

37. SYSTEM OF CIRCLES

1. Find the minimum radius of the circle which is orthogonal to both the circles

$$x^2 + y^2 + 4x + 3 = 0 \text{ and } x^2 + y^2 - 12x + 35 = 0$$

[AP EAMCET 18-09-20_Shift-2]

- | | |
|----------------|----------------|
| 1. 1 | 2. 4 |
| 3. $\sqrt{17}$ | 4. $\sqrt{15}$ |

2. The radical axis of the co axial system of circles with limiting points (1, 2) and (-2, 1) is

[AP EAMCET 21-09-20_Shift-2]

- | | |
|--------------|--------------|
| 1. $x+3y=0$ | 2. $2x+3y=0$ |
| 3. $3x+2y=0$ | 4. $3x+y=0$ |

3. The radical axes of any two circles isto the joining their centres

[AP EAMCET 21-09-20_Shift-2]

1. Parallel
2. Perpendicular
3. Intersecting but perpendicular
4. Can't be determined

4. If the circles $x^2 + y^2 - 2x - 2y - 7 = 0$ and $x^2 + y^2 + 4x + 2y + k = 0$ cut orthogonally, then the length of their common chord is ____ units.

[AP EAMCET 22-09-20_Shift-1]

- | | |
|--------------------------|---------------------------|
| 1. 2 | 2. 5 |
| 3. $\frac{6}{\sqrt{13}}$ | 4. $\frac{12}{\sqrt{13}}$ |

5. If the circle $x^2 + y^2 + 6x - 2y + k = 0$ bisects the circumference of the circle

$$x^2 + y^2 + 2x - 6y - 15 = 0, \text{ then 'k' is equal to}$$

[AP EAMCET 22-09-20_Shift-2]

- | | |
|-------|--------|
| 1. 21 | 2. -21 |
| 3. 23 | 4. -23 |

6. If $x^2 + y^2 - 6x - 8y + 12 = 0$ and

$$x^2 + y^2 - 4x + 6y + k = 0$$

cut orthogonally, then k

- | | |
|--------|--------|
| 1. -24 | 2. 24 |
| 3. -48 | 4. -42 |

7. The equation of the common chord of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 - 5x - 6y + 4 = 0$ is ____

- | | |
|----------------------|----------------------|
| 1. $3x - 3y + 5 = 0$ | 2. $7y + 9x - 3 = 0$ |
| 3. $7x - 9y + 3 = 0$ | 4. $7x + 9y - 3 = 0$ |

8. If the angle between the circles

$$x^2 + y^2 - 4x - 6y + k = 0 \text{ and}$$

$$x^2 + y^2 + 8x - 4y + 11 = 0$$

is $\frac{\pi}{3}$, then a value of k is

[TS EAMCET 09-09-20_Shift-1]

- | | |
|-------|-------|
| 1. -3 | 2. 36 |
| 3. 3 | 4. 2 |

9. $r_1, r_2 > 0$ and if C_1, C_2 are centres of the two circles having only two common tangents and $C_1C_2 = r_1 + r_2$, then which of the following is correct. [TS EAMCET 09-09-20_Shift-2]

1. r_1, r_2 are the radii of the two circles
2. The common chord may divide the line joining the centres in the ratio $r_1 : r_2$
3. r_1, r_2 are always the distances of the centres from the common tangent of the two circles
4. External centre of similitude divides C_1C_2 in the ratio $r_1 : r_2$

10. Let $S \equiv x^2 + y^2 - 6x - 6y + 4 = 0$ and

$S' \equiv x^2 + y^2 - 2x - 4y + 3 = 0$ be two circles. The centre of a circle of radius $\sqrt{14}$ and having same radical axis with either of $S=0$ or $S'=0$ is

[TS EAMCET 09-09-20_Shift-2]

- | | |
|----------|---|
| 1. (3,3) | 2. $\left(\frac{-19}{5}, \frac{-2}{5}\right)$ |
| 3. (1,2) | 4. $\left(\frac{2}{5}, \frac{3}{5}\right)$ |

11. The circle $S=0$ cuts the circles

$$C_1 = x^2 + y^2 - 8x - 2y + 16 = 0 \text{ and}$$

$C_2 = x^2 + y^2 - 4x - 4y - 1 = 0$ orthogonally. If the common chord of $S=0$ and

$C_1 = 0$ is $2x + 13y - 15 = 0$, then the centre of $S=0$ is

[TS EAMCET 10-09-20_Shift-1]

1. $\left(-\frac{11}{3}, \frac{7}{6}\right)$
2. $\left(\frac{11}{3}, -\frac{7}{6}\right)$
3. $\left(\frac{2}{13}, \frac{11}{15}\right)$
4. $\left(\frac{11}{15}, -\frac{2}{13}\right)$

12. The equation of the circle passing through the points of intersection of the two orthogonal circles $S_1 = x^2 + y^2 + kx - 4y - 1 = 0$, and

$S_2 = 3x^2 + 3y^2 - 14x + 23y - 15 = 0$ and passing through the point $(-1, -1)$ is

[TS EAMCET 10-09-20_Shift-1]

1. $x^2 + y^2 - 8x - 2y - 12 = 0$
2. $3x^2 + 3y^2 + 18x - 12y = 0$
3. $5x^2 + 5y^2 - 22x + 15y - 17 = 0$
4. $x^2 + y^2 - 5x + 14y + 7 = 0$

13. Suppose the circle $S: x^2 + y^2 + 2gx + 2fy + c = 0$ cuts orthogonally the two circles

$$S' = x^2 + y^2 - 4x - 6y + 11 = 0 \text{ and}$$

$S'' = x^2 + y^2 - 10x - 4y + 21 = 0$. If the centres of $S=0$ lies on the bisector of the angle between the positive coordinate axes, then

$$2g + 2f + c = \text{[TS EAMCET 10-09-20_Shift-2]}$$

1. 12
2. 8
3. 4
4. 0

14. If the circles $S_1: x^2 + y^2 = 16$ intersects another circle S_2 of radius 5 units such that the common chord is of maximum length and slope $\frac{3}{4}$, then the centre of the circle S_2 is

[TS EAMCET 10-09-20_Shift-2]

1. $\left(\frac{-9}{5}, \frac{12}{5}\right)$ or $\left(\frac{9}{5}, -\frac{12}{5}\right)$
2. $\left(\frac{7}{5}, -\frac{12}{5}\right)$ or $\left(-\frac{7}{5}, \frac{12}{5}\right)$
3. $\left(-\frac{9}{5}, -\frac{12}{5}\right)$ or $\left(\frac{9}{5}, \frac{12}{5}\right)$
4. $\left(\frac{12}{5}, \frac{9}{5}\right)$ or $\left(-\frac{12}{5}, -\frac{9}{5}\right)$

15. The centre of the circle passing through the points intersection of the circles

$$(x+3)^2 + (y+2)^2 = 25 \text{ and}$$

$$(x-2)^2 + (y-3)^2 = 25 \text{ and cutting the circle}$$

$$(x+1)^2 + (y-2)^2 = 16 \text{ orthogonally is}$$

[TS EAMCET 11-09-20_Shift-1]

1. $\left(\frac{-27}{2}, \frac{-25}{2}\right)$
2. $(0, 0)$
3. $\left(\frac{16}{3}, \frac{-25}{4}\right)$
4. $\left(\frac{4}{7}, \frac{3}{7}\right)$

16. L_1 and L_2 are the common tangents to two circles. If L_1 touches the two circles at $A(1, 1)$ and $B(0, 1)$ and L_2 touches the two circles at $C\left(\frac{3}{4}, \frac{4}{5}\right)$, $D\left(-\frac{1}{5}, \frac{7}{5}\right)$, then the equation of the radical axis of the two circles is

[TS EAMCET 11-09-20_Shift-2]

1. $2x - 6y = 7$
2. $2x + y + 7 = 0$
3. $2x + 6y = 7$
4. $x = y$

17. The centre of the smallest circle which cuts the circles $x^2 + y^2 - 2x - 4y - 4 = 0$ and $x^2 + y^2 - 10x + 12y + 52 = 0$ orthogonally is

[TS EAMCET 11-09-20_Shift-2]

1. $(1, 2)$
2. $(-3, 2)$
3. $(3, -2)$
4. $(3, 4)$

18. If $x^2 + y^2 - a^2 + \lambda(x \cos \alpha + y \sin \alpha - p) = 0$ is the smallest circle through the points of intersection of $x^2 + y^2 = a^2$ and $x \cos \alpha + y \sin \alpha = p$, ($0 < p < a$), then $\lambda =$ [TS EAMCET 11-09-20_Shift-2]

1. 1
2. -p
3. -2p
4. -3p

19. The equation of the tangents drawn from the origin to the circle $x^2 + y^2 + 2gx + 2fy + g^2 = 0$

[TS EAMCET 14-09-20_Shift-2]

1. $x = 0, (g^2 + f^2)x - 2gfy = 0$
2. $x = 0, (g^2 - f^2)x - 2gfy = 0$
3. $y = 0, (g^2 - f^2)y - 2gfy = 0$
4. $y = 0, (g^2 + f^2)y - 2gfy = 0$

20. If $2x + y = 0$ is the equation of the chord of a circle $x^2 + y^2 - 2x - 6y + 3 = 0$, then the circle with this chord as diameter passes through the point [TS EAMCET 14-09-20_Shift-2]

1. $(-3, 2)$
2. $(5, -2)$
3. $(-5, 3)$
4. $(-2, 1)$

21. If the radical axis of the circles

$$x^2 + y^2 + 2\alpha x + 2\beta y + c = 0 \text{ and } x^2 + y^2 + \frac{3}{2}x + 4y + c = 0$$

touches the circle $x^2 + y^2 + 2x + 2y + 1 = 0$, then $4\alpha\beta - 8\alpha - 3\beta + 10 =$

[TS EAMCET 14-09-20_Shift-2]

1. 2
2. -2
3. 4
4. -4

22. The radius of the circle whose center lies at $(1, 2)$, while cutting the circle

$$x^2 + y^2 + 4x + 16y - 30 = 0 \text{ orthogonally,}$$

is _____ units.

