4. $\left(-\infty,\frac{17}{8}\right)$

94. If the quadratic equations $x^2-7x+3c=0$ and $x^2+x-5c=0$ have a common root, then for non-zero real value of c the sign of the expression x^2-3x+c is

[TS EAMCET 18-07-2022_Shift-2]

1. negative for all $x\in R$
2. positive for all $x\in(1,3)$
3. negative for all $x\in(1,3)$
4. positive for all $x\in R$

95. Let $f(x)=\frac{6x^2-18x+21}{6x^2-18x+17}$. If m is the maximum value of $f(x)$ and $f(x)>n\ \forall x\in R$. Then 14m – 7n =

[TS EAMCET 18-07-2022_Shift-2]

1. -1
2. 23
3. 35
4. 42

96. If α,β are the roots of the equation $x^2-2\sqrt{3}x+4=0$ then $\alpha^6+\beta^6=$

[TS EAMCET 19-07-2022_Shift-I]

1. 128
2. -64
3. 64
4. -128

97. When $b=17$, it is found that the roots of the equation $x^2+bx+c=0$ are -2 and -15. If α,β are the rots of the same equation when $b=13$ then $|\alpha-\beta|=$

[TS EAMCET 19-07-2022_Shift-I]

1. 7
2. 13
3. 17
4. 30

98. Let x be the real number.
Match the following:

	List – I		List - II
A.	The maximum value of $2x^2+4x+5$	I.	-1
B.	The maximum value of $\frac{x^2+4x+1}{x^2+x+1}$	II.	1
C.	If $1\le\frac{3x^2-5x+6}{x^2+1}\le 2,$ $\forall x\in[a,b]$ then $b=$	III.	2
D.	If $1\le\frac{3x^2-5x+6}{x^2+1}\le 2,$ $\forall x\in[a,b]$ then $a=$	IV.	3
		V.	4

the correct match is

[TS EAMCET 19-07-2022_Shift-I]

1. A-IV, B-III, C-II, D-V
2. A-IV, B-V, C-II, D-III
3. A-IV, B-III, C-V, D-II
4. A-III, B-V, C-IV, D-I

99. If α,β are the roots of a quadratic equation $x^2+bx+c=0$ such that $\alpha^2+\beta^2=5$ and $\alpha^3+\beta^3=9$, then b+c=

[TS EAMCET 20-07-2022_Shift-1]

1. -5 2. -1
3. 1 4. 5

100. The set of all real values of the expression $\frac{x^2-x+2}{x^2+x-2}$ for all $x\in\mathbb{R}-\{-2,1\}$ is

[TS EAMCET 20-07-2022_Shift-1]

1. $(-2,3)$ 2. $\left[\frac{7}{9},\infty\right)$
3. $(-\infty,-1]\cup\left[\frac{7}{9},\infty\right)$ 4. $(-\infty,-1]$

101. **Statement (I):** The set of solutions of $|x|^2-4|x|+3<0$ is the interval (–3, 3).
Statement (II): If x < 3 or x > 5 then $x^2-8x+15>0$.
Which of the above statements is(are)true?

[TS EAMCET 20-07-2022_Shift-2]

1. Statement I is true, but Statement II is false
2. Statement II is true, but Statement I is false
3. Both statement I and Statement II are true
4. Both statement I and Statement II are false

102. If $6x-x^2+12$ attains its extreme value β at $x=\alpha$ then $\beta=$

[TS EAMCET 20-07-2022_Shift-2]

1. 7α 2. 5α
3. 3α 4. α

103. Let α be a common root of the equations $x^3-2x-25\lambda=0, 3x^3-8x-\frac{175}{3}\lambda=0$ and $\lambda>0$. Then $\lambda=$

[TS EAMCET 20-07-2022_Shift-2]

1. $\frac{3}{\sqrt{5}}$ 2. $\frac{\sqrt{3}}{5\sqrt{5}}$
3. $\frac{3}{5\sqrt{5}}$ 4. $\frac{3\sqrt{5}}{5}$

104. If the values of k for which the equation $x^2+2(k+2)x+6k+7=0$ has equal roots are k_1 and k_2, then $k_1^2+k_2^2=$

[15th May 2023 Shift 1]

1. 8 2. 9
3. 10 4. 12

105. If $\left(3+2\sqrt{2}\right)^{x^2-4}+\left(3-2\sqrt{2}\right)^{x^2-4}=6$, then $x^4+x^2+5=$ **[15th May 2023 Shift 1]**

1. -30 2. -35
3. 30 4. 35

106. If the equation $x^4+ax^3+bx^2+cx+d=0$ has three equal roots, then that root is

[15th May 2023 Shift 1]

1. $\frac{6c-ab}{8b-3a^2}$ 2. $\frac{ab-6c}{8b+3a^2}$
3. $\frac{6c-ab}{3a^2-4b}$ 4. $\frac{6c-ab}{3a^2-8b}$

107. α *and* β are the roots of the equation $x^2 - ax + b = 0$, If $\alpha^2+\beta^2$ and $\alpha^3+\beta^3$ are the roots of the equation $Ax^2+Bx+C=0$, then C=

[15th May 2023 Shift 2]

1. $a^5 - 5a^3b + 6ab^2$
2. $a^5 + 5a^3b - 6ab^2$
3. $a^5 - 5a^3b - 6ab^2$
4. $a^5 + 5a^3b + 6ab^2$

108. The minimum value of $f(x) = \frac{x^2-2x+3}{x^2-4x+7}$

[15th May 2023 Shift 2]

1. $1+\frac{1}{\sqrt{3}}$
2. $\frac{3-\sqrt{3}}{3}$
3. $2-\frac{1}{\sqrt{3}}$
4. $3-\frac{1}{\sqrt{3}}$

109. If cot x cot y=a and $x+y=\frac{\pi}{6}$, then the quadratic equation satisfying cot x and cot y is

[15th May 2023 Shift 2]

1. $t^2+(1-a)\sqrt{3t}+a=0$
2. $\sqrt{3}t^2+(1-a)t+a\sqrt{3}=0$
3. $\sqrt{3}t^2+(a-1)t+a\sqrt{3}=0$
4. $t^2+(a-1)\sqrt{3}t+a=0$

1. -1
2. -2
3. 2
4. 1

110. If α and β are the roots of the equation $x^2+x+1=0$, then the quadratic equation whose roots are α^{2023} *and* β^{1012} is

[16th May 2023 Shift 1]

1. $x^2+x+1=0$
2. $x^2-x+1=0$
3. $x^2-x+2=0$
4. $x^2+x+2=0$

111. If α *and* β are the roots of the equation $ax^2+bx+c=0$, then the equation whose roots are $\alpha+\beta$ *and* $\frac{1}{\alpha}+\frac{1}{\beta}$ is

[16th May 2023 Shift 1]

1. $acx^2-(ab+bc)x+b^2=0$
2. $acx^2+(ab+bc)x-b^2=0$
3. $acx^2+(ab+bc)x+b^2=0$
4. $acx^2-(ab+bc)x-b^2=0$

112. If c and d are the roots of $x^2+ax+b=0$, then a root of $x^2+(4c+a)x+(b+2ac+4c^2)=0$ is

[16th May 2023 Shift 2]

1. d+2c
2. d+c
3. d–c
4. d–2c

113. The set $\left\{x\in R: 16(2^x) > 16^{\frac{-1}{x}}\right\}=$

[17th May 2023 Shift 1]

1. $\{x\in R: x>0\}$
2. $\{x\in R: x<0\}$
3. R
4. $\{x\in R: x>2\}$

114. The set $\{x\in R: 4+11x-3x^2>0\}$ is the interval

[17th May 2023 Shift 1]

1. $\left(-\frac{1}{3},4\right)$
2. $\left(\frac{1}{3},4\right)$
3. $\left(-4,\frac{1}{3}\right)$
4. $\left(-4,\frac{-1}{3}\right)$

115. For $X\in R$, the minimum value of $\frac{x^2+2x+5}{x^2+4x+10}$ is

[17th May 2023 Shift 2]

1. $\frac{1}{2}$
2. $\frac{4}{3}$
3. $\frac{3}{4}$
4. $-\frac{1}{2}$

116. If α and β are the roots of the equation $2^{6x}-3(2^{3x+2})+32=0$ with $\beta<1$, then $2\alpha+3\beta=$

[17th May 2023 Shift 2]

1. –3
2. –4
3. 3
4. 4

117. If α, β *and* γ are the roots of the equation $x^3-ax^2+bx-c=0$ then $\alpha^{-2}+\beta^{-2}+\gamma^{-2}=$

[17th May 2023 Shift 2]

