23. RATE MEASURE

1. A kind of bacteria grows by t^3 in t seconds. Time taken for the rate of growth of the bacteria to become 1200 per second is ____

[AP EAMCET 17-09-20_Shift-2]

1. 10 sec　　2. 20 sec
3. 40 sec　　4. 400 sec

2. The sides of an equilateral triangle are increasing at the rate of 2 cm.s-1. How fast does its area increases when its side is 10cm?

[AP EAMCET 17-09-20_Shift-2]

1. $10\sqrt{3}\ cm^2.s^{-1}$　　2. $5\sqrt{3}\ cm^2.s^{-1}$
3. $\sqrt{3}\ cm^2.s^{-1}$　　4. $2\sqrt{3}\ cm^2.s^{-1}$

3. If $f(x)=x^4-x^3+7x^2+14$, then what is the value of $f'(5)$ **[AP EAMCET 18-09-20_Shift-1]**

1. 594　　2. 549
3. 954　　4. 495

4. In a bank, the principal increases continuously at the rate of 6% per year. Then the time required to double 6000 rupees is __

[AP EAMCET 21-09-20_Shift-1]

1. $\frac{50}{3}\log 2$　　2. $\frac{50}{3}\log 6$
3. $\frac{50}{3}\log 3$　　4. $\frac{50}{3}\log 12$

5. The population of a city grows at the annual rate of 3%.what percentage increase is expected in 5 years? **[AP EAMCET 21-09-20_Shift-2]**

1. 12.9%　　2. 13.9%
3. 14.9%　　4. 15.9%

6. The curve $y=ax^3+bx^2+cx+5$ touches the x-axis at P(-2,0) and cuts y-axis at a point Q where gradient is 3.then the values of a,b,c are

[AP EAMCET 21-09-20_Shift-2]

1. $a=\frac{-1}{2},b=\frac{-3}{4},c=3$　　2. $a=\frac{1}{2},b=\frac{3}{4},c=3$
3. $a=1,b=2,c=3$　　4. $a=-1,b=-2,c=3$

7. Displacement 's' of a particle, in meters, at any time 't' seconds is expressed as $s=\frac{t^3}{3}-6t$, find the acceleration at a time when the velocity vanishes. **[AP EAMCET 22-09-20_Shift-1]**

1. $6m.s^{-2}$　　2. $2\sqrt{6}m.s^{-2}$
3. $12m.s^{-2}$　　4. $6\sqrt{6}m.s^{-2}$

8. The radius of a circle is increasing at a rate of 0.1 cm. s^{-1}. Then the rate of change of area, when its radius is 5 cm is-------

[AP EAMCET 22-09-20_Shift-1]

1. $\pi^2 cm^2.s^{-1}$　　2. $\pi\ cm^2.s^{-1}$
3. $2\pi\ cm^2.s^{-1}$　　4. $\frac{\pi}{2}\ cm^2.s^{-1}$

9. If displacement $s=5\sin(2t)$, then the velocity at the end of $\frac{\pi}{3}$ seconds is

[AP EAMCET 22-09-20_Shift-2]

1. 5　　2. $-5\sqrt{3}$
3. $5\sqrt{3}$　　4. -5

10. The side of equilateral triangle is increasing at the rate of 2 cm/s, then the rate at which the area is increasing when the side of the triangle is 20 cm is

1. $5\sqrt{3}\,cm^2/s$　　2. $10\sqrt{3}\,cm^2/s$
3. $25\sqrt{3}\,cm^2/s$　　4. $20\sqrt{3}\,cm^2/s$

11. If the area of a circle increases at the rate of $\frac{1}{\sqrt{\pi}}$ sq. units/sec, then the rate (in units/sec)at which the perimeter of the circle changes,when perimeter is $\sqrt{\pi}$ units,is

[TS EAMCET 10-09-20_Shift-1]

1. 2　　2. 4
3. $\frac{1}{\sqrt{\pi}}$　　4. $\sqrt{\pi}$

12. A tank in the shape of the rectangular parallelepiped has volume 27 cubic meters. This tank is filled with water such that the rate of change of level of the water is thrice the rate of change water quantity falling in the tank, then the height of the tank (in meters) is

[TS EAMCET 10-09-20_Shift-2]

1. 9　　2. 18
3. 81　　4. 243

13. The radius of a sphere is changing. At an instant of time the rate of change in its volume and its surface area are equal. Then the value of radius at that instant is?

