22. TANGENTS & NORMALS

1. The equation of tangent to the curve $y=5x^2-3x+7$ at the point (-1, 4) is _____

[AP EAMCET 17-09-20_Shift-2]

1. $13x-y-9=0$ 2. $13x+y-9=0$
3. $13x+y+9=0$ 4. $13x+2y+5=0$

2. Find the equation of the normal to the curve $y=\frac{(x-7)}{(x-2)(x-3)}$ at the point where it cuts the x-axis **[AP EAMCET 18-09-20_Shift-1]**

1. $20x+y+140=0$ 2. $x-20y-140=0$
3. $x+20y+140=0$ 4. $20x+y-140=0$

3. The equation of the normal to the curve $x=a\cosh(t), y=b\sinh(t)$ at any point t is

[AP EAMCET 18-09-20_Shift-1]

1. $ax+by=a^2+b^2$
2. $ax\operatorname{sech}(t)+by\operatorname{cosech}(t)=a^2+b^2$
3. $ax\operatorname{sech}(t)-by\operatorname{cosech}(t)=a^2-b^2$
4. $\frac{ax}{\sinh(t)}+\frac{by}{\cosh(t)}=a^2+b^2$

4. The curve $3y^2=2ax^2+6b$ passes through the point P(3,-1) and the gradient of the curve at P is "-1", then the value of a and b are

[AP EAMCET 18-09-20_Shift-1]

1. $a=\frac{1}{2}, b=-1$ 2. $a=\frac{-1}{2}, b=1$
3. $a=\frac{1}{2}, b=1$ 4. $a=\frac{-1}{2}, b=-1$

5. The curves $y=4x^2+2x-8$ and $y=x^3-x+13$ touch each other at the point

[AP EAMCET 18-09-20_Shift-2]

1. (34,3) 2. (3,34)
3. (-3,34) 4. (-34,3)

6. If the subnormal at any point on the curve $y^n=ax$ is constant then the value of n is

[AP EAMCET 18-09-20_Shift-2]

1. 1 2. 2
3. 3 4. 4

7. The equation of the normal to the circle $x^2+y^2=16$ at the point $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ is

[AP EAMCET 18-09-20_Shift-2]

1. $x+y=0$ 2. $x-y=\frac{\sqrt{3}}{4}$
3. $x-y=0$ 4. $x+y=\frac{\sqrt{3}}{4}$

8. The tangent at the point (1,2) to the curve $y^2=4x$ makes an angle θ with the positive direction of x-axis, then $\theta=$

[AP EAMCET 18-09-20_Shift-2]

1. 60^0 2. 30^0
3. 90^0 4. 45^0

9. The equation of the tangent to the parabola $y^2=12x$ at (3,-6) is

[AP EAMCET 21-09-20_Shift-2]

1. $x-y+9=0$ 2. $x+y+3=0$
3. $x+y-3=0$ 4. $x=3$

10. The equation of the normal to the curve y=sinx at the point (0,0) is

[AP EAMCET 22-09-20_Shift-1]

1. x=0 2. y=0
3. x-y=0 4. x+y=0

11. A point on the curve $x=3\cos\theta$, $y=2\sin\theta$ at which the tangent is perpendicular to x-axis is

[AP EAMCET 22-09-20_Shift-1]

1. (3, 0) 2. (0, 3)
3. (0, -3) 4. (-3, 3)

12. The equation of the curve passing through (1,2) and whose tangent at any point (x ,y) makes an angle $\tan^{-1}(2x+3y)$ with the x-axis is **[AP EAMCET 22-09-20_Shift-2]**

1. $6x+9y+2=26e^{3x-3}$ 2. $6x+9y-2=26e^{3x-3}$
3. $6x+9y+2=26e^{3x+3}$ 4. $6x+9y-2=26e^{3x+3}$

13. Find the equation of normal to the curve $y=x^3-3x$, which is parallel to the line 2x+18y=9?

[AP EAMCET 22-09-20_Shift-2]

1. x+9y=20 only 2. x+9y=40 only
3. $x+9y=\pm 20$ 4. $x+9y=\pm 40$

14. Tangent at any point θ on the curve $x=35\sec\theta$, $y=35\tan\theta$ is ____

[AP EAMCET 22-09-20_Shift-2]

1. $y\sin\theta = x+35\cos\theta$
2. $y\sin\theta = x-35\cos\theta$
3. $y\cos\theta = x-35\sin\theta$
4. $y\cos\theta = x+35\sin\theta$

15. The tangent to the curve $y=e^{2x}$ at the point $(0,1)$ meets the x-axis at ____

[AP EAMCET 23-09-20_Shift-1]

1. (2, 0)
2. (0, 0)
3. $\left(\frac{-1}{2},0\right)$
4. $\left(\frac{1}{2},0\right)$

16. If the normal to the curve $y=x+\frac{2}{x}$ at the point where abscissa is 2, meets the coordinate axes in points A&B, find the length of AB.