1. $\frac{b^2-3ac}{c^2}$
2. $\frac{b^2-ac}{c^2}$
3. $\frac{b^2-2ac}{c^2}$
4. $\frac{b^2-4ac}{c^2}$

118. If the roots of the equation $3x^2 + 4kx + 3 = 0$ are non – real, then k lies in the interval **[18th May 2023 Shift 2]**

1. $\left[-2,\frac{-3}{2}\right]$
2. $\left[\frac{3}{2},2\right]$
3. $\left(\frac{-3}{2},\frac{3}{2}\right)$
4. (2,3)

119. If cosecθ, and cot θ are the roots of $cx^2 + bx + a = 0 (bc \neq 0)$, then $b^2(b^2 - 4ac) =$ **[18th May 2023 Shift 2]**

1. $-2c^4$
2. $2c^4$
3. $-c^4$
4. C^4

120. The sum of the fourth powers of the roots of the equation $16x^2 - 10x + 1 = 0$ is **[18th May 2023 Shift 2]**

1. $\frac{257}{4096}$
2. $\frac{257}{2048}$
3. $\frac{257}{1024}$
4. $\frac{257}{512}$

121. The number of elements in the set $S = \{x \in Z : x^2 - 7x + 6 \leq 0 \text{ and } x^2 - 3x > 0\}$ is **[19th May 2023 Shift 1]**

1. ∞
2. 2
3. 3
4. 4

122. If one root the equation $4x^2 - 2x + k - 4 = 0$ is the reciprocal of the other, then the value of k is **[12TH MAY 2023 SHIFT-1]**

1. -8
2. 8
3. -4
4. 4

123. If $(x-2)$ is a common factor of the expressions $x^2 + ax + b$ and $x^2 + cx + d$, then $\frac{b-d}{c-a} =$ **[12TH MAY 2023 SHIFT-1]**

1. 1
2. 2
3. 3
4. 4

124. The set of all values of x which satisfy both the inequations $x^2 - 1 \leq 0$ and $x^2 - x - 2 \geq 0$ simultaneously is **[12TH MAY 2023 SHIFT -2]**

1. $(-1,2)$
2. $(-1,1)$
3. $(-2,-1)$
4. $\{-1\}$

125. For all real values of x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is **[12TH MAY 2023 SHIFT -2]**

1. 0
2. $\frac{1}{3}$
3. 1
4. 3

126. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. If the other roots of the first and second equations are integers and are in the ratio 4:3, then their common root is **[12TH MAY 2023 SHIFT -2]**

1. 4
2. 3
3. 2
4. 1

127. If α and β are the roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15} =$ **[12TH MAY 2023 SHIFT -2]**

1. -512
2. -256
3. 256
4. 512

128. If $x^2 + 3x - 2k = 0$ and $x^2 - 2x - 7k = 0$ have a non-zero common root, then the positive root of the equation $kx^2 + (k+2)x - (k+1) = 0$ is **[13TH MAY 2023 SHIFT-1]**

1. 2
2. $\frac{2}{5}$
3. 3
4. $\frac{3}{5}$

129. The values of $\frac{x^2 - 2x + 1}{x^2 + x - 1}$ do not lie in the interval **[13TH MAY 2023 SHIFT-1]**

1. $\left(-\frac{4}{5},0\right)$
2. $\left(-\infty,-\frac{4}{5}\right)$
3. $(0,\infty)$
4. $\left(\frac{4}{5},\infty\right)$

130. If $x^2 + 2px - 2p + 8 > 0$ for all real values of x, then the set of all possible values of p is **[EAPCET 14-05-23 SHIFT-1]**

1. (2,4)
2. $(-\infty,-4)$
3. $(2,\infty)$
4. $(-4,2)$

131. If $R-(\alpha,\beta)$ is the range of $\frac{x+3}{(x-1)(x+2)}$, then the sum of the intercepts of the line $\alpha x+\beta y+1=0$ on the coordinate axes is [EAPCET 14-05-23 SHIFT-1]

1. -8
2. 10
3. 8
4. 9

132. The quadratic equation whose roots are $\sin^2 18^0 \, and \cos^2 36^0$ is [EAPCET 14-05-23 SHIFT-1]

1. $16x^2-12x-1=0$
2. $16x^2-12x+4=0$
3. $16x^2-12x+1=0$
4. $16x^2+12x+1=0$

133. If α,β,δ *are the roots of the equation* $x^4+x^2+1=0$ such that $\alpha+\beta=-1, \gamma+\delta=1$, $\alpha^2=\beta$ *and* $\gamma^2=-\delta$, *then* $\alpha^{2023}+\beta^{2023}+\gamma^{2022}+\delta^{2022}=$

[EAPCET 13-05-23 SHIFT-2]

1. 1
2. 0
3. $1+3\omega$
4. $\omega-2\omega^2$

134. Let the equations $ax^2-7x+c=0$ and $ax^2+5x-c=0$ have a common root and $ac\neq 0$. If 3 is a root of $ax^2-7x+c=0$ other than the common root, then the common root of the given equations is

[EAPCET 13-05-23 SHIFT-2]

1. 3
2. 1/2
3. 2
4. 1/3

135. The set of all values of x for which inequalities $x^2-7x+10\geq 0$ and 2x+3-x^2>0 hold simultaneously is

[EAPCET 13-05-23 SHIFT-2]

1. $(-\infty,2]$
2. $(3,\infty)$
3. $(-1,2]$
4. $[2,3)$

KEY

1)	3	2)	1	3)	4	4)	4	5)	3
6)	3	7)	3	8)	3	9)	3	10)	1
11)	3	12)	4	13)	4	14)	2	15)	4
16)	1	17)	2	18)	1	19)	3	20)	3
21)	3	22)	2	23)	2	24)	1	25)	1
26)	4	27)	4	28)	3	29)	1	30)	1
31)	4	32)	1	33)	3	34)	2	35)	2
36)	3	37)	2	38)	1	39)	4	40)	2
41)	1	42)	4	43)	1	44)	2	45)	4
46)	3	47)	3	48)	3	49)	2	50)	3
51)	3	52)	2	53)	2	54)	2	55)	1
56)	1	57)	4	58)	2	59)	3	60)	4
61)	3	62)	2	63)	1	64)	2	65)	2
66)	1	67)	3	68)	2	69)	4	70)	3
71)	1	72)	3	73)	2	74)	4	75)	3
76)	3	77)	4	78)	2	79)	3	80)	2
81)	2	82)	2	83)	1	84)	1	85)	1
86)	4	87)	2	88)	4	89)	1	90)	2
91)	2	92)	2	93)	3	94)	4	95)	2
96)	4	97)	1	98)	3	99)	2	100)	3
101)	2	102)	1	103)	3	104)	3	105)	4
106)	4	107)	1	108)	2	109)	2	110)	1
111)	3	112)	4	113)	1	114)	1	115)	1
116)	4	117)	3	118)	3	119)	4	120)	1
121)	3	122)	2	123)	2	124)	4	125)	2
126)	3	127)	2	128)	4	129)	1	130)	4
131)	2	132)	3	133)	1	134)	2	135)	3