[TS EAMCET 11-09-20_Shift-2]

1. 1
2. 2
3. $\frac{3}{2}$
4. 3

14. The volume of a sphere is increasing at the rate of 4π c.c/sec. When its volume is 288π c.c, the rate of increase (in cm/sec) in its radius is

[TS EAMCET 11-09-20_Shift-2]

1. $\frac{1}{36}$
2. $\frac{1}{6}$
3. $\frac{1}{7}$
4. $\frac{1}{49}$

15. Let $'x'$ *and* $'y'$ be the sides of two squares such that $y = x - x^2$. The rate of change of area of the second square with respect to area of the first square is___

[AP EAMCET 19-08-2021_Shift-2]

1. $1-3x+2x^2$
2. $1+3x-2x^2$
3. $2x$
4. $x+2x^3-3x^2$

16. The diameter and altitude of a right circular cone, at a certain instant, were found to be 10cm and 20cm respectively. If its diameter is increasing at a rate of 2cm/s , then at what rate must its altitude change, in order to keep its volume constant? **[AP EAMCET 20-08-2021_Shift-1]**

1. 4 *cm/s*
2. 6 *cm/s*
3. *-4 cm/s*
4. *-8 cm/s*

17. The volume of a spherical balloon is increasing at the rate of 30cc per minute. Find the rate of change of surface area of the balloon, when its radius is 6cm. **[AP EAMCET 20-08-2021_Shift-2]**

1. $5cm^2.\min^{-1}$
2. $30cm^2.\min^{-1}$
3. $10cm^2.\min^{-1}$
4. $20cm^2.\min^{-1}$

18. The volume of a spherical ball is increasing at a rate of $4\pi\ cm^3s^{-1}$. The rate at which its radius increases, when its volume is $288\pi\ cm^3$, is_____ $cm.s^{-1}$. **[AP EAMCET 23-08-2021_Shift-2]**

1. $\frac{1}{6}$
2. $\frac{1}{36}$
3. $\frac{1}{9}$
4. $\frac{1}{24}$

19. For the curve $y = 5x - 2x^3$, if 'x' increases at the rate of 2 units/sec, the rate of change in the slope of the curve at $x = 3$ is ______/sec.

[AP EAMCET 24-08-2021_Shift-2]

1. 72
2. 27
3. -72
4. -27

20. A particle moves along a straight line according to the law $s = \frac{1}{3}t^3 - 3t^2 + 9t + 17$, where 's' is in meter and 't' is in second. Its velocity decreases in ______ **[AP EAMCET 25-08-2021_Shift-1]**

1. $0 < t < 5$
2. $0 < t < 3$
3. $t > 5$
4. $t > 3$

21. If $s = 60t - 5t^2$ denotes the distance covered by a particle in time 't', then the distance it covers before coming to rest is ____ units.

[AP EAMCET 25-08-2021_Shift-2]

1. 120
2. 720
3. 240
4. 180

22. If the radius of a spherical balloons is increasing at the rate of 5 inch per minute, then the rate at which the volume increases (in cube inches per minute)when the radius is 10 inches is

[TS EAMCET 04-08-2021_Shift-2]

1. 100π
2. 1000π
3. 2000π
4. 25000π

23. A particle moves in a straight line such that its displacement S (in mts) at a time $t(in \sec)$ is given by $S(t) = t^3 - 4t^2 + 7t$. The instantaneous velocity v at $t = 4$ is

[TS EAMCET 05-08-2021_Shift-1]

1. $21 m/\sec$
2. $23 m/\sec$
3. $20 m/\sec$
4. $19 m/\sec$

24. The volume of a spherical balloon is increasing at the rate of $2cm^3/\sec$. When its radius is 4cm,the rate of change of its surface area $(in\ cm^2/\sec)$ is

[TS EAMCET 05-08-2021_Shift-2]

1. 1
2. 2
3. 3
4. 4

25. A closed cylinder of given volume will have least surface area when the ratio of its height and base radius is

[AP EAMCET 04-07-2022_Shift-2]

1. 2:1
2. 1:2
3. 2:3
4. 3:2

26. For a particle moving on a straight line it is observed that the distance 'S' at a time 't' is given by S=6t-$\frac{t^3}{2}$. The maximum velocity during the motion is

[AP EAMCET 06-07-2022_Shift-2]