1. $\frac{2}{\sqrt{5}}$
2. $\frac{7}{2}$
3. $\frac{7\sqrt{5}}{2}$
4. $\frac{3\sqrt{5}}{2}$

17. Let α , β be two roots of the quadratic equation $x^2+ax-b=0, b\neq 0$. If the straight line $x\cos\theta+y\sin\theta=C$ touches the curve $\left(\frac{x}{\alpha}\right)^n+\left(\frac{y}{\beta}\right)^n=2$ at the point (α,β), then $\left(\frac{a}{b}\right)^2+\frac{2}{b}=$ **[TS EAMCET 09-09-20_Shift-1]**

1. $\frac{1}{2C^2}$
2. $\frac{4}{C^2}$
3. $\frac{2}{C^2}$
4. $\frac{1}{C^2}$

18. The perpendicular distance from the origin to the normal drawn at any point on the curve $x=a(\cos\theta+\theta\sin\theta), y=a(\sin\theta-\theta\cos\theta)$ is

[TS EAMCET 09-09-20_Shift-2]

1. $a\theta$
2. a^2
3. a
4. $\frac{a}{\theta}$

19. Let a be the fixed positive real number and n be an arbitrary constant. For the curve $y=\frac{x^n}{a^{n-1}}$, if the length of the subnormal at any point (α,β) is proportional to α^2 then n=

[TS EAMCET 10-09-20_Shift-1]

1. 2
2. 1
3. 0
4. $\frac{3}{2}$

20. If $\frac{k}{\alpha^3}$ is the length of the sub normal at any point $P(\alpha,y)$ on the curve $x^2-a^2=\frac{x^2y^2}{a^2}$ then k= **[TS EAMCET 10-09-20_Shift-2]**

1. a
2. a^2
3. $\frac{3a}{2}$
4. a^4

21. If the tangent and normal drawn to the curve $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ at $P\left(\theta=\frac{\pi}{2}\right)$ cuts the X-axes at A and B respectively, then the area (in sq.units) of ΔPAB

[TS EAMCET 11-09-20 Shift-1]

1. $\frac{a^2}{\sqrt{2}}$
2. $\frac{\sqrt{2}}{a^2}$
3. a^2
4. $2a^2$

22. $x_1,x_2\in N$ If a line having slope 2 is a tangent to the curve $y=x^4-6x^3+13x^2-10x+5$ at points $P(x_1,y_1)$ and $Q(x_2,y_2)$ then $x_1x_2+y_1y_2=$

[TS EAMCET 11-09-20_Shift-1]

1. 17
2. -5
3. 13
4. -10

23. The distance between the origin and the normal to the curve $y=e^{2x}+x^2$ drawn at $x=0$ is ____ units

[AP EAMCET 19-08-2021_Shift-1]

1. 2
2. $\frac{2}{\sqrt{3}}$
3. $\frac{2}{\sqrt{5}}$
4. $\frac{1}{2}$

24. If the curves $\frac{x^2}{a^2}+\frac{y^2}{4}=1$ and $y^3=16x$ intersect at right angles, then $a^2=$

[AP EAMCET 19-08-2021_Shift-2]

1. $\frac{2}{3}$ 2. $\frac{2}{\sqrt{3}}$

3. $\frac{4}{3}$ 4. $\frac{3}{4}$

25. If $y=4x-6$ is a tangent to the curve $y^2=ax^4+b$ at $(3,6)$, then the values of a and b are **[AP EAMCET 20-08-2021_Shift-1]**

1. $a=\frac{4}{9}\&b=\frac{-4}{9}$ 2. $a=0\&b=\frac{4}{9}$

3. $a=\frac{-4}{9}\&b=\frac{-4}{9}$ 4. $a=\frac{4}{9}\&b=0$

26. The slope of the normal to the curve $y=\frac{x}{x^2+1}$ at $x=-4$ is

[AP EAMCET 23-08-2021_Shift-1]

1. $\frac{-289}{15}$ 2. $\frac{-15}{16}$

3. $\frac{289}{15}$ 4. $\frac{15}{16}$

27. If the tangent to the curve $2y^3=ax^2+x^3$ at the point (a,a) cuts off intercepts α & β on the co-ordinate axes, where $\alpha^2+\beta^2=61$, then the value of $|a|$ is__

[AP EAMCET 23-08-2021_Shift-1]

1. 14 2. 30

3. 20 4. 25

28. The sum of the lengths of the subtangent and the subnormal drawn at $\theta=\frac{\pi}{3}$ on the cycloid $x=a(\theta-\sin\theta);y=a(1-\cos\theta)$ is ______

[AP EAMCET 23-08-2021_Shift-1]

1. $2\sqrt{a}$ 2. $(2\sqrt{3})a$

3. $\frac{2a}{\sqrt{3}}$ 4. $\frac{a}{\sqrt{3}}$

29. If $\sin^{-1}(a)$ is the acute angle between the curves $x^2+y^2=4x$ and $x^2+y^2=8$ at the point (2, 2), then $a=$ ______

[AP EAMCET 23-08-2021_Shift-2]

1. 1 2. $\frac{1}{\sqrt{2}}$

3. 0 4. $\frac{-1}{\sqrt{2}}$

30. Find the coordinates of a point in the curve $y=x^2-3x+2$, at which the tangent drawn to this curve perpendicular to the line $y=x$.

[AP EAMCET 23-08-2021_Shift-2]

1. $(0,2)$ 2. $(1,0)$

3. $(-1,6)$ 4. $(2,-3)$

31. The point on the curve $y=x^3$, at which the tangent to the curve is parallel to the x–axis is

[AP EAMCET 24-08-2021_Shift-1]

1. (2, 2) 2. (3, 3)

3. (4, 4) 4. (0, 0)

32. Find the area of the triangle formed by the x - axis and the tangent and the normal to the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(\frac{a}{\sqrt{2}},\frac{b}{\sqrt{2}}\right)$.