[AP EAMCET 19-08-2021_Shift-1]

1. $\sqrt{41}$
2. $\sqrt{31}$
3. $\sqrt{21}$
4. $\sqrt{11}$

23. The point which has same power with respect to each of the circles $x^2 + y^2 - 8x + 40 = 0$, $x^2 + y^2 - 5x + 16 = 0$ and $x^2 + y^2 - 8x + 16y + 160 = 0$ is _____

[AP EAMCET 19-08-2021_Shift-1]

1. $\left(-8, \frac{-15}{2}\right)$
2. $\left(8, \frac{-15}{2}\right)$
3. $\left(8, \frac{15}{2}\right)$
4. $\left(-8, \frac{15}{2}\right)$

24. The equation of the circle, which cuts orthogonally each of three circles

$$x^2 + y^2 - 2x + 3y - 7 = 0,$$

$$x^2 + y^2 + 5x - 5y + 9 = 0 \text{ and}$$

$$x^2 + y^2 + 7x - 9y + 29 = 0 \text{ is _____}$$

[AP EAMCET 19-08-2021_Shift-2]

1. $x^2 + y^2 - 16x - 18y - 4 = 0$
2. $x^2 + y^2 = a^2$
3. $x^2 + y^2 - 16x = 0$
4. $y^2 - x^2 + 2x = 0$

25. The length of the common chord of the circles

$$x^2 + y^2 + 3x + 5y + 4 = 0 \text{ and}$$

$$x^2 + y^2 + 5x + 3y + 4 = 0 \text{ is _____ units}$$

[AP EAMCET 20-08-2021_Shift-1]

1. 3
2. 2
3. 6
4. 4

26. Find the equation of the circle which passes through the point $(1, 2)$ and the points of intersection of the circles $x^2 + y^2 - 8x - 6y + 21 = 0$ and $x^2 + y^2 - 2x - 15 = 0$.

[AP EAMCET 20-08-2021_Shift-1]

1. $x^2 + y^2 - 18x - 12y + 27 = 0$
2. $2(x^2 + y^2) - 18x - 12y + 27 = 0$
3. $3(x^2 + y^2) - 18x - 12y + 27 = 0$
4. $4(x^2 + y^2) - 18x - 12y + 27 = 0$

27. The equation of radical axis of the circles
 $x^2 + y^2 + 4x + 6y + 7 = 0$ &
 $4x^2 + 4y^2 + 8x + 12y - 9 = 0$

Is [AP EAMCET 20-08-2021 Shift-2]

1. $x + y + 1 = 0$
2. $8x + 12y = 0$
3. $8x + 12y + 37 = 0$
4. $2x + 3y + 7 = 0$

28. The radical axis of the circles

$$S_1: x^2 + y^2 - 4x + 6y - 10 = 0 \text{ and}$$

$S_2: x^2 + y^2 + 2x - 6y + 2 = 0$ cut the circle S_1 in

[AP EAMCET 20-08-2021 Shift-2]

1. two real and distinct points
2. one real point
3. imaginary points
4. can't be determined

29. If $(4, 7)$ & $(-2, -1)$ are ends of a diameter of a circle which intersects X-axis at A and B, then AB is equal to

[AP EAMCET 23-08-2021 Shift-1]

- 5
- 6
- 7
- 8

- ³⁰. If L_1 represents the radical axis of circles

$$x^2+y^2-4x-6y+5=0 \text{ and } x^2+y^2-2x-4y-1=0$$

and L_2 represents the radical axis of

$$x^2 + y^2 + 2x + 2y - 7 = 0 \text{ and}$$

$x^2 + y^2 + x + y + 9 = 0$, then _____

[AP EAMCET 23-08-2021_Shift-1]

1. L_1 is parallel to L_2
2. L_1 is perpendicular to L_2
3. L_1 & L_2 intersect at an angle 30°
4. L_1 & L_2 intersect at $(1, 7)$

- 31 Which circle among the following bisects the circumference of the circle

$$x^2 + y^2 - 8x - 6y + 23 = 0?$$

[AP EAMCET 23-08-2021_Shift-1]

1. $x^2 + y^2 - 6x - 4y + 9 = 0$
2. $x^2 + y^2 + 6x + 4y - 9 = 0$
3. $x^2 + y^2 - 6x + 4y - 9 = 0$
4. $x^2 + y^2 + 6x - 4y + 9 = 0$

32. If the circles $x^2 + y^2 - 6x - 8y - 12 = 0$ and $x^2 + y^2 - 4x + 6y + k = 0$ are perpendicular to each other, then 'k' equals _____

[AP EAMCET 24-08-2021_Shift-1]

- 4
- 0
- 2
- 12

33. Find the equation of a circle which passes through the point (1,2) and the points of intersection of the circles $x^2 + y^2 - 8x - 6y + 21 = 0$ and $x^2 + y^2 - 2x - 15 = 0$.

[AP EAMCET 24-08-2021 Shift-1]

1. $x^2 + y^2 - 6x - 4y + 9 = 0$
2. $x^2 + y^2 - 18x - 12y + 27 = 0$
3. $2(x^2 + y^2) - 18x + 12y + 27 = 0$
4. $4(x^2 + y^2) - 3x + 12y + 16 = 0$

34. If $S = x^2 + y^2 + 2x + 17y + 4 = 0$,

$$S'' = x^2 + y^2 + 7x + 6y + 11 = 0 \text{ and}$$

$S'' = x^2 + y^2 - x + 22y + 3 = 0$ are three circles.

then the length of tangent from their radical center to $S = 0$ is _____ units.

[AP EAMCET 24-08-2021 Shift-2]

1. $\sqrt{53}$
2. $\sqrt{57}$
3. $\sqrt{15}$
4. $\sqrt{17}$

35. The angle between circles

$$x^2 + y^2 + 2x + 4y + 1 = 0 \text{ and}$$

$$x^2 + y^2 - 2x + 6y - 3 = 0 \text{ is}$$

[AP EAMCET 25-08-2021_Shift-2]

1. $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$
2. $\cos^{-1}\left(\frac{3}{\sqrt{31}}\right)$
3. $\cos^{-1}\left(\sqrt{\frac{3}{31}}\right)$
4. $2\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$

36. The locus of the centres of the circles that are passing through the intersection of the circles

$$x^2 + y^2 = 1 \text{ and } x^2 + y^2 - 2x + y = 0$$

[AP EAMCET 25-08-2021_Shift-2]

1. A line whose equation is $x + 2y = 0$
2. A circle
3. A parabola
4. A line whose equation is $2x - y = 0$

37. Choose the correct option regarding the following statements:

Statement – 1: The length of common chord

of circles $x^2 + y^2 + ax + by + c = 0$ and

$$x^2 + y^2 + bx + ay + c = 0 \text{ equals } \frac{\sqrt{(a+b)^2 - 8c}}{2}$$

Statement–2: If two circles intersect at two distinct points, then their radical axis is their common chord.

[AP EAMCET 23-08-2021_Shift-2]

1. Both statements are true and statement–2 is a correct explanation for statement – 1.
2. Both statements are true but statement – 2 is not a correct explanation for statement – 1.
3. Statement – 1 is true, Statement – 2 is false.
4. Statement – 1 is false, statement – 2 is true.

38. Find the equation of a circle which cuts the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ orthogonally, while passing through (3, 0) and touching the y– axis.