SOLUTIONS

1. Let

$$x=\frac{2\alpha}{3-4\alpha}$$

$$3x-4\alpha x=2\alpha$$

$$\alpha=\frac{3x}{4x+2}$$

Put $\alpha=\frac{3x}{4x+2}$ *in* $x^2+7x+3=0$

$$9x^2+21x(4x+2)+3(4x+2)^2=0$$

$$a=141, b=90, \; c=12$$

$$a+b+c=243$$

2. Given equations have

Common root

Let α be the common root

$$\alpha^2=\frac{bc_1-b_1c}{b_1-b}, \quad \alpha=\frac{c-c_1}{b_1-b}$$

$$\left(\frac{c-c_1}{b_1-b}\right)^2=\frac{bc_1-b_1c}{b_1-b}$$

$$(c-c_1)^2=(bc_1-b_1c)(b_1-b)$$

3. $\alpha_1+\alpha_2=-a,\ \alpha_1\alpha_2=1$

$\alpha_3+\alpha_4=-b,\ \alpha_3\alpha_4=1$

$(\alpha_1+\alpha_3)(\alpha_1+\alpha_4)(\alpha_2+\alpha_3)(\alpha_2+\alpha_4)$

$=\left[\alpha_1^2-b\alpha_1+1\right]\left[\alpha_2^2-b\alpha_2+1\right]$

α_1,α_2 are the roots of $x^2+ax+1=0$

$\Rightarrow(-a\alpha_1-b\alpha_1)(-a\alpha_2-b\alpha_2)=(a+b)^2$

4. If $x^2-x-6\ge0\Rightarrow x\le-2$ *or* $x\ge3$

And $x^2-x-6=x+2\ \Rightarrow x=4,-2$

If $x^2-x-6<0\Rightarrow-2<x<3$

And $-(x^2-x-6)=x+2\ \Rightarrow x=2$

5. Let $y=\dfrac{x^2+34x-71}{x^2+2x-7}$

$\Rightarrow y\left(x^2+2x-7\right)=x^2+34x-71$

$\Rightarrow(y-1)x^2+(2y-34)x+(71-7y)=0$

Since x complex $\Rightarrow\Delta<0$

$(2y-34)^2-4(y-1)(71-7y)<0$

$\Rightarrow8y^2-112y+360<0$

$y^2-14y+45<0\Rightarrow y\in(5,9)$

$a=5,b=9$

6. Given quadratic equation is

$ax^2+ax+c=0$

Let roots pk,qk

$(p+q)k=-1\Rightarrow p+q=\dfrac{-1}{k}$

$(pk)(qk)=pqk^2=\dfrac{c}{a}\Rightarrow pq=\dfrac{c}{ak^2}$

$\therefore\dfrac{\sqrt{p}}{\sqrt{q}}+\dfrac{\sqrt{q}}{\sqrt{p}}=\dfrac{p+q}{\sqrt{pq}}=\dfrac{\frac{-1}{k}}{\sqrt{\frac{c}{ak^2}}}=-\sqrt{\dfrac{a}{c}}$

7. $\alpha+\beta=1\ \&\ \alpha^2+\beta^2=13$

$\Rightarrow(\alpha+\beta)^2-2\alpha\beta=13\ \Rightarrow1-2\alpha\beta=13$

$\alpha\beta=-6$

Required quadratic equation is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-x-6=0$

8. $(\cos p-1)x^2+(\cos p)x+\sin p=0$

Roots are real $\Rightarrow\Delta\ge0$

$\cos^2p-4(\cos p-1)(\sin p)\ge0$

$-1\le\cos p\le1\Rightarrow\cos^2p\ge0$ *And* $\cos p-1\le0$

Equality holds if $\sin p>0$

$p\in(0,\pi)$

9. $x=1\Rightarrow a=25$

$x=7\Rightarrow a=7$

$\therefore$ no of values of a is 2

10. 2b = a + c

And $\alpha+\beta=\dfrac{2b}{a}=\dfrac{a+c}{a}=1+\dfrac{c}{a}$

$\alpha\beta=\dfrac{c}{a}=1.\dfrac{c}{a}$ $\qquad\therefore\alpha=1$ *and* $\beta=\dfrac{c}{a}$

11. $3^{x^2-x}=25-4^{x^2-x}$

Satisfies when

$x^2-x=2$

$x^2-x-2=0$

$x=-1$ *and* 2

12. $x^2+bx+c=0$

Since $b,c\in R$

$2+4i,2-4i$ are roots

-b = 4$\Rightarrow b=-4$

$c=(2+4i)(2-4i)=4+16=20$

$(b,c)=(-4,20)$

13. Given: $\alpha+\beta=\dfrac{-q}{p},\alpha\beta=\dfrac{r}{p}$

p,q,r in A.P$\Rightarrow2q=p+r$

And also given $\dfrac{1}{\alpha}+\dfrac{1}{\beta}=4$

$\Rightarrow\alpha+\beta=4\alpha\beta\Rightarrow\dfrac{-q}{p}=4\dfrac{r}{p}\Rightarrow q=-4r$

Now ,

$|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{\dfrac{52r^2}{81r^2}}=\dfrac{2\sqrt{13}}{9}$

14. Given $(8-t)^2<\left(t^2-3t-10\right)$

$\Rightarrow-13t+74<0$

$\Rightarrow13t-74>0$

$t>\dfrac{74}{13}$

$\therefore t \in \left(\frac{74}{13}, \infty\right)$

15. $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= -p^3 - 3q(-p)$

$= 3pq - p^3$

$\alpha^4 + \alpha^2\beta^2 + \beta^4 = (\alpha^2+\beta^2)^2 - \alpha^2\beta^2$

$= \left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - \alpha^2\beta^2$

$= \left[p^2 - 2q\right]^2 - q^2$

$(p^2 - q)(p^2 - 3q)$

16. $x > 0 \Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow (x-3)(x-2) = 0$

$\Rightarrow x = 3\ (or)\ 2$

$x < 0 \Rightarrow x^2 + 5x + 6 = 0$

$\Rightarrow (x+3)(x+2) = 0 \Rightarrow x = -2, -3$

4 solutions

17. $2x^2 + 5x + k = 0$

roots are rational and equal

$\Delta = 0$

$25 - 8k = 0 \Rightarrow k = \frac{25}{8}$

18. $x^2 - 6x + 12$

$b^2 - 4ac = 36 - 48 < 0.$

$\frac{6 \pm i2\sqrt{3}}{2} = 3 \pm i\sqrt{3}$

We can't express it as product of two polynomials. So it is irreducible over Q only it is reducible over C, because

$x^2 - 6x + 12 = (x - 3 + i\sqrt{3})(x - 3 - i\sqrt{3}).$

19. Let α be a common root

$\therefore 2a\alpha^2 - 3b\alpha + 4c = 0.....(1)$

$3\alpha^2 - 4\alpha + 5 = 0......(2)$

From (1) and (2)

$\frac{2a}{3} = \frac{3b}{4} = \frac{4c}{5} = \lambda$

$a = \frac{3\lambda}{2}, b = \frac{4\lambda}{3}, c = \frac{5\lambda}{4}$

$\therefore \frac{a+b}{b+c} = \frac{34}{31}$

20. $3x^2 - 16x + 20 > 0$

$x \in (-\infty, 2) \cup \left(\frac{10}{3}, \infty\right)$

(R) is wrong

21. Roots of $ax^2 + bx + c = 0$ are imaginary

$\Rightarrow b^2 - 4ac < 0$

$3a^2x^2 + 6abx + 2b^2$

minimum value $= \frac{-12a^2b^2}{12a^2} = -b^2 > -4ac$

22. $\sin^4 x - (k+3)\sin^2 x - (k+4) = 0$

$\sin^2 x = \frac{k + 3 \pm \sqrt{(k+3)^2 + 4(k+4)}}{2}$

$= \frac{k + 3 \pm (k+5)}{2} = k + 4, -1$

$\sin^2 x \neq -1$

$0 \le k + 4 \le 1$

$-4 \le k \le -3$

23. $y = x^2 + 9x + 20,\ (roots\ \alpha_1, \alpha_2)$

Roots are -4,-5

$y = x^2 + bx + c,\ (roots\ \alpha_3, \alpha_4)$

$|\alpha_1 - \alpha_3| = 8, |\alpha_2 - \alpha_4| = 8$

$|-4 - \alpha_3| = 8, |-5 - \alpha_4| = 8$

$\alpha_3 = 4, -12, \alpha_4 = 3, -13$

Case:1: $-13 < -12 < -5 < -4$

$y = x^2 + 25x + 156$

b =25, c = 156

case:2: -5 < -4 <3 < 4

$y = x^2 - 7x + 12$

b = -7, c = 12

case:3: -12 < -5 < -4 < 3

$y = x^2 + 9x - 36$

b = 9, c = -36

Sum of possible values of b and c is 159

24. Conceptual

25. Given $\lambda x^2 + 13x + 7 = 0$

Roots are rational numbers

Discriminantk must be a perfect square

$\Delta = 169 - 28\lambda \geq 0$

$if\ \lambda = 0 \rightarrow \Delta = 169$

$if\ \lambda = -2 \rightarrow \Delta = 225$

$if\ \lambda = 6 \rightarrow \Delta = 1 \quad \therefore S = \{0, -2, 6\}$

$sum\ of\ elements\ in\ S = 4$

26. $\alpha = \frac{4(-5)(1) - (-2)^2}{4(-5)} = \frac{6}{5}$

$\beta = \frac{4(1)(r) - (-2)^2}{4(1)} = r - 1$

$5\alpha x^2 + \beta x + 6 > 0$

$6x^2 + (r-1)x + 6 > 0$

$b^2 - 4ac < 0 \Rightarrow r^2 - 2r - 143 < 0$

$(r-13)(r-(-11)) < 0 \Rightarrow r \in (-11, 13)$

27. Let $3^x = t$

$\frac{9t^2 + 6t + 4}{9t^2 - 6t + 4} = y$

$9t^2(1-y) + 6t(1+y) + 4(1-y) = 0$

$\Delta \geq 0$

$36(1+y)^2 - 36.4(1-y^2) \geq 0$

$y \in \left[\frac{1}{3}, 3\right]$ Minimum value =1/3

28. $\left.\begin{matrix} p+q=-7 \\ pq=3 \end{matrix}\right\}$ for equation $x^2 + 7x + 3 = 0$