1. 3
2. 6
3. 9
4. 12

KEY

1)	2	2)	1	3)	4	4)	1	5)	4
6)	1	7)	2	8)	2	9)	4	10)	4
11)	1	12)	3	13)	2	14)	1	15)	1
16)	4	17)	3	18)	2	19)	3	20)	2
21)	4	22)	3	23)	2	24)	1	25)	1
26)	2								

SOLUTIONS

1. $y = t^3$

$\frac{dy}{dx} = 3t^2 = 1200$

$\Rightarrow t^2 = 400$

$\Rightarrow t = 20$ sec

2. Given side length (a)=10

$\frac{da}{dt} = 10$

Area of equilateral(A)=$\frac{\sqrt{3}}{4}a^2$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4}.2a.\frac{da}{dt}$

$= \frac{\sqrt{3}}{4}.2(10).2$

$= 10\sqrt{3}$

3. $f'(x) = 4x^3 - 3x^2 + 14x$

$f'(5) = 500 - 75 + 70 = 495$

4. Principal = 6000

Increasing rate=6% per year

$\frac{dP}{dt} = \frac{6}{100}P$

$\frac{1}{P}dP = \frac{6}{100}dt$

$\int_{6000}^{12000} \frac{1}{P}dP = \frac{6}{100}\int dt$

$[\log P]_{6000}^{12000} = \frac{3}{50}(t)$

$t = \frac{50}{3}[\log 12000 - \log 6000]$

$= \frac{50}{3}\log 2$

5. Let consider a population p(0) of a given city in year 0
In year 1, the population would have increased 3% which is =1+0.03=1.03
So p(1)(year 1 population)=p(0)(1.03)
For p(2)=p(0) $(1.03)^2$

.....

.....

P(5)=p(0) $(1.03)^5$=15.9%
So your population in 5 years would have grown =15.9%

6. Given $y = ax^3 + bx^2 + cx + 5 \rightarrow (1)$
The point P(-2,0) passes through eq(1)
We get
$8a - 4b + 2c - 5 = 0 \rightarrow (2)$
Eq.(1) diff w.r.to 'x', we get
$\frac{dy}{dx} = 3ax^2 + 2bx + c$

$\left(\frac{dy}{dx}\right)_{at\, P(-2,0)} = 12a - 4b + c = 0 \rightarrow (3)$

(0,y) $\Rightarrow y = ax^3 + bx^2 + cx + 5$ *we get* $y = 5$

$Q(0,5)$ *and* $\frac{dy}{dx} = 3(Given)$

$\left(\frac{dy}{dx}\right)_{at\, Q(0,5)} = 3ax^2 + 2bx + c$

$3 = 0 + 0 + c$

$\therefore c = 3$

$$\left.\begin{array}{l} from(3),\ 12a-4b+3=0 \\ from(2),\ 8a-4b+1=0 \end{array}\right\} solving$$

$$we\ get\ a=\frac{-1}{2}, b=\frac{-3}{4}$$

$$\therefore a=\frac{-1}{2}, b=\frac{-3}{4}, c=3$$

7. $s=\frac{t^3}{3}-6t \Rightarrow \frac{ds}{dt}=\frac{3t^2}{3}-6$

$$\Rightarrow v=t^2-6=0$$

$$t=\sqrt{6}$$

$$a=\frac{dv}{dt}=2t=2\sqrt{6}$$

8. r=5cms

$$\frac{dr}{dt}=0.1cm/\sec$$

$$A=\pi r^2$$

$$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$$

$$=2\pi(5)(0.1)cm/\sec$$

$$=\pi cms^{-1}$$

9. Displacement S=5 sin2t

$$velocity\ \frac{dS}{dt}=10\cos 2t$$

$$\left(\frac{dS}{dt}\right)_{t=\frac{\pi}{3}}=10\cos\frac{2\pi}{3}=10\left(\frac{-1}{2}\right)=-5$$