[AP EAMCET 23-08-2021_Shift-2]

1. $x^2 + y^2 + 6x + 6y + 9 = 0$
2. $x^2 + y^2 - 6x - 6y + 9 = 0$
3. $x^2 + y^2 - 6x + 6y - 9 = 0$
4. $x^2 + y^2 + 6x - 6y - 9 = 0$

39. If the radical centre of the circles

$$x^2 + y^2 - 8x - 2y + 8 = 0, x^2 + y^2 + 6x + 8y - 24 = 0,$$

And $x^2 + y^2 - 2x + 2y + 2 = 0$ is (a, b) , then $a + b =$

[TS EAMCET 04-08-2021_Shift-2]

1. 34
2. 10
3. -15
4. -24

40. The equation of the circle passing through (1,1) and through the points of intersection of the circles $x^2 + y^2 + 13x - 3y = 0$ and

$$2x^2 + 2y^2 + 4x - 7y - 25 = 0 \text{ is}$$

[TS EAMCET 04-08-2021_Shift-1]

1. $4x^2 + 4y^2 + 30x - 13y - 25 = 0$
2. $2x^2 + 2y^2 + 15x - 19y = 0$
3. $4x^2 + 4y^2 + 25x + 12y - 45 = 0$
4. $4x^2 + 4y^2 + 13x - 30y + 9 = 0$

41. If the circles $(x - 2)^2 + (y - 3)^2 = 25$ and $25x^2 + 25y^2 - 40x - 70y - 160 = 0$ touch

internally at (α, β) then $\alpha + \beta =$

[TS EAMCET 04-08-2021_Shift-1]

1. 0
2. -2
3. -1
4. 1

4. $(-3, -4)$

50. The radical centre of the three circles
 $x^2 + y^2 - 1 = 0$, $x^2 + y^2 - 8x + 15 = 0$ and
 $x^2 + y^2 + 10y + 24 = 0$ is

[AP EAMCET 05-07-2022_Shift-1]

1. $\left(2, \frac{-5}{2}\right)$
2. $\left(2, \frac{5}{2}\right)$
3. $\left(-2, \frac{5}{2}\right)$
4. $\left(-2, \frac{-5}{2}\right)$

51. The radical centre of the circles

$$x^2 + y^2 + 3x + 2y + 1 = 0,$$

$$x^2 + y^2 - x + 6y + 5 = 0 \text{ and}$$

$$x^2 + y^2 + 5x - 8y + 15 = 0 \text{ is}$$

[AP EAMCET 06-07-2022_Shift-2]

1. (3,2)
2. (-3,-2)
3. (2,3)
4. (-2,-3)

52. The equation of circle passing through (0,0) and cutting orthogonally the circles

$$x^2 + y^2 + 6x - 15 = 0 \text{ and } x^2 + y^2 - 8y - 10 = 0 \text{ is}$$

[AP EAMCET 07-07-2022_Shift-1]

1. $2(x^2 + y^2) - 10x + 5y = 0$
2. $2(x^2 + y^2) + 10x - 5y = 0$
3. $2(x^2 - y^2) + 10x + 5y = 0$
4. $2(x^2 - y^2) - 10x - 5y = 0$

53. Suppose A(2,3) and B are the points of intersections of two circles. The points P lying on one circle and Q lying on the other circle are such that BP and BQ constitute the diameters of the circles. If the slopes of the radical axis and PQ are $\frac{3}{4}$ and a/b respectively, then the value of $3a+4b$ is [AP EAMCET 07-07-2022_Shift-2]

1. 1
2. 0
3. 2
4. -1

54. The straight line $x \cos \alpha + y \sin \alpha = p$ cuts the circle $x^2 + y^2 - a^2 = 0$ at A and B. then the equation of circle having AB as diameter is

[AP EAMCET 08-07-2022_Shift-1]

1. $x^2 + y^2 - a^2 + p(x \cos \alpha + y \sin \alpha - p) = 0$
2. $x^2 + y^2 - a^2 - p(x \cos \alpha + y \sin \alpha + p) = 0$
3. $x^2 + y^2 - a^2 + 2p(x \cos \alpha + y \sin \alpha - p) = 0$
4. $x^2 + y^2 - a^2 - 2p(x \cos \alpha + y \sin \alpha - p) = 0$

55. Number of circles intersecting

$$x^2 + y^2 = 4, x^2 + y^2 - 2x - 3 = 0 \text{ and } x^2 + y^2 - 2y - 3 = 0$$

orthogonally is

[AP EAMCET 08-07-2022_Shift-2]

1. 0
2. 1
3. 2
4. ∞

56. If the common chord of the circles $x^2 + y^2 + 4y = 0$ and $x^2 + y^2 - 4x - 5 = 0$ is the diameter of the circle $S = 0$ then the abscissa of the centre of the circle $S = 0$ is

[TS EAMCET 18-07-2022_Shift-1]

1. $-\frac{13}{8}$
2. $\frac{3}{8}$
3. $\frac{3}{4}$
4. $-\frac{13}{4}$

57. If $\left(0, \frac{3}{4}\right)$ is the radical centre of the circles

$$S \equiv x^2 + y^2 + \alpha x + 6y = 0,$$

$$S' \equiv x^2 + y^2 + 2\alpha x + \alpha y + 6 = 0 \text{ and}$$

$$S'' \equiv x^2 + y^2 + 6\alpha x - \alpha y + 3 = 0 \text{ then the distance between the radical centre and the centre of the circle } S' = 0 \text{ is}$$

[TS EAMCET 18-07-2022_Shift-2]

1. 8
2. 15
3. $\frac{\sqrt{65}}{4}$
4. $\frac{\sqrt{5}}{4}$

67. If the angle between the circles

$$x^2 + y^2 + 2x - 4y + 1 = 0 \text{ and}$$

$$x^2 + y^2 - 4x - 2y + c = 0 \text{ is } \frac{\pi}{4}, \text{ then } c =$$

[17th May 2023 Shift 1]

1. 3
2. -13
3. -3 or 13
4. -31 or -3

68. If the circles $x^2 + y^2 - 4x + 2fy + 1 = 0$ and

$$x^2 + y^2 + 2gx - 4y - 1 = 0 \text{ cut orthogonally, then}$$

$$r_1^2 + r_2^2 - 8 =$$

[18th May 2023 shift -1]

1. g^2
2. $-f^2$
3. $2g^2$
4. $-2f^2$

69. If A and B are the points of intersection of the circles $x^2 + y^2 - 4x + 6y - 3 = 0$ and $x^2 + y^2 + 2x - 2y - 2 = 0$, then the distance between A and B is

[18th May 2023 Shift 2]

1. $\frac{13}{10}$
2. $\frac{\sqrt{41}}{3}$
3. $\frac{\sqrt{231}}{5}$
4. $\frac{26}{5}$

70. The line $x + y + 2 = 0$ intersects the circle

$$x^2 + y^2 + 4x - 4y - 4 = 0 \text{ in two points A and}$$

B. let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a different circle passing through the points A and B. If the distance of the centre of $S=0$

from AB is $\sqrt{2}$, then $g + f + c =$

[19th May 2023 Shift 1]

1. 12
2. 8
3. 6
4. 0

71. The equation of the circle whose diameter is the common chord of the circles

$$x^2 + y^2 + 2x + 3y + 1 = 0 \text{ and}$$

$$x^2 + y^2 + 4x + 3y + 2 = 0 \text{ is}$$

[12th MAY 2023 SHIFT -2]

1. $2x^2 + 2y^2 + 2x + 6y + 1 = 0$
2. $x^2 + y^2 - 2x + 3y - 1 = 0$
3. $x^2 + y^2 + 2x + 3y - 4 = 0$
4. $2x^2 + 2y^2 - x + 2y + 1 = 0$

72. The equation of the line perpendicular to the radical axis of two circles

$$x^2 + y^2 - 5x + 6y + 12 = 0, x^2 + y^2 + 6x - 4y - 14 = 0$$

and passing through (1,1) is

[12th MAY 2023 SHIFT -2]

1. $2x + 3y - 5 = 0$
2. $x + y - 2 = 0$
3. $10x + 11y - 21 = 0$
4. $11x + 10y - 21 = 0$

73. If the angle between the circles

$$x^2 + y^2 - 2x - 4y + c = 0 \text{ and}$$

$$x^2 + y^2 - 4x - 2y + 4 = 0 \text{ is } 60^\circ, \text{ then } c =$$

[12th MAY 2023 SHIFT -2]

1. $\frac{3 \pm \sqrt{5}}{2}$
2. $\frac{6 \pm \sqrt{5}}{2}$
3. $\frac{7 \pm \sqrt{5}}{2}$
4. $\frac{9 \pm \sqrt{5}}{2}$

74. If the angle between the circles

$$x^2 + y^2 - 2x + 2y + 1 = 0 \text{ and}$$

$$x^2 + y^2 + 2x - 2y + k = 0 \text{ is } \frac{\pi}{3}, \text{ then}$$

[13th MAY 2023 SHIFT-1]

1. k is a rational number but not an integer
2. k is an irrational number
3. There is no real number k satisfying the given condition
4. k is an integer

75. Let the line $x - y + 1 = 0$ intersect the circle

$$x^2 + y^2 + 2x + 2y + 1 = 0 \text{ in two points A and B. If AB is the diameter of the circle}$$

$$x^2 + y^2 + 2gx + 2fy + c = 0 \text{ then}$$

$$g + f =$$

[13th MAY 2023 SHIFT-1]

1. $3c$
2. $2c$
3. c
4. 0

76. If the angle between the circles

$$x^2 + y^2 - 4x - 6y + k = 0 \text{ and}$$

$$x^2 + y^2 + 8x - 4y + 11 = 0 \text{ is } \frac{\pi}{2}, \text{ then the}$$

value of k is [EAPCET 14-05-23 SHIFT-1]

1. -3
2. 3
3. -15
4. 15

- 77 If the radical centre of the given three circles $x^2 + y^2 = 1$, $x^2 + y^2 - 2x - 3 = 0$ and $x^2 + y^2 - 2y - 3 = 0$ is $C(\alpha, \beta)$ and r is the sum of the radii of the given circles, then the circle with $C(\alpha, \beta)$ as centre and r as radius is
- [EAPCET 14-05-23 SHIFT-1]