[AP EAMCET 24-08-2021_Shift-1]

1. $\frac{ab}{4}\sqrt{a^2+b^2}$ 2. $4ab$

3. $\frac{b}{4a}(a^2+b^2)$ 4. $\frac{ab}{2}\sqrt{a^2+b^2}$

33. If the relation p (subnormal length) = q (sub tangent length)2 hold true for the curve $by^2=(x+a)^3$, then the value of $\frac{p}{q}=$

[AP EAMCET 24-08-2021_Shift-2]

1. $\frac{8}{27}$ 2. $\frac{8b}{27}$

3. $\frac{8}{27b}$ 4. $\frac{27}{8b}$

34. The point on the curve $x^2+y^2=a^2, y\geq 0$, at which the tangent is parallel to the x–axis, is ______

[AP EAMCET 24-08-2021_Shift-2]

1. $(a,0)$
2. $(-a,0)$
3. $(0,a)$
4. $(0,-a)$

35. If two curves $x=y^2, xy=a^3$ cut each other orthogonally at a point, then $a^2=$ ______

[AP EAMCET 25-08-2021_Shift-1]

1. $\frac{1}{3}$
2. $\frac{1}{2}$
3. 2
4. 3

36. The point at which the tangent line to the curve of $y=\frac{16}{x}-x^2$ is horizontal, is

[TS EAMCET 04-08-2021_Shift-2]

1. $(2,4)$
2. $(-2,-12)$
3. $(1,15)$
4. $(-1,-17)$

37. The equation of the tangent to the curve $y=\pi e^{\frac{-x}{\pi}}$ at the point where it crosses Y-axis is

[TS EAMCET 04-08-2021_Shift-2]

1. $\pi x+2y=2\pi$
2. $2x+\pi y=\pi^2$
3. $x-y+\pi=0$
4. $x+y=\pi$

38. If the tangent at point P on the curve $y=4x^4+x$ is perpendicular to the tangent to the same curve at (0,0),then the point P is

[TS EAMCET 04-08-2021_Shift-1]

1. $\left(\frac{-1}{2},\frac{-1}{4}\right)$
2. $\left(\frac{1}{2},\frac{3}{4}\right)$
3. $(1,5)$
4. $(-1,3)$

39. The angle between the curves $2x^2+y^2=20$ and $4y^2-x^2=8$ at a point where they intersect in the 4th quadrant is

[TS EAMCET 04-08-2021_Shift-1]

1. 0
2. $\frac{\pi}{2}$
3. $\frac{\pi}{4}$
4. $\frac{\pi}{6}$

40. The equation of the tangent to the curve $xy^5+2x^2y-x^3+y+1=0$ at $x=0$ is

[TS EAMCET 05-08-2021_Shift-1]

1. $3x+4y+4=0$
2. $y=x-1$
3. $5x+7y+7=0$
4. $x+y+1=0$

41. If the two curves $x=y^2$ and $xy=K$ cut each other at right angles, then a possible value of K is **[TS EAMCET 05-08-2021_Shift-1]**

1. $\frac{1}{8}$
2. $\frac{1}{2}$
3. $\frac{1}{\sqrt{2}}$
4. $\frac{1}{2\sqrt{2}}$

42. If the tangent to the curve $xy+ax+by=0$ at $(1,1)$ makes an angle $\tan^{-1}2$ with X–axis, then $\frac{a+b}{ab}=$ **[TS EAMCET 05-08-2021_Shift-2]**

1. 0
2. $\frac{1}{2}$
3. $\frac{-1}{2}$
4. 2

43. The ratio of the length of the subnormal to the square of the length of the subtangent at any point P on the curve $y^2=(2x+1)^3$ is

[TS EAMCET 06-08-2021_Shift-2]

1. 27
2. $\frac{1}{9}$
3. 9
4. $\frac{8}{27}$

44. If θ is the acute angle between the curves $x^2+y^2=2020\sqrt{2}$ and $x^2-y^2=2020$, then $\frac{\sin\theta+\cos\theta}{\tan\theta}=$

[TS EAMCET 06-08-2021_Shift-2]

1. $\sqrt{2}$
2. $\frac{3+\sqrt{3}}{2}$
3. $\frac{3+\sqrt{3}}{4}$
4. $\frac{3+\sqrt{3}}{6}$

45. The equation of the normal drawn to the curve $y=\sin 3x$ at $x=\frac{\pi}{4}$ is

[TS EAMCET 06-08-2021_Shift-1]

1. $y=\frac{\sqrt{3}}{2}\left(x+\frac{6-\pi}{4}\right)$
2. $y=\frac{\sqrt{2}}{3}\left(x+\frac{6-\pi}{4}\right)$
3. $y=\frac{\sqrt{3}}{2}\left(x-\frac{6-\pi}{4}\right)$
4. $y=\frac{\sqrt{2}}{3}\left(x-\frac{6-\pi}{4}\right)$

46. Suppose A,B,C and D are the 4 intersection points of the curves $\frac{x^2}{18}+\frac{y^2}{8}=1$ and $x^2-y^2=5$ in 1st ,2nd ,3rd and 4th quadrants respectively. If $\theta_1,\theta_2,\theta_3$ and θ_4 respectively are the angles between the curves at A,B,C and D, then

[TS EAMCET 06-08-2021_Shift-1]

1. $\theta_1\neq\theta_2\neq\theta_3\neq\theta_4$
2. $\theta_1=\theta_2;\theta_3=\theta_4;\theta_2\neq\theta_3$
3. $\theta_1=\theta_3;\theta_2=\theta_4;\theta_3\neq\theta_2$
4. $\theta_1=\theta_2=\theta_3=\theta_4$

47. If the curves $y=x^3-3x^2-8x-4$ and $y=3x^2+7x+4$ touch each other at a point P then the equation of common tangent at P is

[AP EAMCET 04-07-2022_Shift-1]

1. $x-y+1=0$
2. $2x-y+1=0$
3. $x+y+1=0$
4. $2x+y+1=0$

48. The number of those tangents to the curve $y^2-2x^3-4y+8=0$ which pass through the point (1,2) is

[AP EAMCET 04-07-2022_Shift-2]