Given $\frac{3p}{1-2p}, \frac{3q}{1-2q}$ are roots of

$lx^2 + mx + n = 0$

Now, sum of roots $\frac{3p}{1-2q} + \frac{3q}{1-2p} = \frac{-m}{l}$

On simplifying we get $\frac{m}{l} = \frac{57}{27} ....(1)$

Product of roots $\left(\frac{3p}{1-2p}\right)\left(\frac{3q}{1-2q}\right) = \frac{n}{l}$

On simplifying we get

$\frac{n}{l} = 1.....(2)$

From (1) and (2)

$\left.\begin{matrix} l:m:n = 27:57:27 \\ \Rightarrow l:m:n = 9:19:9 \\ l - m + n = 9 - 19 + 9 = -1 \end{matrix}\right\}(\because GCD\ of\ l, m, n\ is\ 1)$

29. Given equations

$3x^2 - 7x + 2 = 0; kx^2 + 7x - 3 = 0$

w.k.t.,

$(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$

Now, $(2k+9)^2 = (21+7k)(21-14) \Rightarrow k = 6$

30. $\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

$a\alpha^2 + b\alpha + c = 0 \Rightarrow \alpha(a\alpha + b) = -c$

$\Rightarrow aa + b = \frac{-c}{\alpha}$

$\frac{1}{a\alpha + b} = \frac{-\alpha}{c} \Rightarrow \frac{\beta}{a\alpha + b} = \frac{-\alpha\beta}{c}, \frac{\alpha}{a\beta + b} = \frac{-\alpha\beta}{c}$

$\left(\frac{\alpha}{a\beta + b}\right)^3 - \left(\frac{\beta}{a\alpha + b}\right)^3 = \left(\frac{-\alpha\beta}{c}\right)^3 - \left(\frac{-\alpha\beta}{c}\right)^3 = 0$

31. $\max .of \left\{x \in R / \sqrt{x+2} > \sqrt{8 - x^2}\right\}$

$x + 2 \geq 0 \Rightarrow x \geq -2 \Rightarrow x \in [-2, \infty)$

$8 - x^2 \geq 0 \Rightarrow x^2 - 8 \leq 0 \Rightarrow x \in \left[-2\sqrt{2}, 2\sqrt{2}\right]$

$x + 2 > 8 - x^2 \Rightarrow x^2 + x - 6 > 0$

$\Rightarrow x \in (-\infty, -3) \cup (2, \infty)$

required interval $= \left[2, 2\sqrt{2}\right]$

Max .of x $= 2\sqrt{2}$

32. $y = \frac{x^2 + 14x + 9}{x^2 + 2x + 3}$

$\Rightarrow (y-1)x^2 + (2y-14)x + (3y-9) = 0$

$\Delta \geq 0 \Rightarrow y^2 + y - 20 \leq 0 \Rightarrow y \in [-5, 4]$

33. $12 - x - x^2 \geq 0, x + 10 \neq 0, 2x + 9 \neq 0$ and $2x + 9 \leq x + 10$

$\Rightarrow -4 \leq x \leq 3, x \neq -10, x \neq \frac{-9}{2}$ and $x \leq 1$

$\therefore x \in [-4, 3] \cap (-\infty, 1] = [-4, 1] \cup \{3\}$

34. $\left(x^2+5x+5\right)^{x+5}=1$

$x+5=0$ & $x^2+5x+5=1$

$x=-5 \quad (x+4)(x+1)=0$

$x+5=1$ & $x^2+5x+5=1$

$x=-4$ & $(x+4)(x+1)=0$

$x=-5,-4,-1$

35. $1+x^2=\sqrt{3}x \Rightarrow x+\frac{1}{x}=\sqrt{3}, x^2+\frac{1}{x^2}=1,$

$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)=0$

$\sum_{n=1}^{24}\left(x^n-\frac{1}{x^n}\right)^2=\sum_{n=1}^{24}\left[\left(x^n\right)^2+\left(\frac{1}{x^n}\right)^2-2\right]$

$=\left(x+\frac{1}{x}\right)+\left(x^2+\frac{1}{x^2}\right)+\ldots\ldots+\left(x^{24}+\frac{1}{x^{24}}\right)-48$

$=1-1+0+1-1+0+\ldots.+0-48=-48$

36. $11x^2+12x-13=0$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}=2.54$

37. $7x^2-13x+a=0....(1)$

The root of equation (1) are rational

So, $\Delta=b^2-4ac$ be a perfect square

$\Delta=(13)^2-28(a)$ By verification $a=6$

38. Verification

39. The roots of $x^2+x+1=0$ are ω,ω^2

$\alpha=\omega,\beta=\omega^2$

$\alpha^{2021}=\omega^{2021}=\omega^2$

$\beta^{2021}=\left(\omega^2\right)^{2021}=\omega$

Required equation is

$x^2-\left(\omega^2+\omega\right)x+\omega^2.\omega=0$

$x^2+x+1=0$

40. $f(10-x)=3x^2+4x-5$ and

$f(x)=px^2+qx+r$

Let $10-x=1 \Rightarrow x=9$

$f(1)=274, f(1)=p+q+r$

41. $x^2+ax+b=0$ & $x^2+bx+a=0$ have a common root .

$x=1$ satisfies the both equations

$a+b+1=0 \Rightarrow a+b=-1$

42. $\frac{x^2-bx}{ax-c}=\frac{m-1}{m+1}$

$(m+1)x^2-\left[b(m+1)+a(m-1)\right]x+c(m-1)=0$

Sum of the roots is zero

$\frac{b(m+1)+a(m-1)}{m+1}=0 \Rightarrow m=\frac{a-b}{a+b}$

43. $x^2-5|x|-14=0$

(i) If $x>0, x^2-5x-14=0$

$(x-7)(x+2)=0$

(ii) If $x<0,\ x^2+5x-14=0$

$(x+7)(x-2)=0$

$x=-7\ (x<0)$

All the roots are real.

44. $\frac{\left(x^2+1\right)^3}{x^3}+\frac{x^2+1}{3x}=0$

$\left(x+\frac{1}{x}\right)^3+\frac{1}{3}\left(x+\frac{1}{x}\right)=0$

$\left(x+\frac{1}{x}\right)\left[\left(x+\frac{1}{x}\right)^2+\frac{1}{3}\right]=0$

$x+\frac{1}{x}=0, \left(x+\frac{1}{x}\right)^2+\frac{1}{3}=0$ not possible

$x^2+1=0$ not possible.

There exists No. real roots.

45. $x^2+px+q=0$

$\alpha+\alpha^2=-p,\ \alpha^3=q \Rightarrow \alpha=(q)^{1/3}$

$q^{2/3}+pq^{1/3}+q=0$

$$\left(q^{2/3}+pq^{1/3}\right)^3=-q^3$$

$$q^2+p^3q+3pq(-q)=-q^3$$

$$q+p^3-3pq=-q^2\Rightarrow p\left(3q+p^2\right)=-q(q+1)$$

46. $x^2-ax+b=0$ & $x^2+bx-a=0$ have common root.

$x^2-ax+b=0$

$\underline{x^2+bx-a=0}$

$-(a+b)x+(b+a)=0$

$(a+b)(x-1)=0\Rightarrow a+b=0$ (or) $x=1$

if $x=1,\ 1-a+b=0\Rightarrow a-b=1$

$a+b=0$ (or) $a-b=1$

47. The roots of $x^2-x+1=0$ are $-\omega,-\omega^2$

$\alpha=-\omega,\ \beta=-\omega^2$

$\alpha^{2009}=(-\omega)^{2009}=-\omega^2$

$\beta^{2009}=\left(-\omega^2\right)^{2009}=-\omega$

$\alpha^{2009}+\beta^{2009}=-\omega^2-\omega=1$

48. $\left(\frac{1}{4}\right)x^2+bx+c=0\Rightarrow\Delta=b^2-4\left(\frac{1}{4}\right)c=b^2-c$

If b^2-c be a perfect square then the roots of given equation are integers.

b^2-c be a perfect square.

49. $x^2+mx+n=0$ has real roots $m^2-4n\ge 0$

$m^2\ge 4n$

If $m=0$, then $n=0$

If $m=1$,then $n=0$

If $m=2$,then $n=0,1$

If $m=3$,then $n=0,1,2$

If $m=4$,then $n=0,1,2,3,4$

If $m=5$,then $n=0,1,2,3,4,5,6$

If $m=6$,then $n=0,1,2,3,4,5,6,7,8,9$

If $m=7,8,9,10$, then

$n=0,1,2,3,4,5,6,7,8,9,10$

Total number of ordered pairs = 73.