1. $(x-1)^2 + (y-1)^2 = 2$

2. $(x-1)^2 + (y+1)^2 = 4$

3. $(x-2)^2 + (y-2)^2 = 25$

4. $(x+1)^2 + (y+1)^2 = 25$

- 78 A circle $S \equiv x^2 + y^2 + 2gx + 2fy + 4 = 0$ cuts the circle $x^2 + y^2 - 4x - 4y - 4 = 0$ orthogonally and makes an angle of 60° with the circle $x^2 + y^2 + 4x + 4y + 4 = 0$. Then the radius of the circle $S=0$ is

[EAPCET 13-05-23 SHIFT-2]

1. 4

2. 3

3. 5

4. 1

- 79 If the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ cuts each of the three circles $x^2 + y^2 + 4x + 4y + 7 = 0$, $x^2 + y^2 - 4x + 4y + 7 = 0$ and $x^2 + y^2 - 4x - 4y + 7 = 0$ orthogonally, then the equation of the tangent drawn at the point $(\sqrt{3}, 2)$ to the circle $S = 0$ is

[EAPCET 13-05-23 SHIFT-2]

1. $(\sqrt{3}-1)x + 4y + (\sqrt{3}-1) = 0$

2. $\sqrt{3}x + 2y - 7 = 0$

3. $(\sqrt{3}+2)x + 3y + (\sqrt{3}+1) = 0$

4. $\sqrt{3}x - 2y + 7 = 0$

KEY

1)	4	2)	4	3)	2	4)	4	5)	4
6)	1	7)	4	8)	1	9)	2	10)	2
11)	3	12)	3	13)	1	14)	1	15)	3
16)	3	17)	3	18)	3	19)	3	20)	4

21)	3	22)	4	23)	2	24)	1	25)	4
26)	3	27)	3	28)	1	29)	4	30)	1
31)	1	32)	2	33)	3	34)	2	35)	1
36)	1	37)	4	38)	2	39)	2	40)	1
41)	2	42)	1	43)	4	44)	3	45)	2
46)	2	47)	4	48)	1	49)	3	50)	1
51)	1	52)	1	53)	2	54)	4	55)	1
56)	2	57)	3	58)	2	59)	2	60)	1
61)	1	62)	4	63)	3	64)	4	65)	2
66)	4	67)	1	68)	3	69)	3	70)	2
71)	1	72)	3	73)	3	74)	2	75)	3
76)	3	77)	4	78)	1	79)	2	80)	

SOLUTIONS

1. Radical axis is $S - S' = 0$

$$\Rightarrow x = 2$$

$$\therefore C(2, 0)$$

$$\text{Minimum radius is } \sqrt{4 - 24 + 35} = \sqrt{15}$$

2. Limiting points $(1, 2)$ and $(-2, 1)$ so radical axis perpendicular bisector of the line joining the

$$\text{limiting points } P = \left(\frac{-1}{2}, \frac{3}{2} \right)$$

$$\text{Line passing through } P = \left(\frac{-1}{2}, \frac{3}{2} \right) \text{ and having}$$

$$\text{slope '-3' is } 3x + y = 0$$

- 3 The radical axis of the two circles are perpendicular to the joining their centres

$$4 \quad 2(gg' + ff') = c + c' \quad K=1 \quad d^2 = r_1^2 + r_2^2 \quad d = \sqrt{5}$$

$$x^2 + y^2 + 6x - 2y + k = 0 \dots (1)$$

$$5. \quad x^2 + y^2 + 2x - 6y - 15 = 0 \dots (2)$$

$$\text{Equation of radical axis is } (1) - (2) = 0$$

$$4x + 4y + k + 15 = 0$$

$$\text{Centre of equation (2) lies on radical axis}$$

$$4(-1) + 4(3) + k + 15 = 0$$

$$k = -23$$

- 6 The two circles cut orthogonally

$$2gg' + 2ff' = c + c' \quad \text{We get } k = -24$$

$$S \equiv x^2 + y^2 + 2x + 3y + 1 = 0$$

$$S' \equiv x^2 + y^2 - 5x - 6y + 4 = 0$$

Equation of common chord, $S - S' = 0$
 $\Rightarrow 7x + 9y - 3 = 0$

$$8 \quad x^2 + y^2 - 4x - 6y + k = 0 \quad x^2 + y^2 + 8x - 4y + 11 = 0$$

$$C_1 = (2, 3), r_1 = \sqrt{13 - k} \quad C_2 = (-4, 2), r_2 = 3$$

$$d = C_1C_2 = \sqrt{37}$$

$$\cos \theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1r_2}$$

$$k^2 + 39k + 108 = 0$$

$$K = -36, -3$$

9 Conceptual

$$S \equiv x^2 + y^2 - 6x - 6y + 4 = 0$$

$$10 \quad S' \equiv x^2 + y^2 - 2x - 4y + 3 = 0$$

Required circle is

$$s'' = x^2 + y^2 + 2gx + 2fy + c = 0$$

$$s - s'' = (2g + 6)x + (2f + 6)y + c - 4 = 0 \rightarrow (1)$$

$$s' - s'' = (2g + 2)x + (2f + 4)y + c - 3 = 0 \rightarrow (2)$$

(1) and (2) samelines

$$\frac{2g+6}{2g+2} = \frac{2f+6}{2f+4} = \frac{c-4}{c-3}$$

$$\frac{g+3}{g+1} = \frac{f+3}{f+2} = \frac{c-4}{c-3} = k$$

$$g = \frac{k-3}{1-k}, f = \frac{2k-3}{1-k}, c = \frac{-3k+4}{1-k}$$

$$r^2 = 14$$

$$g^2 + f^2 - c = 14$$

$$12k^2 - 17k = 0$$

$$k = 0, k = \frac{17}{12}$$

$$\text{If } k = 0 \text{ then } g = -3, f = -3, c = 4$$

$$\text{If } k = \frac{17}{12} \text{ then } g = \frac{19}{5}, f = \frac{2}{5}$$

$$\therefore \text{Centre} = (-g, -f) = \left(\frac{-19}{5}, \frac{-2}{5}\right)$$

$$11 \quad C + \lambda L = 0$$

$$x^2 + y^2 + (-8 + 2\lambda)x + (-2 + 13\lambda)y + (16 - 15\lambda) = 0 \dots (1)$$

$$x^2 + y^2 - 4x - 4y - 1 = 0 \dots (2)$$

Circles (1) and (2) cuts orthogonally

$$2(\lambda - 4)(-2) + 2\left(\frac{13\lambda - 2}{2}\right)(-2) = 16 - 15\lambda - 1$$

$$5 - 15\lambda = 0$$

$$\lambda = \frac{1}{3}$$

$$(1) \Rightarrow 3x^2 + 3y^2 - 22x + 7y + 33 = 0$$

$$x^2 + y^2 - \frac{22}{3}x + \frac{7}{3}y + 11 = 0$$

$$g = -\frac{11}{3}, f = \frac{7}{6}$$

$$\text{Centre} = \left(\frac{11}{3}, -\frac{7}{6}\right)$$

$$12. \quad g_1 = \frac{k}{2}; f_1 = -2; c_1 = -1$$

$$g_2 = -\frac{7}{3}; f_2 = \frac{23}{6}; c_2 = -\frac{15}{3}$$

$$\text{Given } 2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

$$2\left(\frac{k}{2}\right)\left(-\frac{7}{3}\right) + 2(-2)\left(\frac{23}{6}\right) = -1 - \frac{15}{3}$$

$$k = -4$$

Required equation of circle is

$$S + \lambda S' = 0$$

$$x^2 + y^2 - 4x - 4y - 1 +$$

$$\lambda(3x^2 + 3y^2 - 14x + 23y - 15) = 0 \rightarrow (1)$$

$$(x, y) = (-1, -1)$$

$$\lambda = \frac{1}{2}$$

$$(1) \Rightarrow 5x^2 + 5y^2 - 22x + 15y - 17 = 0$$

13. Orthogonal condition

$$\Rightarrow c = -4g - 6f - 11 \text{ and } c = -10g - 4f - 21$$

$$\Rightarrow 3g - f + 5 = 0 \rightarrow (1)$$

$$\text{Also } (-g, -f) \text{ lies on } y = x \Rightarrow -g = -f \rightarrow (2)$$

$$\text{From (1) \& (2) } \Rightarrow g = f = -\frac{5}{2}$$

$$\therefore c = 14 \Rightarrow 2g + 2f + c = 4$$

14 $c_1 = (0, 0), c_2 = (h, k)$

$$(x - h)^2 + (y - k)^2 = 5^2$$

$$S \equiv x^2 + y^2 - 2hx - 2ky + h^2 + k^2 = 25$$

$$S_1 \equiv x^2 + y^2 = 16$$

$$S - S_1 = 0 \Rightarrow 2hx + 2ky - h^2 - k^2 = -9$$

$$\text{Slope of chord} = \frac{3}{4}$$

$$\text{Slope of required line} = \frac{-4}{3}$$

passing through $(0, 0)$

$$y = -\frac{4}{3}x \Rightarrow \frac{y}{x} = -\frac{4}{3}$$

$$P = \left| \frac{-h^2 - k^2 + 9}{4h^2 + 4k^2} \right| = 0$$

$$h^2 + k^2 = 9$$

$$h^2 + \frac{16h^2}{9} = 9 \Rightarrow h = \pm \frac{9}{5}$$

$$\therefore \left(\frac{9}{5}, -\frac{12}{5} \right) \text{ or } \left(-\frac{9}{5}, \frac{12}{5} \right)$$