1. 0
2. 2
3. 1
4. 3

49. If the straight line $x\cos\alpha+y\sin\alpha=p$ touches the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a,b) on it and $\frac{1}{a^2}+\frac{1}{b^2}=\frac{k}{p^2}$ then k=

[AP EAMCET 04-07-2022_Shift-2]

1. 4
2. 5
3. 6
4. 7

50. Condition that 2 curves $y^2=4ax$, $xy=c^2$ cut orthogonally is

[AP EAMCET 04-07-2022_Shift-2]

1. $c^2=16a^2$
2. $c^2=32a^2$
3. $c^4=16a^4$
4. $c^4=32a^4$

51. If the normal drawn at a print P on the curve $3y-6x-5x^3$ passes through (0, 0) then the positive integral value of the abscissa of the point P is

[AP EAMCET 05-07-2022_Shift-1]

1. 1
2. $2/3$
3. $1/3$
4. $-2/3$

52. The line joining the points (0, 3) and (5, –2) is a tangent to the curve $y=\frac{C}{x+1}$ then C=

[AP EAMCET 05-07-2022_Shift-1]

1. 1
2. –2
3. 4
4. 5

53. If the two curves $y=a^x$ and $y=b^x$ intersect at angle α, then tan α =

[AP EAMCET 05-07-2022_Shift-2]

1. $\frac{\log a-\log b}{1+\log a\log b}$
2. $\frac{\log a+\log b}{1-\log a\log b}$
3. $\frac{\pi}{4}$
4. $\frac{\pi}{2}$

54. The area of the triangle formed by the tangent to the curve $xy=a^2$ at(x_1, y_1) on it and the axes is **[AP EAMCET 05-07-2022_Shift-2]**

1. a^2 sq.units
2. $\frac{3a^2}{2}$ sq.units
3. $2a^2$ sq.units
4. $4a^2$ sq.units

55. If the slope of the line through (0, 0) which is tangent to the curve $y=x^2+x+16$ is m, then the value of m–4 is

[AP EAMCET 05-07-2022_Shift-2]

1. 9
2. 10
3. 12
4. 13

56. If the line ax + by + c = 0 is a normal to the curve xy=1, Then

[AP EAMCET 05-07-2022_Shift-2]

1. $a > 0, b > 0$
2. $a > 0, b < 0$
3. $a > 0, b = 0$
4. $a < 0, b < 0$

57. m is the slope of a tangent to the curve $e^y = 1+x^2$ $at\ x=1$ then $m=$

[AP EAMCET 06-07-2022_Shift-1]

1. $\frac{2}{\log 2}$
2. $\log 2$
3. 2
4. 1

58. V is the set of points on the curve $y^3-3xy+2=0$ where the tangent is vertical then V=

[AP EAMCET 06-07-2022_Shift-1]

1. Φ
2. $\{(1,0)\}$
3. $\{(1,1)\}$
4. $\{(0,0)(1,1)\}$

59. The equation of tangent of the curve $y=\sqrt{9-2x^2}$ at the point where the ordinate and abscissa are equal is **[AP EAMCET 06-07-2022_Shift-1]**

1. $2x+y-3\sqrt{3}=0$
2. $2x+y+3\sqrt{3}=0$
3. $2x-y-3\sqrt{3}=0$
4. $2x-y+3\sqrt{3}=0$

60. The angle between the tangents drawn at (0,0) to the curves $y^3-x^2y+5y-2x=0$ and $x^4-x^3y^2+5x+2y=0$ is

[AP EAMCET 06-07-2022_Shift-2]

1. $\pi/6$
2. $\pi/4$
3. $\pi/3$
4. $\pi/2$

61. The normal to the curve x= a (1+ cosθ), y= a sin θat θ always passes through a fixed point, then the fixed point is

[AP EAMCET 06-07-2022_Shift-2]

1. (a,0)
2. (0,a)
3. (0,0)
4. (a,a)

62. The curve $y=ax^3+bx^2+cx+5$ touches x-axis at $P(-2,0)$, then c=

[AP EAMCET 07-07-2022_Shift-1]

1. 4a+5
2. 4a-5
3. 5-4a
4. 0

63. The curves $y=x^2-1, y=8x-x^2-9$

[AP EAMCET 07-07-2022_Shift-1]

1. intersect at right angles at(2,3)
2. touch each other at (2,3)
3. intersect at 45^0
4. intersect at 60^0

64. Equation of tangent to the curve $y=x+\frac{4}{x^2}$ which is parallel to x-axis is

[AP EAMCET 07-07-2022_Shift-2]

1. y=8
2. y=0
3. y=3
4. y=2

65. The normal to the curve y=f(x) at the points (3,4) makes an angle $\frac{3\pi}{4}$ with positive x-axis then f'(3)=

[AP EAMCET 07-07-2022_Shift-2]

1. 3
2. 2
3. 1
4. 4

66. The sum of the intercepts made by a tangent drawn to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at (a, b) on the coordinate axes is

[AP EAMCET 08-07-2022_Shift-1]

1. a + b
2. a^2+b^2
3. 2(a-b)
4. 2(a+b)

67. The angle between the curve $2y=e^{-x/2}$ and the y-axis is $\tan^{-1}(k)$ then k =

[AP EAMCET 08-07-2022_Shift-1]

1. 1
2. 2
3. 3
4. 4

68. The angle made by the tangent at $\theta=\frac{\pi}{3}$ on the curve $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ with x-axis is **[AP EAMCET 08-07-2022_Shift-2]**