50. x^2+px+1 is a factor of ax^3+bx+c

	a	0	b	c
$-p$	0	$-ap$	ap^2	0
-1	0	0	$-a$	ap
	a	$-ap$	$b+ap^2-a$	$c+ap$

$c+ap=0$ and $b-a+ap^2=0$

$p=-\frac{c}{a}$, $b-a+a\left(-\frac{c}{a}\right)^2=0\Rightarrow a^2-c^2=ab$

51. $x^2+bx+c=0$ has real roots

$b^2-4c\ge 0\Rightarrow b^2\ge 4c$

If $b=2$, then $c=1$

If $b=3$, then $c=1,2$

If $b=4$, then $c=1,2,3,4$

If $b=5,6$, then $c=1,2,3,4,5,6$

Required probability $=\frac{19}{6^2}=\frac{19}{36}$

52. $x^2<4x+77\ \&\ x^2>4$

$x^2-4x-77<0$ & $x^2>4$

$(x-11)(x+7)<0$ & $(x+2)(x-2)>0$

$x\in(-7,11)$ & $x\in(-\infty,-2)\cup(2,\infty)$

$x\in(-7,-2)\cup(2,11)$

Smallest negative integer is -6

53. $x^2-2cx+ab=0$

Roots are real and unequal $\Delta>0$

$4c^2-4ab>0\Rightarrow c^2>ab$

$x^2-2(a+b)x+a^2+b^2+2c^2=0$

$\Delta=4(a+b)^2-4\left(a^2+b^2+2c^2\right)$

$=8ab-8c^2=8\left(ab-c^2\right)<0$

So roots are imaginary

54. $\frac{\alpha}{\alpha+1},\frac{\beta}{\beta+1}$ are roots of $x^2+7x+3=0$

$$\left(\frac{\alpha}{\alpha+1}\right)^2+\frac{7\alpha}{\alpha+1}+3=0$$

$\alpha^2+7\alpha(\alpha+1)+3(\alpha+1)^2=0$

$\alpha^2+7\alpha^2+7\alpha+3\alpha^2+6\alpha+3=0$

$11\alpha^2+13\alpha+3=0$

$\therefore\alpha,\beta$ are roots of $11x^2+13x+3=0$

55. $y=\frac{x^2+14x+9}{x^2+2x+3}$

$yx^2+2xy+3y=x^2+14x+9$

$(y-1)x^2+(2y-14)x+3y-9=0$

Equation have real roots

$\therefore \Delta \geq 0$

$b^2-4ac\geq 0$

$(2y-14)^2-4(y-1)(3y-9)\geq 0$

$\Rightarrow (y+5)(y-4)\leq 0 \Rightarrow -5\leq y\leq 4$

56. Given, $x^2-5x-14>0$

$\Rightarrow x^2-7x+2x-14>0$

$\Rightarrow (x+2)(x-7)>0$

$\Rightarrow x\in(-\infty,-2)\cup(7,\infty)$

Clearly, $\alpha=-2, \beta=7 \Rightarrow \frac{\alpha}{\beta}=\frac{-2}{7}$

57. Let $y=\frac{(x+2)(x+5)}{(x+6)}$

$\Rightarrow y(x+6)=x^2+7x+10$

$\Rightarrow x^2+(7-y)x+(10-6y)=0$

$\forall x\in R, \Delta\geq 0$

$\Rightarrow (7-y)^2-4(10-6y)\geq 0$

$\Rightarrow y^2+10y+9\geq 0$

$\Rightarrow y\in(-\infty,-9]\cup[-1,\infty)$

$\therefore y\notin(-9,-1)$

58. Given,

$x=2+2^{2/3}+2^{1/3}$

$\Rightarrow x-2=2^{2/3}+2^{1/3}$

C.O.B.S

$\Rightarrow (x-2)^3=2^2+2+3.2^{\frac{2}{3}+\frac{1}{3}}(x-2)$

$\Rightarrow x^3-6x^2+6x=-6+8=2$

59. Conceptual

60. Maximum value $=\frac{4ac-b^2}{4a}=\frac{-4-9}{-4}=\frac{13}{4}$

A is false and B is true

61. $f(x)\,\&\,g(x)$ have a common root of

$(c_1a_2-c_2a_1)^2=(a_1b_2-a_2b_1)(b_1c_2-b_2c_1)$

$\Rightarrow a=-3$

$f(x)+g(x)=2x^2-x-1=0$

Sum of roots $=\frac{-b}{a}=\frac{1}{2}$

62. $x^2-10x+24\leq 0$

$(x-4)(x-6)\leq 0$

$x\in[4,6]$

$\therefore \alpha=4, \beta=6$

$\alpha=$ Min. value of x^2+bx+5

$4=\frac{20-b^2}{4}\Rightarrow 16=20-b^2\Rightarrow b^2=4$

$\beta=$ Max. Value of $-x^2+ax+5$

$6=\frac{-20-a^2}{-4}\Rightarrow a^2=4$

$\therefore a^2b^2=16$

63. $a=\frac{-5}{2(1)}=\frac{-5}{2}$

$M=$ Minimum value of $x^2+5x-2=\frac{-35}{4}$

$\frac{M}{a}=\frac{7}{2}=3.5$

64. $f(x)=\frac{x^2+x+1}{x^2-x+1}$

Maximum value and minimum value exists at

$x=\pm\sqrt{\frac{c}{a}}=\pm\sqrt{\frac{1}{1}}=\pm 1$

$l=1, m=-1, l+m=0$

65. 2,3 be the roots of $2x^3+mx^2-13x+n=0$

$f(2)=f(3)=0$

$16+4m-26+n=0$

$4m+n-10=0$.........(1)

$54+9m-39+n=0$

$9m+n+15=0$.......(2)

Solving (1) and (2), we get $m=-5, n=30$

66. Let y = 0, $f(x)=f(x)+f(0)+0f(0)=0$

$\therefore C=0$

Let $y=-x$

$f(0)=f(x)+f(-x)-x^2$

$0=ax^2+bx+ax^2-bx-x^2$

$x^2(2a-1)=0$

$2a-1=0$

$a=\frac{1}{2}$

$a+b+c=0^3$

$\frac{1}{2}+b+0=3$

$\therefore b=\frac{3}{2}$

$\sum_{n=1}^{10} f(n)=\frac{1}{2}\sum_{n=1}^{10}(n^2+5n)$

67. $3^{x+1}+3^{-x+1}=10$

$3.3^x+\frac{3}{3^x}=10$

Put $3^x=t$

$3.(3x)^2-10.3^x+3=0$

$3t^2-10t+3=0$

$\therefore t=\frac{1}{3} or\, t=3$

$\therefore x=1 or -1$

No .of positive real roots =1

68. Put $\frac{1-x}{x}=t$

$\frac{1}{\sqrt{t}}+\sqrt{t}=\frac{13}{6}$

$6t-13\sqrt{t}+6=0$

$3\sqrt{t}(2\sqrt{t}-3)-2(2\sqrt{t}-3)=0$

$t=\frac{4}{9}\quad ,t=\frac{9}{4}$

$\therefore x=\frac{9}{13}, x=\frac{4}{13}$

No of real root =2

69. $4^x-\frac{3^x}{\sqrt{3}}=3^x\sqrt{3}-\frac{4^x}{2}$

$\frac{3}{2}4^x=\frac{4}{\sqrt{3}}3^x$

$\frac{4^{x-1}}{4^{1/2}}=\frac{3^{x-1}}{\sqrt{3}}$

$4^{x-\frac{3}{2}}=3^{x-\frac{3}{2}}$

$\therefore x=\frac{3}{2}$

70. $f(0)=b$

$f(f(0))=0$

$f(b)=0$

$b^2+ab+b=0$

$\therefore a+b=-1$

71. Put

$|x-2|=t$

$t^2+t-2=0$

$t^2+2t-t-2=0$

$t(t+2)-1(t+2)=0$

$t=1 \; or \; t=-2$

$|x-2|=1 or |x-2|=-2$

(not possible)