15. Given

$$S \equiv x^2 + y^2 + 6x + 4y - 12 = 0$$

$$S' \equiv x^2 + y^2 - 4x - 6y - 12 = 0$$

$$\text{Here } L \equiv S - S' = 0 \Rightarrow x + y = 0$$

Required circle is

$$S + \lambda L = 0$$

$$\Rightarrow x^2 + y^2 + (6 + \lambda)x + (4 + \lambda)y - 12 = 0$$

Which cuts orthogonally the other circle.

$$x^2 + y^2 + 2x - 4y - 11 = 0$$

By using orthogonality condition, we can write

$$2 \left[3 + \frac{\lambda}{2} - 4 - \lambda \right] = -23$$

$$\Rightarrow \lambda = 21$$

Required circle is

$$x^2 + y^2 + 27x + 25y - 12 = 0$$

$$\text{and its centre is } \left(\frac{-27}{2}, \frac{-25}{2} \right)$$

16. Equation of radical axis passing through the mid points of AB and mid points of CD

17 The locus of centre of a circle which cuts orthogonally given two circles is radical axes. Equation of radical axis $x - 2y - 7 = 0$. It is diameter of smallest circle. By verification centre $(3, -2)$ lies on $x - 2y - 7 = 0$

18 $x^2 + y^2 - a^2 + \lambda x \cos \alpha + \lambda y \sin \alpha - p\lambda = 0$

$$x^2 + y^2 + \lambda x \cos \alpha + \lambda y \sin \alpha - (p\lambda + a^2) = 0$$

If it is a small circle then intersecting line is diameter.

$$\Rightarrow \lambda = -2p$$

19. Equation of tangents are

$$S_1^2 = S S_{11}$$

$$(gx + fy + g^2)^2 = g^2(x^2 + y^2 + 2gx + 2fy + g^2) = 0$$

$$\Rightarrow y[(g^2 - f^2)y - 2gfy] = 0$$

20. $S \equiv x^2 + y^2 - 2x - 6y + 3 = 0$ $L = 2x + y$

Equation of a circle be $S + \lambda L = 0$

Centre $\left(1 - \lambda, \frac{6 - \lambda}{2}\right)$

This centre lies on $L=0$

$$2 - 2\lambda + \frac{6 - \lambda}{2} = 0$$

$$\Rightarrow \lambda = 2$$

equation of circle be

$$x^2 + y^2 + 2x - 4y + 3 = 0$$

Which satisfies $(-2, 1)$

21 Equation of radical axis is

$$\left(2\alpha - \frac{3}{2}\right)x + (2\beta - 4)y = 0$$

Which touches the circle

$$x^2 + y^2 + 2x + 2y + 1 = 0 \text{ then } r = d$$

$$1 = \frac{\left(2\alpha - \frac{3}{2}\right)(-1) + (2\beta - 4)(-1)}{\sqrt{\left(2\alpha - \frac{3}{2}\right)^2 + (2\beta - 4)^2}}$$

After simplification we get

$$4\alpha\beta - 8\alpha - 3\beta + 10 = 4$$

22. Given centre $(1, 2)$

$$S_1 \equiv x^2 + y^2 + 4x + 16y - 30 = 0$$

$$S_2 \equiv x^2 + y^2 + 2gx + 2fy + c = 0$$

Orthogonally $2gg' + 2ff' = c + c'$

$$\Rightarrow c = -6, r = \sqrt{1 + 4 + 6} = \sqrt{11}$$

23. Given that

$$S \equiv x^2 + y^2 - 8x + 40 = 0$$

$$S' \equiv x^2 + y^2 - 5x + 16 = 0$$

$$S'' \equiv x^2 + y^2 - 8x + 16y + 160 = 0$$

$$S - S' = 0 \Rightarrow x = 8$$

$$S' - S'' = 0 \Rightarrow y = \frac{-15}{2} \therefore (x, y) = \left(8, \frac{-15}{2}\right)$$

24. IPE

$$25 \quad S - S' = 0 \Rightarrow x - y = 0$$

$$C_1\left(\frac{-3}{2}, \frac{-5}{2}\right), r_1 = \frac{3}{\sqrt{2}}, d = \frac{1}{\sqrt{2}}$$

$$\text{Common chord} = 2\sqrt{r^2 - d^2} = 4$$

$$S \equiv x^2 + y^2 - 8x - 6y + 21 = 0$$

$$26 \quad S' \equiv x^2 + y^2 - 2x - 15 = 0$$

$$P(1, 2), S + \lambda S' = 0 \Rightarrow \lambda = \frac{1}{2}$$

$$2(x^2 + y^2) - 18x - 12y + 27 = 0$$

$$S \equiv x^2 + y^2 + 4x + 6y + 7 = 0$$

$$27. \quad S' \equiv 4x^2 + 4y^2 + 8x + 12y - 9 = 0$$

$$S - S' = 0 \Rightarrow 8x + 12y + 37 = 0$$

$$|r_1 - r_2| < C_1 C_2 < r_1 + r_2$$

28

It has two real and distinct points

$$29 \quad x^2 + y^2 - 2x - 6y - 15 = 0, y = 0$$

$$x^2 - 2x - 15 = 0 \Rightarrow x = -3, 5$$

$$A(-3, 0), B(5, 0) \Rightarrow AB = 8$$

$$S \equiv x^2 + y^2 - 4x - 6y + 5 = 0$$

30

$$S' \equiv x^2 + y^2 - 2x - 4y - 1 = 0$$

$$S - S' = 0 \Rightarrow L_1 \equiv x + y - 3 = 0$$

$$S'' \equiv x^2 + y^2 + 2x + 2y - 7 = 0$$

$$S''' \equiv x^2 + y^2 + x + y + 9 = 0$$

$$S'' - S''' = 0 \Rightarrow L_2 \equiv x + y - 16 = 0$$

L_1 is parallel to L_2

$$\text{Centre} = (4, 3)$$

31

$$S - S' = 0 \Rightarrow 2x + 2y - 14 = 0$$

$$C \text{ lies on } 2x + 2y - 14 = 0$$

$$\therefore \text{Required eqn is } x^2 + y^2 - 6x - 4y + 9 = 0$$

$$32. S \equiv x^2 + y^2 - 6x - 8y - 12 = 0$$

$$S' \equiv x^2 + y^2 - 4x + 6y + k = 0, \theta = 90^\circ$$

$$0 = c_1 + c_2 - 2(g_1g_2 + f_1f_2) \Rightarrow k = 0$$

$$33. S \equiv x^2 + y^2 - 8x - 6y + 21 = 0$$

$$S' \equiv x^2 + y^2 - 2x - 15 = 0$$

$$P(1, 2), S + \lambda S' = 0 \Rightarrow \lambda = \frac{1}{2}$$

$$2(x^2 + y^2) - 18x - 12y + 27 = 0$$

34. IPE

35. IPE

$$36. S - S' = 0 \Rightarrow 2x - y - 1 = 0, S + \lambda L = 0$$

$$(\alpha, \beta) = \left(-\lambda, \frac{\lambda}{2}\right) \Rightarrow \alpha = -\lambda, \beta = \frac{\lambda}{2}$$

$$\Rightarrow \beta = \frac{-\alpha}{2} \Rightarrow 2\beta = -\alpha$$

$$\Rightarrow \alpha + 2\beta = 0 \Rightarrow x + 2y = 0$$

$$37. (i) \Rightarrow S \equiv x^2 + y^2 + ax + by + c = 0$$

$$S' \equiv x^2 + y^2 + bx + ay + c = 0$$

$$S - S' = 0 \Rightarrow x - y = 0, C\left(\frac{-a}{2}, \frac{b}{2}\right)$$

$$r = \frac{\sqrt{a^2 + b^2 - 4c}}{2}, d = \frac{|b - a|}{2\sqrt{2}}$$

$$2\sqrt{r^2 - d^2} = \frac{\sqrt{(a+b)^2 - 8c}}{2\sqrt{2}}$$

(ii) conceptual

38. IPE problem

$$39. \text{ Given circles } x^2 + y^2 - 8x - 2y + 8 = 0 \dots (1)$$

$$x^2 + y^2 + 6x + 8y - 24 = 0 \dots (2)$$

$$x^2 + y^2 - 2x + 2y + 2 = 0 \dots (3)$$

Radical axis of (1) and (2) is

$$7x + 5y - 16 = 0 \dots (A)$$

Radical axis of (2) and (3) is

$$4x + 3y - 13 = 0 \dots (B)$$

Radical centre is $(-17, 27) = (a, b)$

$$a + b = 10$$

$$40. R.A, L = 22x + y + 25 = 0$$

$$S + \lambda L = 0$$

$$x^2 + y^2 + 13x - 3y + \lambda(22x + y + 25) = 0$$

$$12 + \lambda(48) = 0$$

$$\lambda = \frac{-1}{4}$$

$$\Rightarrow 4x^2 + 4y^2 + 30x - 13y - 25 = 0$$

$$41. \left(\frac{4-6}{2}, \frac{7-9}{2}\right) = (-1, -1) = (\alpha, \beta)$$

$$\therefore \alpha + \beta = -1 - 1 = -2$$

42. Diagram

$$S = x^2 + y^2 + 2x + 3y + 1 = 0$$

$$S' = x^2 + y^2 + 4x + 3y + 2 = 0$$

Equation of the circle $S + \lambda S' = 0$

$$x^2 + y^2 + 2x + 3y + 1$$

$$+ \lambda(x^2 + y^2 + 4x + 3y + 2) = 0 \dots (1)$$

It passes through $(-1, 1)$

$$\lambda = \frac{-4}{3}$$

Put λ value in (1)