1. $\frac{\pi}{3}$
2. $\frac{\pi}{6}$
3. $\frac{2\pi}{3}$
4. $\frac{5\pi}{6}$

69. If the angle between the curves $y^2=4x$ and $y=e^{\frac{-x}{2}}$ is θ, then $\operatorname{cosec}^2\left(\frac{\theta}{2}\right)=$

[AP EAMCET 08-07-2022_Shift-2]

1. 2
2. 3
3. $\sqrt{3}$
4. $\sqrt{2}$

70. The area of the triangle formed by the tangent and the normal drawn to the curve $y^2=4x$ at (1, 2) with Y–axis is (in square units)

[TS EAMCET 18-07-2022_Shift-1]

1. 4
2. 3
3. 2
4. 1

71. Consider two families of curves $y^2=4ax$ (a is a parameter) and $x^2+\frac{y^2}{2}=c^2$ (c is parameter). If one curve from each family is chosen, then the angle between those two curves is

[TS EAMCET 18-07-2022_Shift-1]

1. π
2. $\frac{\pi}{4}$
3. $\frac{3\pi}{4}$
4. $\frac{\pi}{2}$

72. If θ is the angle between the curves $y^2=4x$ and $x^2+y^2=5$ then $|\tan\theta|=$

[TS EAMCET 18-07-2022_Shift-2]

1. 5
2. 4
3. 3
4. 2

73. The equation of the normal to the curve $4x^2+9y^2=36$ at the point $P\left(\frac{7\pi}{4}\right)$ is

[TS EAMCET 19-07-2022_Shift-I]

1. $2x-3y-6\sqrt{2}=0$
2. $2x+3y=0$
3. $3\sqrt{2}x+2\sqrt{2}y-5=0$
4. $3\sqrt{2}x-2\sqrt{2}y-13=0$

74. If θ is the acute angle between the curves $x^2+y^2=4$ and $y^2=3x$ then $\tan\theta=$

[TS EAMCET 19-07-2022_Shift-I]

1. $\frac{5}{\sqrt{3}}$
2. $\frac{\sqrt{3}}{4}$
3. $\frac{4}{\sqrt{3}}$
4. $\frac{\sqrt{3}}{5}$

75. The equation of the normal to the curve $\sin y=\sqrt{3}x\sin\left(\frac{\pi}{6}+y\right)$ at $x=0$, is

[TS EAMCET 19-07-2022_Shift-2]

1. $2x+\sqrt{3}y=0$
2. $2x+y=0$
3. $x+2y=0$
4. $\sqrt{3}x+2y=0$

76. **Assertion(A):** The curves $y^2=4x$ and $x^2=-2y$ intersect at (1,2) orthogonally
Reason(R): If the product of the slopes of the tangents drawn to two curves at their point of intersection is –1. Then the curves are said to cut each other orthogonally.
The correct option among the following is

[TS EAMCET 19-07-2022_Shift-2]

1. (A) is true, (R) is true and (R) is the correct explanation for (A)
2. (A) is true, (R) is true and (R) is not the correct explanation for (A)
3. (A) is true but (R) is false
4. (A) is false but (R) is true

77. $y=x^2$ is the given curve. Imagine that this curve is dragged along the positive X–axis to a distance of 'a' units. If the acute angle between the curves at two positions is θ then

[TS EAMCET 20-07-2022_Shift-1]

1. $\theta=\frac{\pi}{2}$
2. $\tan\theta=\frac{2|a|}{|1-a^2|}$
3. $\cos\theta=\frac{2|a|}{|1-a^2|}$
4. $\theta=0$

78. The equation of the tangent to the curve $x^2+y-7=4x$ at the point(1, 10) is

[TS EAMCET 20-07-2022_Shift-2]

1. $y=2x+8$
2. $y=x+8$
3. $y=-2x-14$
4. $y=x-4$

79. If θ is the angle between the curves $x^2-y^2=4$ and $y^2=3x$ then tan θ =

[TS EAMCET 20-07-2022_Shift-2]

1. $\frac{5}{3\sqrt{3}}$
2. $\frac{5}{6\sqrt{3}}$
3. $\frac{5}{18}$
4. $\frac{5}{6}$

80. If the tangent drawn to the curve $y=x^3-ax^2+x+1$ at each point $x\in\mathbb{R}$, is inclined at an acute angle with the positive direction of X-axis, then the set of all possible values of 'a' is **[15th May 2023 Shift 1]**

1. $\mathbb{R}-(-\sqrt{3},\sqrt{3})$
2. $[-3, 3]$
3. $\mathbb{R}$
4. $(-\sqrt{3},\sqrt{3})$

81. The number of points on the curve $y=2t^2+3t-5$ and $x=t^3-4t^2-3t$ such that the normals drawn at them on the curve are parallel to X-axis is **[15th May 2023 Shift 1]**

1. 1
2. 4
3. 3
4. 2

82. If the angle between the curve $y=e^{2(1+x)-4}$ and $x^2y=1$ at the point (1,1) is θ,then $|\sin\theta|+|\cos\theta|=$ **[15th May 2023 Shift 2]**

1. 7/5
2. 3/5
3. 8/7
4. 6/5

83. If the points of contact of the tangents drawn from (0,0) to the curve $y=x^2+3x+4$ are (α,β) and (γ,δ), then β+δ =

[15th May 2023 Shift 2]

1. 7
2. 25
3. 16
4. 13

84. If the tangent drawn at A(2,1) to the curve $x=1+\frac{1}{y^2}$ meets the curve again at B, then

[16th May 2023 Shift 1]

1. The tangent drawn at B coincides with the tangent drawn at A
2. The angle between the tangents drawn at A and B is neither 0 nor $\frac{\pi}{2}$
3. The tangent drawn at A and the tangent drawn at B are perpendicular to each other
4. Then tangent drawn at A is parallel to the tangent drawn at B

85. The points on the curve $y^2=x+\sin x$ at which the normal is parallel to the Y-axis lie on **[16th May 2023 Shift 1]**

1. a line parallel to Y-axis
2. A circle with centre at origin
3. a parabola
4. a pair of lines bisecting the angle between the coordinate axes .