$x-2=\pm1$

$x=3, \quad x=1$

Sum of the real root =4

72. Let α,β be the roots of $x^2+ax+b=0$

and γ,δ be the root of $x^2+bx+a=0$

$|\alpha-\beta|=|\gamma-\delta|$

$(\alpha+\beta)^2-4\alpha\beta=(\gamma+\delta)^2-4\gamma\delta$

$a^2-4b=b^2-4a$

$a^2+4a=b^2+4b$

$a^2-b^2+4(a-b)=0$

$a+b+4=0$

73. $a,b,c \in z$

$(x+a)(x+1991)+1=0$

$(x+a)(x+1991)=-1$

$\therefore x+a=1,\ x+1991=-1$

(or) $x+a=-1\ ,x+1991=1$

$x+a=-1,\ x+1991=+1$

Case(i) $\Rightarrow a=-1+1990$

$=1989$

Case (2) $x+a=1\ ;\ x+1991=-1$

$x=-1992$

$-1992+a=1$

$a=1993$

$\therefore a=1989\ or\ 1993$

74. $x^2-2(1-3m)x+7(3+2m)=0$

it has distinct roots

$\therefore \Delta>0$

$4(1-3m)^2-4.7(3+2m)>0$

$1+9m^2-6m-21-14m>0$

$9m^2-20m-20>0$

No of element in S is infinite

75. By trial and error method

If $x=-4,\ x^4-2x^3+x-380$

$=256+128-4-380$

$=384-384=0$

-4	1	-2	0	1	-380
	0	-4	24	-96	380
	1	-6	24	-95	0

$x^3-6x^2+24x-95=0$

Put $x=5$

Then $125-150+120-95$

$=245-245=0$

$(x+4)(x-5)(x^2-x+19)=0$

But roots of $x^2-x+19=0$ are imaginary

$\therefore$ real root are 5, -4

Sum of real roots $=1$

76. Put

$x=-5+4i$

$x+5=4i$

S.O.B.S

$x^2+10x+41=0$

By synthetic division. We can write

$x^4+9x^3+35x^2-x+4$

$=(x^2-x+4)(x^2+10x+41)-160$

$=0(x^2-x+4)-160=-160$

77. $\alpha+\beta=10,\qquad \alpha\beta=-8$

$$\frac{a_{10}-8a_8}{5a_9}=\frac{\alpha^{10}-\beta^{10}-8(\alpha^8-\beta^8)}{5(\alpha^9-\beta^9)}$$

$$=\frac{\alpha^8(\alpha^2-8)-\beta^8(\beta^2-8)}{5(\alpha^9-\beta^9)}$$

$$=\frac{10(\alpha^9\ \beta^9)}{5(\alpha^9-\beta^9)}=2$$

78. $\Delta=0$

$(2m+1)^2-4m=0$

$4m^2+1+4m-4m=0$

$4m^2+1=0$

Root are imaginary

79. $f(x)=ax^2+bx+c$

$f(1)+2f(2)=0$

$a+b+c+2[4a+2b+c]=0$

$9a+56+3c=0\qquad (1)$

$2f(1)+f(2)=0$

$2(a+b+c)+4a+2b+c=0$

$6a+4b+3c=0\qquad (2)$

$(1)-(2)=3a+b=0$

80. Put $x^{\frac{1}{3}}=t$

$t^2+t-2=0$

$(t+2)(t-1)=0$

$t+2=0, \quad t-1=0$

$x^{\frac{1}{3}}=-2, \quad x^{\frac{1}{3}}=1$

$x=-8, \quad x=1$

Sum of squares of roots =65

81. $x^2-(a+c)x+ac+2x^2-2(b+d)x+2bd=0$

$\Delta=[a+c+2b+2d]^2-4(3)(ac+2bd)$

$=[(a+2d)-(c+2b)]^2+4(a+2d)(c+2b)$

$-12(ac+2bd)$

on simplifying this $\Delta>0$

Roots are real & district

82. $(ac^n)^{\frac{1}{n+1}}+(a^nc)^{\frac{1}{n+1}}+b=0$

$(ac^n)^{\frac{1}{n+1}}+(a^nc)^{\frac{1}{n+1}}=-b$

83. $f(x)=\dfrac{x^2+ax+b}{x^2-ax+b}$ then range

$[f(-\sqrt{b}),f(\sqrt{b})]$

$f(x)=\dfrac{x^2+x+1}{x^2-x+1}$ Rare $=[f(-1),f(1)]$

$=\left[\dfrac{1}{3},3\right]$

84. $3(x_1^5+x_2^5)=11(x_1^2+x_2^3)$

$\dfrac{x_1^5+x_2^5}{x_1^3+x_2^3}=\dfrac{11}{3}$

$\Rightarrow x_1^2+x_2^2-\dfrac{x_1^2x_2^2(x_1+x_2)}{(x_1+x_2)(x_1^2-x_1x_2+x_2^2)}=\dfrac{11}{3}$

$5-\dfrac{x_1^2x_2^2}{5-x_1x_2}=\dfrac{11}{3}$

$\Rightarrow 3x_1^2x_2^2+4x_1x_2-20=0$

$\therefore x_1x_2=\dfrac{-10}{3}$ or $x_1x_2=2$

If $x_1x_2=2$ then

$(x_1+x_2)^2-2x_1x_2=5$

$(x_1+x_2)^2=5+2(2)=9$

$x_1+x_2=\pm3$

Equation is $x^2\pm3x+2=0$

If $x_1x_2=\dfrac{-10}{3}$ then

$x_1^2+x_2^2=5$

$(x_1+x_2)^2-2x_1x_2=5$

$(x_1+x_2)^2=5+2x_1x_2=\dfrac{-5}{3}$

(not possible)

85. $\alpha+\beta=\dfrac{-b}{a}, \alpha\beta=\dfrac{c}{a}$

Equation whose root are $\sqrt{5}\alpha,\sqrt{5}\beta$

$x^2-(\sqrt{5}\alpha+\sqrt{5}\beta)x+\sqrt{5}\alpha.\sqrt{5}\beta=0$

$x^2-\sqrt{5}\left(\dfrac{-b}{a}\right)x+5\alpha\beta=0$

$ax^2+\sqrt{5}bx+5c=0$

86. If $a^2+b^2+c^2=p$ then

Extreme value of $ab+bc+ca=\left[\dfrac{-p}{2},p\right]$

but $a^2+b^2+c^2=1$

$\because$ Extreme value of ab+bc +ca are $\left[-\dfrac{1}{2},1\right]$

87. If $ax^3+bx+c=(x^2+px+1)\times g(x)$

$g(x)=\dfrac{ax^3+bx+c}{x^2+px+1}$

$=\dfrac{(x^2+px+1)(ax-ap)+(b-a+ap^2)x+c+ap}{x^2+px+1}$

since x^2+px+1 is a factor

Remainder =0

$(b-a+ap^2)x+c+ap=0$

$p^2=\dfrac{a-b}{a}, p=\dfrac{c}{a}$

$\left(-\dfrac{c}{a}\right)^2=\dfrac{a-b}{a}$

$\dfrac{c^2}{a^2}=\dfrac{a-b}{a}$

$c^2=a^2-ab$

$a^2-c^2=ab$

88. $\alpha+\beta=11$

$\alpha^2+\beta^2=61$

$(\alpha+\beta)^2-2\alpha\beta=61$

$121-61=2\alpha\beta$

$\alpha\beta=30$

Rev equation is $x^2-11x+30=0$

89. $\alpha^2+(\alpha+2)^2=290$

$\alpha^2+\alpha^2+4\alpha-286=0$

$\alpha^2+2\alpha-143=0$

$\alpha=11, \alpha=-13$ (not possible)

$\alpha=11$ is odd

$\therefore$ no of positive even integers = 0

90. $Let\ y=\dfrac{x}{x^2-5x+9}$

$yx^2-5xy+9y=x$

$yx^2-(5y+1)x+9y=0$

$\Delta\geq 0$

91. conceptual

92. $x^2-8x+15\geq 0$

$(x-3)(x-5)\geq 0$

$A=(-\infty,3]U[5,\infty)$

$\dfrac{x-3}{2x-5}-\dfrac{x-6}{2x-11}<0$

$\dfrac{(2x-11)(x-3)-(x-6)(2x-5)<0}{(2x-5)(2x-11)}$

$\dfrac{\cancel{2x^2}-\cancel{11}_x^x+33-\cancel{2x^2}+17x-15}{(2x-5)(2x-11)}<0$

$\dfrac{18}{(2x-5)(2x-11)}<0$

$B=\left(\dfrac{5}{2},\dfrac{11}{2}\right)$

$\therefore A\cap B=\left(\dfrac{5}{2},3\right)\cup\left[5,\dfrac{11}{2}\right)$

93. $a=-2<0$

Maximum value $=\dfrac{4(-2)(1)-3^2}{4(-2)}=\dfrac{-8-9}{-8}$

$k=\dfrac{17}{8}$

$kx^2+2x+1>0$

$17x^2+16x+8>0$

$\Delta<0$

$\therefore$ Range = R

94. $x^2-7x+3c=0$ (1)

$x^2+x-5c=0$ (2)