10. $\delta a=2\ cm/s$

$$a=20 \Rightarrow \Delta=\frac{\sqrt{3}}{4}a^2$$

$$\delta\Delta=\frac{\sqrt{3}}{4}2a(\delta a)=20\sqrt{3}$$

11. Given

$$\frac{dA}{dt}=\frac{1}{\sqrt{\pi}}$$

$$p=\sqrt{\pi}$$

$$2\pi r=\sqrt{\pi}$$

$$r=\frac{\sqrt{\pi}}{2\pi}$$

$$A=\pi r^2$$

$$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$$

$$\frac{1}{\sqrt{\pi}}=\frac{2\pi\sqrt{\pi}}{2\pi}\frac{dr}{dt}$$

$$\frac{dr}{dt}=\frac{1}{\pi}$$

$$w.k.t\quad p=2\pi r$$

$$\frac{dp}{dt}=2\pi\frac{dr}{dt}=2\pi\left(\frac{1}{\pi}\right)=2$$

12. $lbh=27$

$$v=lbh$$

$$\frac{dv}{dt}=lb\frac{dh}{dt}$$

$$\frac{dv}{dt}=lb.3.\frac{dv}{dt}$$

$$lb=\frac{1}{3}$$

$$v=\frac{1}{3}h$$

$$81=h$$

13. $V=\frac{4}{3}\pi r^3,\ 4\pi r^2=S$

$$\frac{dv}{dt}=\frac{ds}{dt}$$

$$\frac{4}{3}\pi 3r^2\frac{dr}{dt}=4\pi.2r.\frac{dr}{dt}$$

$$r=2$$

14. $V=\frac{4}{3}\pi r^3$

$$\frac{dv}{dt}=4\pi\ \frac{cc}{\sec}\quad V=288\pi$$

$$288\pi=\frac{4}{3}\pi r^3$$

$$r^3=216\Rightarrow r=6$$

$$\frac{dv}{dt}=\frac{4}{3}\pi\, 3r^2\frac{dr}{dt}$$

$$4\pi=4\pi.r^2.\frac{dr}{dt}\Rightarrow\frac{dr}{dt}=\frac{1}{36}$$

15. $A_1 = x^2, A_2 = y^2$

$$\frac{\Delta A_2}{\Delta A_1} = \frac{2y.\Delta y}{2x.\Delta x} = \frac{(x-x^2).(1-2x).\Delta x}{x.\Delta x} = 2x^2 - 3x + 1$$

16. $V = \frac{1}{3}\pi r^2 h.$

$$\Rightarrow \delta r = \frac{\pi}{3}\left[r^2.(\delta h) + h(2r).\delta r\right].$$

$$\Rightarrow 0 = \frac{\pi}{3}\left[(25)(\delta h) + (20)(2)(5)(1)\right]$$

$$\Rightarrow \delta h = -8cm/s.$$

$$V = \frac{4}{3}\pi r^3$$

17. $\Rightarrow \delta v = 4\pi r^2.\delta r \Rightarrow \delta r \Rightarrow \delta r = \frac{5}{24\pi}$

$$A = 4\pi r^2$$

$$\Rightarrow \delta A = 8\pi r.\delta r = 10cm^2/\text{min}.$$

18. $V = \frac{4}{3}\pi r^3 \Rightarrow r = 6$

$$\delta r = 4\pi r^2.\delta r \Rightarrow 4\pi = 4\pi(36).(\delta r)$$

$$\Rightarrow \delta r = \frac{1}{36}cm/s.$$

19. $m = \frac{dy}{dx} = 5 - 6x^2.$

$$\Rightarrow \delta_m = -12x.(\delta x). = -12(3)(2) = -72 units/\text{sec}.$$

20. $s = \frac{1}{3}t^3 - 3t^2 + 9t + 17.$

$$\Rightarrow V = \frac{ds}{dt} = t^2 - 6t + 9 = f(t)$$

$$f'(t) = 0 \Rightarrow t = 3.$$

$$f''(t) = 2 > 0$$

$\therefore f(t)$ has minimum at $t = 3$

$\therefore 0 < t < 3.$

21. $\frac{ds}{dt} = 0 \Rightarrow t = 6.$

$$\therefore s = 60(6) - 5(6)^2 = 180\, units$$

22. Given $\frac{dr}{dt} = 5, r = 10$

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi 3r^2.\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi(100).5 = 2000\pi$$

23. $S(t) = t^3 - 4t^2 + 7t$

$$V = S'(t) = 3t^2 - 8t + 7$$

$$(V(t))_{t=4} = 23$$

24. $\frac{dv}{dt} = 2cm^3/\text{sec}\quad, r = 4cm, v = \frac{4}{3}\pi r^3$

$$\frac{dv}{dt} = \frac{4}{3}\pi.3r^2\frac{dr}{dt}$$

$$2 = 4\pi.16.\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{32\pi}$$

$$S = 4\pi r^2$$

$$\frac{ds}{dt} = 4\pi.2r.\frac{dr}{dt} = 8\pi.4.\frac{1}{32\pi} = 1$$

25. Conceptual

26. $\frac{d^2s}{dt^2} = 0$