86. f(x) is a continuous function on R and y=f(x) is a curve. If (α,β) is a point such that $\beta=f(\alpha)$ and $p\alpha+m\beta+n=0(p\neq 0, m\neq 0)$, then which one of the following is True? [16th May 2023 Shift 2]

1. When $p+mf'(\alpha)=0, px+my+n=0$ intersects the curve y=f(x)
2. $px+my+n$ is always a tangent to the curve y=f(x)
3. When $p+mf'(\alpha)\neq 0, px+my+n=0$ intersects the curve y=f(x)
4. $px+my+n$ =0 is never a tangent to the curve y=f(x)

87. If the slope of the tangent on a curve at any point (x,y) is equal to $\dfrac{y^2-x^2}{2xy}$ then equation of the normal at $\left(\dfrac{1}{2},\dfrac{\sqrt{3}}{3}\right)$ is

[16th May 2023 Shift 2]

1. $\sqrt{3x}+y=\sqrt{3}$
2. $x+\sqrt{3y}=\sqrt{3}$
3. $3x-\sqrt{3y=0}$
4. $x+\sqrt{3y}=0$

88. The ordinates of the points on the curve $y=Tan^{-1}\left(\sin\sqrt{x}\right), 0\leq x\leq 8\pi^2$, at which the tangent is parallel to X – axis are

[17th May 2023 Shift 1]

1. $\pm\dfrac{\pi}{3}$
2. $\pm\dfrac{\pi}{6}$
3. $\pm\dfrac{\pi}{4}$
4. $\pm\dfrac{\pi}{2}$

89. If $\left(a^2-1\right)x+ay+(3-a)=0$ is a normal to the curve xy =1, then the interval in which 'a' lies is **[17th May 2023 Shift 1]**

1. $[-1,1]\cup[2,\infty)$
2. $(-\infty,-1]\cup(0,1]$
3. $[-1,1)\cup(1,\infty)$
4. $(1,\infty)$

90. If θ is the angle made by the normal drawn to the curve $x=e^t\cos t, y=e^t\sin t$ at the point (1,0), with the X–axis then $\theta=$

[17th May 2023 Shift 2]

1. $\pi/2$
2. $\pi/4$
3. $3\pi/2$
4. $3\pi/4$

91. If a normal drawn at a point P to the curve y=sin x passes through the origin, then the locus of P is **[17th May 2023 Shift 2]**

1. $x^2=y^2-y^4$
2. x+y=1
3. $\dfrac{1}{y^2}-\dfrac{1}{x^2}=1$
4. $\dfrac{1}{y^4}+\dfrac{1}{x^4}=1$

92. If the tangent drawn to the curve $y=x^3$ at a point (α,β) cuts again the curve at another point (α_1,β_1) then $\dfrac{\beta_1}{\beta}=$

[17th May 2023 Shift 2]

1. –2
2. 1
3. –8
4. 27

93. If the locus of the points on the curve $x^3y^2+\dfrac{x^2}{y}=5$ at which the tangent is parallel to X-axis is f(x, y)=0, then the point that lies on this curve f(x, y)=0 is

[18th May2023 shift -1]

1. $\left(2,\sqrt[3]{3}\right)$
2. $\left(\sqrt[3]{2},3\right)$
3. $\left(-2,\dfrac{1}{\sqrt[3]{3}}\right)$
4. $\left(-\sqrt[3]{2}\dfrac{1}{\sqrt[3]{3}}\right)$

94. The area (in sq units) of the triangle formed by the normal drawn at the point (1, 0) on the curve $x=e^{\sin y}$ with the coordinate axes is

[18th May2023 shift -1]

1. 1
2. $\dfrac{1}{4}$
3. $\dfrac{1}{2}$
4. $\dfrac{3}{8}$

95. If the tangent drawn to the curve $(x^2+1)(y-3)=x$ at a point P, lying in the first quadrant, is a horizontal line, then the equation of the normal at the point P is [18th May2023 shift -1]

1. $x=\frac{7}{2}$
2. x=1
3. $y=\frac{7}{2}$
4. y=1

96. The slope of the tangent at any point (x, y) on the curve, is equal to the product of the coordinates of that point. If the equation of the normal to the curve at the point $(\sqrt{2},e) is\ ax+by=1, then\ \frac{b}{a}=$ **[18th May2023 shift -1]**

1. $\frac{1}{\sqrt{2}e}$
2. $\frac{e}{\sqrt{2}}$
3. $\sqrt{2}e$
4. $\frac{\sqrt{2}}{e}$

97. The locus of the point on the curve y =sin x where the tangent drawn at that point always passes through the point $(0,\pi)$ is **[18th May 2023 Shift 2]**

1. $x = y - \pi$
2. $\sin x + \cos y + 1 = 0$
3. $x^2(1-y^2)=(y-\pi)^2$
4. $x^2+(y-\pi)^2=0$

98. A(-2,9) and B(1,6) are two points on the curve $y=x^2+5$. The coordinates of the point C on the curve such that the tangent drawn at A is parallel to the chord BC, is [**18th May 2023 Shift 2**]

1. (-5,30)
2. (0,5)
3. (-9,86)
4. (6,41)