Apply condition

$(c_1a_2-c_2a_1)^2=(a_1b_2-a_2b_1)(b_1c_2-b_2c_1)$

$64c^2=8\times 32c$

$\therefore c=4$

$x^2-3x+c=0$

$x^2-3x+4=0$

$\Delta<0$

$x^2-3x+c>0\ \forall x\in R$

95. $f(x)=\dfrac{6x^2-18x+21}{6x^2-18x+17}$

$Let\ y=\dfrac{6x^2-18x+21}{6x^2-18x+17}$

Minimum of f (x) $m=\dfrac{15}{7}$

$f(x)>1$

$n=1$

$14m-7n={}^2\cancel{14}\left(\dfrac{15}{\cancel{7}}\right)-7(1)$

$=23$

96. $f(x)=x^2-2\sqrt{3}x+4$

$f^1(x)=2x-2\sqrt{3}$

$$\begin{array}{r|ccccccc} & 2 & -2\sqrt{3} & & 0 & 0 & 0 & 0 \\ 2\sqrt{3} & 0 & 4\sqrt{3} & 12 & 8\sqrt{3} & 0 & -32\sqrt{3} & -192 \\ -4 & 0 & -0 & -8 & -8\sqrt{3} & -16 & 0 & 64 \\ \hline & 2 & 2\sqrt{3} & 4 & 0 & -16 & -32\sqrt{3} & -128 \\ & \alpha^0+\beta^0 & & & & & & \alpha^6+\beta^6 \end{array}$$

$\therefore \alpha^6+\beta^6=-128$

97. $c=(-2)(-15)=30$

$x^2+17x+30=0$

When b=13

$-\beta_1=|-10+3|=7$

98. (A) minimum value of $2x^2+4x+5$ is 3

(B) Let $y=\frac{x^2+4x=1}{x^2+x+1}$

$\Delta\ge 0$

$\Rightarrow y^2-4\le 0$

$y\in[2,2]$

Maximum value =2

(C) $1\le\frac{3x^2-5x+6}{x^2+1}\le 2$

$\frac{3x^2-5x+6}{x^2+1}-1\ge 0$

$2x^2-5x+5\ge 0$

$\forall x\in R$

$\frac{3x^2-5x+6}{x^2+1}-2\le 0$

$\frac{3x^2-5x+6-2x^2-2\le 0}{x^2+1}$

$x^2-5x+4\le 0$

$x\in[1,4]$

$a=1, b=4$

(D) a =1.

99. $\alpha=1,\beta=2$

$b=-3$

$c=\alpha\beta=2$

$b+c=-3+2=-1$

100. $y=\frac{x^2-x+2}{x^2+x-2}$

$(y-1)x^2+(y+1)x-2(y+1)=0$

$\Delta\ge 0$

$9y^2+2y-7\ge 0$

$(9y-7)(y+1)\ge 0$

$y\in(-\infty,-1]\cup\left[\frac{7}{9},\infty\right)$

101. Statement -1:

$|x|^2-4|x|+3<0$

$(|x|-1)(|x|-3)<0$

$|x|\in(1,3)$

$1<|x|<3$

$\therefore x\in(-\infty,1)U(1,\infty)U(-3,3)$

But $x\in(-3,3)$ which is false

$x^2-8x+15>o$

Statement -2: $(x-3)(x-5)>0$

$\therefore x<3$ or $x>5$ $(True)$

102. $6x-x^2+12$

$\beta=$ maximum value$=\frac{4(-1)(12)-36}{4(-1)}=\frac{-48-36}{-4}$

$=\frac{-84}{-4}=21$

$\alpha=\frac{-b}{2a}=\frac{-6}{2(-1)}=3$

$\therefore\beta=7\alpha$

103. $3\alpha^3-6\alpha-75\lambda=0$ (1)

$3\alpha^3-8\alpha-\frac{175\lambda}{3}=0$ (2)

$(1)-(2)\Rightarrow\alpha=\frac{25\lambda}{3}$

Substitute $\alpha=\frac{25\lambda}{3}$ in given equation (1)

$\frac{635\lambda^2}{27}-\frac{5}{3}=0$

$125\frac{625\lambda^2}{27_9}=\frac{5}{3}\Rightarrow\lambda=\frac{3}{5\sqrt{5}}$

104. 4(k+2)2-4(6k+7)=0

$\Rightarrow$ k=-1 (or) 3

$k_1^2+k_2^2=1+9=10$

105. $x^2=5\Rightarrow x^4=25$

$x^4+x^2+5=35$

106. Let Roots be $\alpha,\alpha,\alpha,\beta$ and use S_1 & S_2

107 α,β are the roots of the equation

$x^2-ax+b=0$

$\alpha+\beta=a$

$\alpha\beta=b$

Equation having the roots

$\alpha^2+\beta^2$ and $\alpha^3+\beta^3$ is

$x^2-\left[(\alpha^2+\beta^2)+(\alpha^3+\beta^3)\right]x+\left[(\alpha^2+\beta^2)(\alpha^3+\beta^3)\right]=0$

Compare with $Ax^2+Bx+c=0$

$c=(\alpha^2+\beta^2)(\alpha^3+\beta^3)$

$=(a^2-2b)(a^3-3ab)$

$\boxed{c=a^5-5a^3b+6ab^2}$

108 $f(x)=y$

$\dfrac{x^2-2x+3}{x^2-4x+7}=y$

$(1-y)x^2+(4y-2)x+(3-7y)=0$

$\Delta\geq 0$

$-3y^2+6y-2\geq 0$

$3y^2-6y+2\leq 0$

$y=\dfrac{3\pm\sqrt{3}}{3}\Rightarrow\dfrac{3-\sqrt{3}}{3}\leq y\leq\dfrac{3+\sqrt{3}}{3}$

Min value = $\dfrac{3-\sqrt{3}}{3}$

109. $\cot x.\cot y=a\rightarrow(1)$

$x+y=\dfrac{\pi}{6}$

$\cot(x+y)=\cot\dfrac{\pi}{6}$

$\dfrac{\cot y.\cot x-1}{\cot y+\cot x}=\sqrt{3}$

$\dfrac{a-1}{\sqrt{3}}=(\cot x+\cot y)\rightarrow(2)$

Equation satisfying cot x and cot y is

$t^2-(\cot x+\cot y)t+(\cot x.\cot y)=0$

$t^2-\dfrac{(a-1)}{\sqrt{3}}t+a=0$

$\sqrt{3}t^2+(1-a)t+a\sqrt{3}=0$

110.

Given α,β are the roots of equation $x^2+x+1=0$

$w.k.t\ \alpha=w,\beta=w^2$ are roots of $x^2+x+1=0$

now q.e $x^2-(\alpha^{2023}+\beta^{1012})x+(\alpha^{2023}.\beta^{1012})=0$

$x^2-(w+w^2)x+w.w^2=0$

111. Given α,β are root of eq $ax^2+bx+c=0$

Now $\alpha+\beta=\dfrac{-b}{a},\alpha\beta=\dfrac{c}{a}$

now $\alpha+\beta+\dfrac{1}{\alpha}+\dfrac{1}{\beta}=\dfrac{-b}{a}+\dfrac{-b}{c}=-b(\dfrac{a+c}{ac})$

$(\alpha+\beta).(\dfrac{1}{\alpha}+\dfrac{1}{\beta})=(\dfrac{-b}{a})(\dfrac{-b}{c})=\dfrac{b^2}{ac}$

$\boxed{acx^2+(ab+bc)x+b^2=0}$

112. $x^2+ax+b=0$

Roots : c, d

c + d = –a, cd = b

113. $\{X\in R/16(2^x)>16^{-1/x}\}$

$16.2^x>16^{-1/x}$

$\Rightarrow 2^x>2^{4\left(\frac{-1}{x}-1\right)}$

$\Rightarrow x>\dfrac{-4}{x}-4$

$\Rightarrow x^2+4x+4>0$

$\Rightarrow(x+2)^2>0$

$\Rightarrow x>-2$

By verification $=\{x\in R/x>0\}$

114. $\{x\in R/4+11x-3x^2>0\}$

$3x^2-11x-4<0$

$(x-4)(3x+1)<0$

$x\in\left(-\dfrac{1}{3},4\right)$

115. $y=\frac{x^2+2x+5}{x^2+4x+10}$

$(y-1)x^2+2(2y-1)x+5(2y-1)=0$

$\Delta \geq 0$

$6y^2-11y+4\leq 0$

$(2y-1)(3y-4)\leq 0$

$y\in\left[\frac{1}{2},\frac{4}{3}\right]$

116. Let $2^{3x}=t$

$t^2-12t+32=0$

$(t-4)(t-8)=0$

$t=4\ or\ 8$

$2^{3x}=2^2\ or\ 2^3$

$3x=2\ or\ 3$

$x=2/3\ or\ 1$

$\alpha=1,\ \beta=2/3$

$2\alpha+3\beta=4.$

117. replace x by $\frac{1}{x}$

$cx^3-bx^2+ax-1=0$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{r^2}=\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{r}\right)^2-2\left(\frac{1}{\alpha\beta}+\frac{1}{\beta r}+\frac{1}{r\alpha}\right)$