$$\Rightarrow x^2 + y^2 + 2x + 3y$$

$$+ 1 - \frac{4}{3}(x^2 + y^2 + 4x + 3y + 2) = 0$$

$$x^2 + y^2 + 10x + 3y + 5 = 0$$

43. Given circle $x^2 + y^2 + 2kx + 4y - 4 = 0 \dots (1)$

Centre $C_1 = (-k, -2), r_1 = \sqrt{k^2 + 4 + 4} = \sqrt{k^2 + 8}$

C_1 lies in 4th quadrant $-k > 0 \Rightarrow k < 0$.

$x^2 + y^2 + 6x - 2y + 6 = 0 \dots (2)$

$C_2 = (-3, 1), r_2 = \sqrt{9 + 1 - 6} = 2$

Circle (1) touches circle (2).

$C_1 C_2 = r_1 \pm r_2$

$\sqrt{(k-3)^2 + 3^2} = \sqrt{k^2 + 8} \pm 2$

S.O.B.S

$(k-3)^2 + 9 = k^2 + 8 + 4 \pm 4\sqrt{k^2 + 8}$

$-3(k-1) = \pm 2\sqrt{k^2 + 8}$

Again S.O.B.S

$9(k-1)^2 = 4(k^2 + 8)$

$\Rightarrow 5k^2 - 18k - 23 = 0$

$\Rightarrow k = \frac{23}{5}, k = -1$

$\therefore \boxed{k = -1}$

44. Let $C(h, k)$ be locus of centre of circles

$d^2 = r_1^2 + r_2^2$

$(h-1)^2 + (k+2)^2 = (h-a)^2 + (k-b)^2 + 9$

$(a-1)h + (b+2)k = \frac{a^2 + b^2 + 4}{2}$

The locus of $P(h, k)$ is

$(a-1)x + (b+2)y = \frac{a^2 + b^2 + 4}{2}$

45. $x^2 + y^2 + \frac{2g\lambda}{1+\lambda}x + \frac{2f}{1+\lambda}y + \frac{k\lambda}{a+\lambda} = 0$

Centre $= \left(\frac{-g\lambda}{1+\lambda}, \frac{-f}{1+\lambda} \right)$

For point circles radius is zero

$\frac{g^2\lambda^2}{1+\lambda^2} + \frac{f^2}{1+\lambda^2} - \frac{k\lambda}{1+\lambda} = 0$

$(g^2 - k)\lambda^2 - k\lambda + f^2 = 0$

$\lambda_1\lambda_2 = \frac{f^2}{g^2 - k}$

$A \left(\frac{-g\lambda_1}{1+\lambda_1}, \frac{-f}{1+\lambda_1} \right), B \left(\frac{-g\lambda_2}{1+\lambda_2}, \frac{-f}{1+\lambda_2} \right)$

Slope of $OA \times$ slope of $OB = -1$

$-g^2 + k = g^2 \Rightarrow g^2 = \frac{k}{2}$

46. Given circles are

$x^2 + y^2 - 2x - 9 = 0, x^2 + y^2 - 4y - 1 = 0$

$C_1 = (1, 0), C_2 = (0, 2) \Rightarrow d = \sqrt{5}$

$r_1 = \sqrt{10}, r_2 = \sqrt{5}$

$\cos \theta = \frac{|d^2 - r_1^2 - r_2^2|}{2r_1r_2} = \frac{1}{\sqrt{2}}$

$\therefore \text{Angle } \theta = \frac{\pi}{4}$

47. Two circles touch at $(0, 0)$ externally.

$\therefore$ Radical axis is their common tangent at $(0, 0)$.

48. Required circle equation is $S + \lambda L = 0$

$(x^2 + y^2 - a^2) + \lambda(x \cos \alpha + y \sin \alpha - P) = 0$

$C = \left(\frac{-\lambda \cos \alpha}{2}, \frac{-\lambda \sin \alpha}{2} \right)$

C lies on $x \cos \alpha + y \sin \alpha = P$

$\Rightarrow \frac{-\lambda}{2} = P \Rightarrow \lambda = -2P$

49 Radical axis $s - s' = 0$

$$\Rightarrow x - y = 0$$

Image of (3,4) w.r.to $x=y$ is (4,3)

50 Radical centre of s, s' is

$$s - s' = 0 \Rightarrow 8x - 16 = 0$$

$$\Rightarrow x = 2$$

Radical centre of s', s''

$$\text{is } s' - s'' = 0$$

$$-8x - 10y - 9 = 0$$

$$\Rightarrow -8(-2) - 10y - 9 = 0$$

$$\Rightarrow y = \frac{-5}{2}$$

$$R.C = \left(2, \frac{-5}{2}\right)$$

51 $s - s' = 0$

$$\Rightarrow 4x - 4y - 4 = 0$$

$$x - y - 1 = 0 \rightarrow (1)$$

$$s' - s'' = 0$$

$$\Rightarrow -6x + 14y - 10 = 0$$

$$\Rightarrow 3x - 7y + 5 = 0 \Rightarrow 3x - 7y + 5 = 0 \rightarrow (2)$$

Solve (1) & (2) $\Rightarrow (3, 2)$

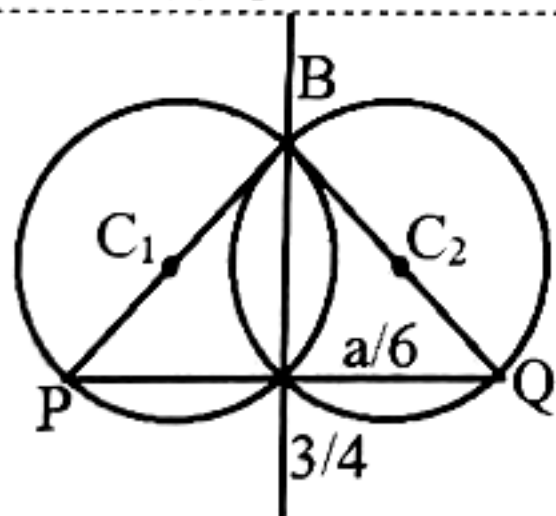
52 $s - s' = 0$

$$\Rightarrow 6x + 8y - 5 = 0 \dots (1)$$

Centre lies on eq(1) we get

$\therefore$ By verification (1) option

53

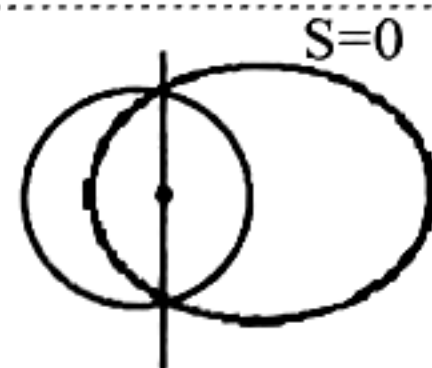


$$\frac{a}{b} = \frac{-4}{3}$$

$$3a = -4b$$

$$3a + 4b = 0$$

54



Req. circle $s + \lambda l = 0$

$$(x^2 + y^2 - a^2) + \lambda(x \cos \alpha + y \sin \alpha - p) = 0$$

$$\rightarrow (1)$$

$$x^2 + y^2 + (\lambda \cos \alpha)x + \lambda \sin \alpha y - \lambda p - a^2 = 0$$

$$\text{Centre} \left(\frac{-\lambda \cos \alpha}{2}, \frac{-\lambda \sin \alpha}{2} \right)$$

$$\frac{-\lambda \cos \alpha}{2} \cdot \cos \alpha - \frac{\lambda \sin \alpha}{2} \cdot \sin \alpha = P$$

$$\Rightarrow -\frac{\lambda}{2}(1) = P \Rightarrow$$

$$\Rightarrow \lambda = -2p$$

$$(1) \Rightarrow x^2 + y^2 - a^2 - 2$$

$$P(x \cos \alpha + y \sin \alpha - P) = 0$$

55 Given

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$c + c' = 2gy' + 2ff'$$

$$C - 4 = 0$$

$$C = 4$$

$$c + c'' = 2gg'' + 2ff''$$

$$g = -\frac{1}{2}, f = -1\frac{1}{2}$$

$$x^2 + y^2 - x - y + 4 = 0$$

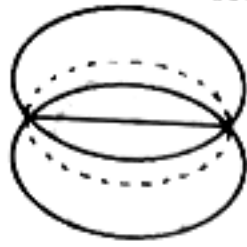
$$r = \sqrt{\frac{1}{4} + \frac{1}{4} - 4} = \sqrt{\frac{1}{2} - 4}$$