99. If all the normals drawn to the curve $y=\frac{1+3x^2}{3+x^2}$ at the points of intersection of $y=\frac{1+3x^2}{3+x^2}$ and y =1 pass through the point (α,β), then $3\alpha+2\beta$=[**18th May 2023 Shift 2**]

1. 4
2. 2
3. -2
4. -4

100. The equation of the normal to the curve y =cosh x drawn at the point nearest to the origin is **[18th May 2023 Shift 2]**

1. $y=0$
2. $x=1$
3. $x=0$
4. $y=1$

101. Let $n \in (0,\infty)$. If all the curves $y=x^n \log x$ for distinct values of n, always have $y = x-1$ as the tangent drawn at a fixed point (α,β), then $\alpha+\beta$= [**18th May 2023 Shift 2**]

1. 0
2. log 2
3. 1
4. Log 3

102. Let $f:R\to R$ be a continuous function. If $px+my+n=0$ is a tangent drawn to the curve y=f(x) at $x=\alpha$, then at $x=0, \frac{d}{dx}\left(f\left(\alpha e^{2x}\right)\right)=$ **[19th May 2023 Shift 1]**

1. 0
2. $\frac{p}{m}$
3. $\frac{-2\alpha m}{p}$
4. $\frac{-2p\alpha}{m}$

103. The equation of the normal at $t=\frac{\pi}{2}$ to the curve $x=2\sin t, y=2\cos t$ is **[12TH MAY 2023 SHIFT-1]**

1. $x=2$
2. $y=0$
3. $y=2x+3$
4. $y=3$

104. If the slope of the tangent drawn to the curve $y=e^{a+bx^2}$ at the point $P(1,1)$ is –2, then the value of 2a –3b is **[12TH MAY 2023 SHIFT-1]**

1. 5
2. 6
3. 7
4. 8

105. If the equation of a tangent drawn to the curve $y=\cos(x+y), -1\le x\le 1+\pi$ is $x+2y=k$, then k=[**12TH MAY 2023 SHIFT -2**]

1. 1
2. $\frac{\pi}{4}$
3. $\frac{\pi}{2}$
4. 2

106. The slope of the normal drawn at a point P to the curve $y = x^3 - 10x^2 + 31x - 30$ is $-\frac{1}{14}$ If the coordinates of P are integers, then the X-intercept of the tangent drawn at P to the given curve is **[13TH MAY 2023 SHIFT-1]**

1. $\frac{-11}{7}$
2. 22
3. $\frac{11}{7}$
4. -22

107. If a line having slope 2 is a tangent to the curve $y = x^4 - 6x^3 + 13x^2 - 12x + 5$ at points $P(x_1, y_1)$ and $Q(x_2, y_2), x_1, x_2 \in N$ then $x_1x_2 - y_1y_2 =$

[EAPCET 14-05-23 SHIFT-1]

1. 17
2. 3
3. -17
4. -13

108. Let m be the slope of the normal L drawn at (1,2) to the curve $x = t^2 - 7t + 7, y = t^2 - 4t - 10$ and $ax + by + c = 0$ be the equation of the normal L. If G.C.D. of (a,b,c) is 1, then $m(a+b+c) =$

[EAPCET 14-05-23 SHIFT-1]

1. 8
2. $\frac{-64}{5}$
3. -8
4. 5

109. For $h, k \in N$, let P(h,k) be the point of intersection of the curves $x^2y - x^3 = 8$ and $y^3 - xy^2 = 32$. If θ is the acute angle between these two curves at P, then $\tan\theta =$

[EAPCET 13-05-23 SHIFT-2]

1. 27/11
2. 1/3
3. $\frac{\pi}{2}$
4. 3

110. If the slope of the tangent drawn at any point (x,y) to the curve $y = f(x)$ is $3x^2 - 5$ and $f(1) = 2$, then the tangent at $(1,2)$ to the curve $y = f(x)$ intersects the curve at the point **[EAPCET 13-05-23 SHIFT-2]**

1. $(2,0)$
2. $(-2,8)$
3. $(3,-2)$
4. $(-1,6)$

KEY

1)	3	2)	4	3)	2	4)	1	5)	2
6)	2	7)	3	8)	4	9)	2	10)	4
11)	1	12)	1	13)	3	14)	2	15)	3
16)	3	17)	2	18)	3	19)	4	20)	4
21)	3	22)	1	23)	3	24)	3	25)	4
26)	3	27)	2	28)	3	29)	2	30)	2
31)	4	32)	3	33)	2	34)	3	35)	2
36)	2	37)	4	38)	1	39)	2	40)	2
41)	4	42)	2	43)	1	44)	1	45)	2
46)	4	47)	1	48)	3	49)	1	50)	4
51)	1	52)	3	53)	1	54)	3	55)	1
56)	2	57)	4	58)	3	59)	1	60)	4
61)	1	62)	1	63)	2	64)	3	65)	3
66)	4	67)	4	68)	3	69)	1	70)	4
71)	4	72)	3	73)	3	74)	1	75)	1
76)	4	77)	2	78)	1	79)	2	80)	4
81)	4	82)	1	83)	3	84)	2	85)	3
86)	3	87)	1	88)	3	89)	2	90)	4
91)	1	92)	3	93)	3	94)	3	95)	2
96)	3	97)	3	98)	1	99)	1	100)	3
101)	3	102)	4	103)	2	104)	1	105)	3
106)	3	107)	4	108)	4	109)	4	110)	2

SOLUTIONS

1. $\frac{dy}{dx} = 10x - 3$

$m = \left(\frac{dy}{dx}\right)_{(-1,4)} = -13$

Equation of tangent

$y - 4 = -13(x+1)$

$\Rightarrow 13x + y + 9 = 0$

2. $y = \frac{(x-7)}{(x-2)(x-3)}$

P(7,0)