$=\frac{b^2}{c^2}-\frac{2a}{c}$

$=\frac{b^2-2ac}{c^2}$

118. Given, $3x^2+4kx+3=0$ are non – real

$b^2-4ac<0$

$16k^2-4(3)(3)<0$

$16k^2-36<0$

$16k^2<36$

$k^2<\frac{36}{16}$

$k=\left(\frac{-3}{2},\frac{3}{2}\right)$

119. Cosec θ, cot θ are the roots of

$cx^2+bx+a=0 \quad (bc\neq 0)$

$\operatorname{cosec}\theta+\cot\theta=\frac{-b}{c}$

$\operatorname{cosec}\theta\times\cot\theta=\frac{a}{c}$

$(\operatorname{cosec}\theta+\cot\theta)^2=\operatorname{cosec}^2\theta+\cot^2\theta+2.\operatorname{cosec}\theta.\cot\theta$

$-2\operatorname{cosec}\theta.\cot\theta+2\operatorname{cosec}\theta.\cot\theta$

$\frac{b^2}{c^2}=(\operatorname{cosec}\theta-\cot\theta)^2+4\operatorname{cosec}\theta.\cot\theta$

$(\operatorname{cosec}\theta-\cot\theta)^2=\frac{b^2-4ac}{c^2}$........(1)

$(\operatorname{cosec}\theta+\cot\theta)^2=\frac{b^2}{c^2}$......... (2)

$(1)\times(2)$

$(\operatorname{cosec}^2\theta-\cot^2\theta)^2=\frac{(b^2-4ac)b^2}{c^4}$

$1=\frac{(b^2-4ac)b^2}{c^4}$

120. Sum of the fourth powers of the roots of the Equation :

$16x^2-10x+1=0\ is\ \alpha^4+\beta^4$

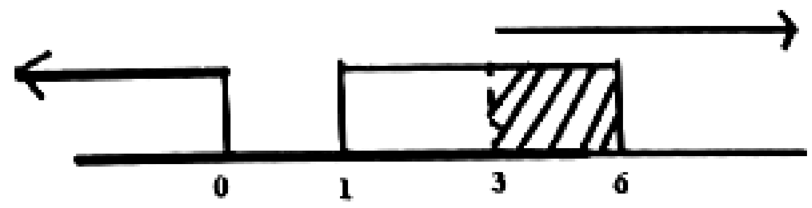
$\left(\frac{25}{64}-\frac{1}{8}\right)^2-\frac{1}{128}$

$\frac{289}{4096}-\frac{1}{128}=\frac{257}{4096}$

121. $x^2-7x+6\leq 0\ \&\ x^2-3x>0$

$\Rightarrow(x-1)(x-6)\leq 0\ \&\ x(x-3)>0$

$\Rightarrow x\in[1,6] \qquad x<0\ or\ x>3$



S = {4,5,6}

122. Given reciprocal equation

$4x^2-2x+k-4=0$

$\therefore 4=k-4$

$k=8$

123. $(x-2)$ *is a common factor of* x^2+ax+6

and x^2+cx+d

$\Rightarrow 2$ *is a common root of* $x^2+ax+b=0$

and $x^2+cx+d=0$

$\therefore \not{4}+2a+b=0$

$\underline{\not{4}}+2c+\underline{d}=0$

$2(a-c)+b-d=0$

$b-d=2(c-a)$

$\frac{b-d}{c-a}=2$

124. $x^2-1\le 0,\quad x^2-x-2\ge 0$

$x\in[-1,1]\to(1)\quad x\le -1$ *or* $x\ge 2\to(2)$

from (1) & (2) $x\in\{-1\}$

125. *let* $y=\frac{1-x+x^2}{1+x+x^2}$

$y^1=0 \Rightarrow \quad x^2-1=0\Rightarrow x=\pm 1$

For x =1 then value of y is 1/3

For x = - 1 then value of y is 3

126. $x^2-6x+a=0, x^2-cx+6=0$

roots are $(\alpha,4\beta)\quad(\alpha,3\beta)$

$4\alpha\beta=a\quad 3\alpha\beta=6$

$a=8\quad \alpha\beta=2$

$x^2-6x+8=0$

x=2, x=4

$\therefore$ common root $\alpha=2$

127 α,β root for $x^2+2x+2=0$

$\alpha=-1+i\quad,\beta=-1-i\quad \alpha^2=-2i,\beta^2=2i$

$\alpha^{15}+\beta^{15}=\left(\alpha^2\right)^7.\alpha+\left(\beta^2\right)^7.\beta$

$=(-2i)^7(-1+i)+(2i)^7(-1-i)=-256$

128. $x^2+3x-2k=0$..........(1)

$x^2-2x-7k=0$..........(2)

(1) and (2) have common root

$\therefore$ common root

$\left(c_1a_2-c_2a_1\right)^2=\left(a_1b_2-a_2b_1\right)\left(b_1c_2-b_2c_1\right)$

By solving we get k = 5

By sub k=5 in $kx^2+(k+2)x-(k+1)=0$

We get $x=3/5$

129. Let $\frac{x^2-2x+1}{x^2+x-1}=y$

$x^2(y-1)+x(y+2)-(y+1)=0$

$\Delta\ge 0$

$5y^2+4y\ge 0$

$y\ge 0,\ y\ge -4/5$

$\therefore\left(\frac{-4}{5},0\right)$

130. $x^2+2px-2p+8>0$

x is Real $\Rightarrow \Delta=(2p)^2-4(-2p+8)<0$

$=4p^2+8p-32<0$

$\Rightarrow\ p^2+2p-8<0$

$(p+4)(p-2)<0$

$p\in(-4,2)$

131. Let $y=\frac{x+3}{(x-1)(x+2)}$

$\Rightarrow y(x^2+x-2)=x+3$

$yx^2+(y-1)x-2y-3=0$

$x\in R\Rightarrow\quad \Delta\ge 0$

$$9y^2+10y+1\geq 0$$

$$(9y+1)\ (y+1)\geq 0$$

$$\left(y+\frac{1}{9}\right)(y+1)\geq 0$$

$$y\in R-\left(-1,-\frac{1}{9}\right)$$

$$\therefore\ \alpha=-1,\ \ \beta=-\frac{1}{9}$$

Consider $\alpha x+\beta y+1=0\Rightarrow -x-\frac{1}{9}y+1=0$

$$\Rightarrow 9x+y=9$$

Sum of intercepts = 1+9=10

132.

Required Q.E. is

$$x^2-(\sin^2 18^0+\cos^2 36^0)x+(\sin^2 18^0.\cos^2 36^0)=0$$

$$x^2-\left(\left(\frac{\sqrt{5}-1}{4}\right)^2+\left(\frac{\sqrt{5}+1}{4}\right)^2\right)x+$$

$$\left(\left(\frac{\sqrt{5}-1}{4}\right)^2\left(\frac{\sqrt{5}+1}{4}\right)^2\right)=0$$

$$\Rightarrow x^2-\frac{12}{16}x+\frac{1}{16}=0$$

$$\Rightarrow\ \ 16x^2-12x+1=0$$

133.

$$x^4+x^2+1=0$$

$$\Rightarrow\left(x^2-x+1\right)\left(x^2+x+1\right)=0$$

$$\Rightarrow \alpha=w,\ \ \ \beta=w^2$$

$$\gamma=-w\ \ f=-w^2$$

Now

$$(w)^{2023}+\left(w^2\right)^{2023}+(-w)^{2022}+\left(-w^2\right)^{2022}$$

$$=w+w^2+1+1$$

$$=0+1=1$$

134. $ax^2-7x+c=0\rightarrow 1$ } *have common root*

$ax^2+5x-c=0$

$\Rightarrow 6x=c$

$3,4$ *a root of* $ax^2-7x+c=0\rightarrow 2$

$9a-21+6x=0$

$a=\dfrac{7-2x}{3}\rightarrow 3$

And *from* 2 & 3

$\Rightarrow\therefore x=1/2\ ,\ 3$

$\therefore$ *common root is* $1/2$

135. $x^2-7x+10\geq 0$ | $2x+3-x^2>0$

$(x-2)(x-5)\geq 0$ | $(x+1)(x-3)<0$

$x\in(-\infty,2]\cup[5,\infty)$ | $x\in(-1,3)$

Common Interval is

$x\in(-1,2]$