But $r < 0$

It does n't exist a circle

Ans : 0

56.



$$\text{Let } s - s' = 0$$

$$\Rightarrow 4x + 4y + 5 = 0 \rightarrow (1)$$

$$\text{Req. circle } s + \lambda s' = 0$$

$$\Rightarrow x^2 + y^2 + 4y + \lambda(x^2 + y^2 - 4x - 5) = 0$$

$$\Rightarrow (1+\lambda)x^2 + (1+\lambda)y^2 + (-4\lambda)x + 4y - 5\lambda = 0$$

$$\text{Centre } \left(\frac{2\lambda}{1+\lambda}, \frac{-2}{1+\lambda} \right)$$

Lies on (1)

$$\frac{8\lambda}{1+\lambda} - \frac{-2}{1+\lambda} + 5 = 0$$

$$\frac{2\left(\frac{3}{13}\right)}{1+\frac{3}{13}} = \frac{6}{16} \Rightarrow 8\lambda - 8 = -5 - 5\lambda$$

$$= \frac{3}{8} \Rightarrow 13\lambda = 3$$

$$\lambda = \frac{3}{13}$$

57. Let $s - s' = 0$

$$\Rightarrow \alpha x + (\alpha - 6)y + 6 = 0$$

$$\left(0, \frac{3}{4}\right) \text{ lies on this}$$

$$\Rightarrow 0 + (\alpha - 6)\left(\frac{3}{4}\right) + 6 = 0$$

$$\Rightarrow 0 + (\alpha - 6)3 + 24 = 0$$

$$\Rightarrow (\alpha - 6)3 + 24 = 0 \Rightarrow \alpha = -2$$

$$s' = x^2 + y^2 - 4x - 2y + 6 = 0$$

$$C = (2, 1), P = \left(0, \frac{3}{4}\right)$$

$$CP = \sqrt{2^2 + \left(1 + \frac{3}{4}\right)^2}$$

$$= \sqrt{4 + \frac{1}{16}} = \frac{\sqrt{5}}{4}$$

58. Given $S \equiv x^2 + y^2 + 2yx + 2fy + c = 0$

$$S' \equiv x^2 + y^2 + 6x - 4y + 4 = 0$$

$$s - s' = 0$$

$$(2g+6)x + (2f+4)y + c - 4 = 0$$

$$x + y = 0 \rightarrow c = 4$$

$$r = 1$$

$$g^2 + f^2 - c = 1$$

$$g^2 + f^2 - 4 = 1$$

$$g^2 + f^2 - 5 = 0$$

$$2g + 6 = 2f + 4$$

$$g = f - 1$$

from eqⁿ (1)

$$(f-1)^2 + f^2 = 5$$

$$f = -1, 2$$

$$g = -2, 1$$

$$g + f = \pm 3$$

59. $2[g(2) + f(-2)] = c + 3$

$$2[g(2) + f(2)] = c + 3$$

$$f = 0, g = 0$$

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{3}$$

60. Given $s = x^2 + y^2 + 2gx + 2fy + c = 0$

$$s' = x^2 + y^2 - 6x - 12y + 1 = 0$$

$$2(gg' + ff') = c + c'$$

$$2(g(-3) + f(-6)) = c + 1$$

$$-6y - 12f = c + 1 \rightarrow (1)$$

$$\text{Centre } (-4, 2) = (-g, -f)$$

$$g = 4, f = -2$$

$$-24 + 24 = c + 1$$

$$c + 1 = 0$$

$$c = -1$$

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{16 + 4 + 1}$$

$$= \sqrt{21}$$

$$61 \quad r_1 = \sqrt{4+16-4} = 4, C(2,4)$$

$$d = \frac{|2+4-3|}{\sqrt{1+1}} = \frac{3}{\sqrt{2}}$$

$$\Rightarrow 2\sqrt{r^2 - d^2} = 2\sqrt{16 - \frac{9}{2}}$$

$$\Rightarrow \sqrt{2} \sqrt{23} \\ = \sqrt{46}$$

62. Given

$$s = x^2 + y^2 + 2gx + 2fy + c = 0$$

Center(h, k)

$$x^2 + y^2 - 2hx - 2ky + c = 0$$

$P(0,0)$

$$s = x^2 + y^2 - 2hx - 2ky = 0$$

$$s' = x^2 + y^2 + 4x + 6y + 12 = 0$$

$$s'' = x^2 + y^2 + 4x - 6y + 9 = 0$$

Orthogonally

$$2[gg' + ff'] = c + c'$$

$$K = \frac{-1}{4}, h = -\frac{21}{8}$$

$$K = 2h = \frac{-1}{4} + \frac{21}{4} = \frac{20}{4} = 5$$

63. Given

$$S = x^2 + y^2 + 2gx - 4y + 4 = 0$$

$$s' = x^2 + y^2 + 6x + 2fy + 12 = 0$$

$$s'' = x^2 + y^2 + 10y + 20 = 0$$

$$s' - s = 0$$

$$(g-3)x - 2g + fy - 4 = 0$$

Point $(-1, -1)$

$$-3g + f - 1 = 0$$

$$s' - s'' = 0$$

$$3x + (f-5)y - 4 = 0$$

point $(-1, -1)$

$$f = -2, g = -2$$

$$\text{Then } g-f = -1+2=1$$

$$64. \quad S = x^2 + y^2 + 2gx + 2fy + c = 0$$

$$S' = x^2 + y^2 - 2x + 2y - 2 = 0$$

$$S'' = x^2 + y^2 + 4x - 6y + 9 = 0$$

$S = 0$ & $S' = 0$ cuts orthogonally

$$\Rightarrow 2g(-1) + 2f(1) = c - 2$$

$$\Rightarrow -2g + 2f = c - 2 \rightarrow 1$$

$S = 0$ & $S'' = 0$ cuts orthogonally

$$\Rightarrow 2g(2) + 2f(-3) = c + 9$$

$$\Rightarrow 4g - 6f = c + 9 \rightarrow 2$$

$(-g, -f)$ lies on $2x+3y-2=0$

$$2g + 3f + 2 = 0 \rightarrow 3$$

$$\text{from 1 and 2} \Rightarrow 6g - 8f = 11 \rightarrow 4$$

$$\text{from 3 and 4} \Rightarrow 4 \rightarrow \cancel{6g} - 8f = 11$$

$$3 \times 3 \rightarrow \cancel{6g} + 9f = -6 \\ \hline -17f = 17$$

$$\boxed{f = -1} \quad \boxed{g = \frac{1}{2}}$$

$$\boxed{c = 1}$$

$$\therefore 2g + f = 1 - 1 = 0 = c - f$$

65

$$A(1,2) \quad r_1 = 3 \quad B(-1,-1) \quad r_2 = r, \theta = \frac{\pi}{3}$$

$$d = AB = \sqrt{4+9} = \sqrt{13}$$

$$\cos \theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1r_2} \Rightarrow \cos \frac{\pi}{3} = \frac{13 - 9 - r^2}{2(3)r}$$

$$\Rightarrow \pm \frac{1}{2} = \frac{4 - r^2}{2(3)r}$$

$$\Rightarrow \pm 3r = 4 - r^2$$

$$r^2 + 3r - 4 = 0 \text{ (or) } r^2 - 3r - 4 = 0$$

$$r = 1 \text{ or } -4 \quad r = 4 \text{ or } -1$$

No. of possible values (r) = 2

$$66 \quad s = x^2 + y^2 - 6x + 2ky + 1 = 0$$

$$s^1 = x^2 + y^2 + 2kx - 6y - 7 = 0$$

$$c(3, -k)$$

$$r = \sqrt{k^2 + 8}$$

$$c^1(-k, 3)$$

$$r^1 = \sqrt{k^2 + 16}$$

$$cc^1 = \sqrt{(k+3)^2 + (k+3)^2} = \sqrt{2}(k+3)$$

$$d^2 = r^2 + r_1^2$$

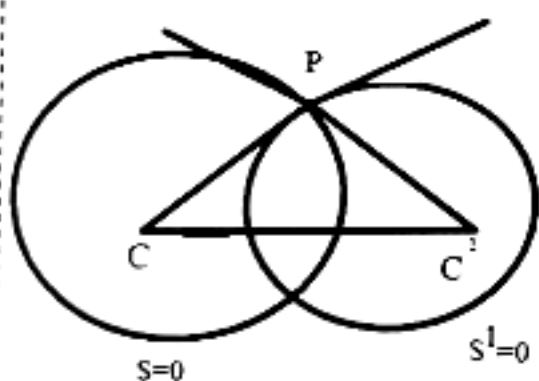
$$2(k+3)^2 = 2k^2 + 24$$

$$k = \frac{1}{2}$$

$$c^1 = \left(\frac{-1}{2}, 3 \right)$$

$$r^1 = \sqrt{\frac{1}{4} + 9 + 7}$$

$$= \frac{\sqrt{65}}{2}$$



$$67. \quad C_1 = (-1, 2), \quad C_2 = (2, 1)$$

$$r_1 = \sqrt{1+4-1}, \quad r_2 = \sqrt{4+1-C}$$

$$r_1 = 2, \quad r_2 = \sqrt{5-C}$$

$$C_1C_2 = \sqrt{9+1}$$

$$d = \sqrt{10}$$

$$\cos 45^\circ = \frac{10-4-5+C}{2(2)\sqrt{5-C}}$$

$$8(5-C) = 1+C^2+2C$$

$$C^2+10C-39=0$$

$$(C-3)(C+13)=0$$

$$C=3 \text{ (or) } C=-13$$

not satisfied

$$68 \quad 2[(-2)(g) + f(-2)] = 0$$

$$g = -f \rightarrow \textcircled{1}$$

$$r_1^2 + r_2^2 - 8 = 4 + f^2 - 1 + g^2 + 5 - 8$$

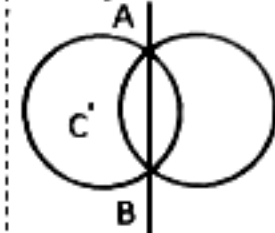
$$= f^2 + g^2$$

$$= 2g^2$$

from $\textcircled{1}$

$$69 \quad x^2 + y^2 - 4x + 6y - 3 = 0 \rightarrow (1)$$

$$x^2 + y^2 + 2x - 2y - 2 = 0 \rightarrow (2)$$



$$(1) - (2) \rightarrow -6x + 8y - 1 = 0$$

$$6x - 8y + 1 = 0$$

$$G = (2, -3), \quad r = \sqrt{4+9+3} = \sqrt{16} = 4$$

$$\text{Length of chord as } AB = 2\sqrt{r^2 - d^2}$$

$$d = \left| \frac{6(2) - 8(-3) + 1}{10} \right| = \frac{37}{10}$$

$$\text{length of chord} = \frac{2}{10} \sqrt{231}$$

$$70 \quad x+y+2=0$$

$$x^2 + y^2 + 4x - 4 + \lambda(x+y+2) = 0$$

$$\left(\frac{-(4+\lambda)}{2}, \frac{-(-4+\lambda)}{2} \right)$$

$$\lambda=1$$

$$71 \quad \text{Given circles are}$$

$$s \equiv x^2 + y^2 + 2x + 3y + 1 = 0 \text{ --- (1)}$$

$$s^1 \equiv x^2 + y^2 + 4x + 3y + 2 = 0 \text{ --- (2)}$$

The common chord of (1) and (2) is

$$2x+1=0 \text{ --- (3)}$$

Required circle is $s + \lambda L = 0$

$$x^2 + y^2 + 2x + 3y + 1 + \lambda(2x+1) = 0$$

$$\left(-(\lambda+1), \frac{-3}{2} \right) \text{ lies on the (3)}$$

Centre

$$\therefore \lambda = \frac{-1}{2}$$

$$\therefore \text{Required circles is } 2x^2 + 2y^2 + 2x + 6y + 1 = 0$$

72 Given circles are

$$x^2 + y^2 - 5x + 6y + 12 = 0 \dots (1)$$

$$x^2 + y^2 + 6x - 4y - 14 = 0 \dots (2)$$

Radical axis of (1) and (2) is

$$11x - 10y + 24 = 0$$

Equation of any line perpendicular to the radical axis is

$$10x + 11y + k = 0$$

This line passing through (1,1)

$$10 + 11 + k = 0$$

$$k = -21$$

Required line is $10x + 11y - 21 = 0$

73 Given circles are

$$x^2 + y^2 - 2x - 4y + c = 0$$

$$x^2 + y^2 - 4x - 2y + 4 = 0$$

Here $g = -1$, $f = -2$, $C = c$

$$g^1 = -2, f^1 = -1, c^1 = 4$$

$$\theta = 60^\circ$$

$$\cos \theta = \frac{c + c^1 - 2gg^1 - 2ff^1}{2\sqrt{g^2 + f^2 - c}\sqrt{g^{1^2} + f^{1^2} - c^1}}$$

$$\therefore C = \frac{7 \pm \sqrt{5}}{2}$$

$$74 \quad \cos \frac{\pi}{3} = \frac{1 + k - 2(-1)(1) - 2(1)(-1)}{2\sqrt{1 + 1 - k}\sqrt{1 + 1 - k}}$$

$$\frac{1}{2} = \frac{1 + k + 2 + 2}{2\sqrt{2 - k}} = \frac{k + 5}{\sqrt{2 - k}} = 1$$

$$k + 5 = \sqrt{2 - k}$$

$$k^2 + 25 + 10k = 2 - k$$

$$k^2 + 10k + k + 23 = 0$$

$$k^2 + 11k + 23 = 0$$

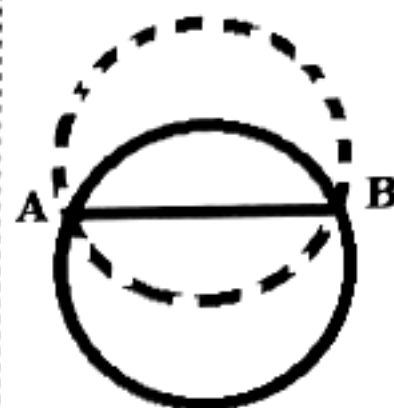
$$k = \frac{-11 \pm \sqrt{121 - 92}}{2}$$

$$k = \frac{-11 \pm \sqrt{29}}{2} \text{ (irrational number)}$$

$$75. S + \lambda L = 0$$

$$(x^2 + y^2 + 2x + 2y + 1) + \lambda(x - y + 1) = 0$$

$$x^2 + y^2 + x(2 + \lambda) + y(2 - \lambda) + 1 + \lambda = 0$$



$$c \left(-\frac{(2 + \lambda)}{2}, \frac{\lambda - 2}{2} \right) \text{ lies on } x - y + 1 = 0$$

$$\frac{-2 - \lambda}{2} + \frac{2 - \lambda}{2} + 1 = 0 \Rightarrow -2 - \lambda + 2 - \lambda + 2 = 0$$

$$-2\lambda + 2 = 0$$

$$2\lambda = 2$$

$$\lambda = 1$$

$$(x^2 + y^2 + 2x + 2y + 1)$$

$$+ 1(x - y + 1) = 0$$

$$x^2 + y^2 + 2x + 2y + 1 + x - y + 1 = 0$$

$$x^2 + y^2 + 3x + y + 2 = 0, 2g = 3$$

$$g = \frac{3}{2}, f = \frac{1}{2}, c = 2$$

$$g + f = \frac{3}{2} + \frac{1}{2} = 2 = c$$

$$76. x^2 + y^2 - 4x - 6y + k = 0$$

$$x^2 + y^2 + 8x - 4y + 11 = 0$$

$$r_1 = \sqrt{4 + 9 - k} = \sqrt{13 - k}$$

$$r_2 = \sqrt{16 + 4 - 11} = \sqrt{9} = 3$$

$$d = \sqrt{36 + 1} = \sqrt{37}$$

$$\theta = \pi/2$$

$$d^2 = r_1^2 + r_2^2$$

$$37 = 13 - k + 9$$

$$k = -15$$

77

$$r_1 = 1$$

$$r_2 = \sqrt{1+3} = 2$$

$$r_3 = \sqrt{1+3} = 2$$

$$x^2 + y^2 - 1 = 0 \rightarrow 1$$

$$x^2 + y^2 - 2x - 3 = 0 \rightarrow 2$$

$$x^2 + y^2 - 2y - 3 = 0 \rightarrow 3$$

$$1-2 \Rightarrow 2x+2=0$$

$$x = -1$$

$$3-1 \Rightarrow y+1=0$$

$$y = -1$$

$$\text{Radical centre} = (-1, -1), (\alpha, \beta)$$

$$r = r_1 + r_2 + r_3 = 1 + 2 + 2 = 5$$

Required circle eqn is

$$(x+1)^2 + (y+1)^2 = 5^2$$

$$78. \quad x^2 + y^2 + 2gx + 2fy + 4 = 0 \quad \text{--- (1) } g, f, 4$$

$$x^2 + y^2 - 4x - 4y - 4 = 0 \quad \text{--- (2) } -2, -2, -4$$

1 & 2 are orthogonal

$$2(g)(-2) + 2f(-2) = 0$$

$$g + f = 0$$

$$x^2 + y^2 + 4x + 4y + 4 = 0 \quad \text{--- (3) } 2, 2, 4$$

angle b/w (1) & (3) is 60°

$$\frac{1}{2} = \frac{8 - 2(2g + 2f)}{2\sqrt{g^2 + f^2 - 4}\sqrt{4 + 4 - 4}}$$

$$g^2 + f^2 - 4 = 16$$

radius of the circle $S = 0$

$$\sqrt{g^2 + f^2 - c} = \sqrt{16} = 4$$

$$79. \quad \text{R.A of (1) \& (2) } x = 0$$

$$\text{R.A of (2) \& (3) } y = 0 \therefore c(0, 0)$$

radius r = length of tangent from $(0, 0)$ to circle (1)

$$= \sqrt{7}$$

$$x^2 + y^2 = 7$$

Eq of tangent at $(\sqrt{3}, 2)$ to $x^2 + y^2 - 7 = 0$ is

$$S_1 = 0$$

$$xx_1 + yy_1 - 7 = 0$$

$$\sqrt{3}x + 2y - 7 = 0$$