Slope of tangent

$\frac{dy}{dx} = \frac{(x^2 - 5x + 6) - (2-7)(2x-5)}{(x^2 - 5x + 6)^2}$

$$m=\left(\frac{dy}{dx}\right)_{(7,0)}=\frac{1}{20}$$

Eq. of normal is $y-0=-20(x-7)$

$20x+y-140=0$

3. $x=a\cosh(t), y=b\sinh(t)$

$\frac{dx}{dt}=a\sinh t, \frac{dy}{dt}=b\cosh t$

Equation of normal is

$$y-b\sinh t=\frac{-a\sinh t}{b\cosh t}(x-a\cosh t)$$

$ax\sec ht+by\cos echt=a^2+b^2$

4. P(3,-1) lies on $3y^2=2ax^2+6b \rightarrow (A)$

$\Rightarrow 6a+2b=1 \rightarrow (1)$

D.w.r.to.x to Eq. (A)

$$6y\frac{dy}{dx}=4ax \Rightarrow \frac{dy}{dx}=\frac{2ax}{3y}$$

$$\left(\frac{dy}{dx}\right)_{P(3,-1)}=\frac{6a}{-3}=-2a$$

$$but\ gradient\left(\frac{dy}{dx}\right)_{P(3,-1)}=-1$$

$\therefore a=1/2$

$b=-1(from(1))$

5. $m_1=m_2\ at\ P(x_1,y_1)$

$\Rightarrow 8x_1+2=3x_1^2-1$

$\Rightarrow x_1=3\ (or)\ \frac{-1}{3}$

$\Rightarrow y_1=34\ (or)\ \frac{-74}{9}$

$\therefore P(3,34)$

6. $S.N=|ym|=\left|y.\frac{a}{ny^{n-1}}\right|$

$=\left|\frac{a}{n}.y^{2-n}\right|$ is a constant

$\Rightarrow 2-n=0 \Rightarrow n=2$

7. Eq. of normal is $y_1x-x_1y=0$

$\Rightarrow x-y=0$

8. $\left(\frac{dy}{dx}\right)_{(1,2)}=m=1$

$\Rightarrow \theta=45^0$

9. Given parabola is $y^2=12x$

Eq. of tangent at P(3,-6) is

$S_1=0$

$y(-6)=2(3)(x+3)$

$\therefore x+y+3=0$

10. $y=\sin x \Rightarrow \frac{dy}{dx}=\cos x$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(0,0)}=1$$

Slope of normal=-1

Eq. of normal is $y-0=-1(x-0)$

$x+y=0$

11. Conceptual

12. Conceptual

13. $y=x^3-3x$

$y^1=3x^2-3$

$$perpendicular\ slope=\frac{-1}{3x^2-3}=slope(2x+18y=9)$$

$3x^2-3=9 \Rightarrow x=\pm 2$

$\Rightarrow y=\pm 2$

14. $\frac{dy}{dx}=\frac{35\sec^2\theta}{35\sec\theta tan\theta}=\frac{1}{\sin\theta}$

$eqn. of\ \tan gent$

$$y-35\tan\theta=\frac{1}{\sin\theta}(x-35\sec\theta)$$

$y\sin\theta=x+35\sin\theta\tan\theta-35\sec\theta$

$=x-35\sec\theta(1-\sin^2\theta)$

$y\sin\theta=x-35\cos\theta$

15. $\frac{dy}{dx}=2e^{2x}, m=2$

$$y-1=2(x-0)$$

$$y=0 \Rightarrow x=\frac{-1}{2}$$

$$\left(\frac{-1}{2},0\right)$$

16. Given curve $y=x+\frac{2}{x}$

$$\frac{dy}{dx}=1-\frac{2}{x^2}$$

$$\left(\frac{dy}{dx}\right)_{x=2}=\frac{1}{2}$$

$\therefore$ slope of normal =-2

Equation of normal is $2x+y-7=0$

It meets the coordinate axes at $A\left(\frac{7}{2},0\right)$ & $B(0,7)$

$$AB=\frac{7\sqrt{5}}{2}.$$

17. $\alpha+\beta=-a \quad \alpha\beta=-b$

$$\left(\frac{x}{\alpha}\right)^n+\left(\frac{y}{\beta}\right)^n=2$$

Slope of tangent to the curve at (α,β) is $\frac{-\beta}{\alpha}$

The equation of tangent is $\beta x+\alpha y+2b=0$.....(1)

$\cos\theta x+\sin\theta y-c=0$....(2)

Eq (1) and (2) represent same line

$$\frac{\cos\theta}{\beta}=\frac{\sin\theta}{\alpha}=\frac{-c}{2b}$$

$$\cos^2\theta+\sin^2\theta=1$$

$$\frac{c^2}{4b^2}\left[(\alpha+\beta)^2-2\alpha\beta\right]=1$$

$$\left(\frac{a}{b}\right)^2+\frac{2}{b}=\frac{4}{c^2}$$

18. $x=a(\cos\theta+\theta\sin\theta),\ y=a(\sin\theta-\theta\cos\theta)$

$$\frac{dy}{dx}=\frac{\cos\theta+\theta\sin\theta-\cos\theta}{-\sin\theta+\theta\cos\theta+\sin\theta}=\text{Tan}\,\theta$$

Equation of normal is

$$y-a(\sin\theta-\theta\cos\theta)=\frac{-\cos\theta}{\sin\theta}[x-a(\cos\theta+\theta\sin\theta)]$$

Perpendicular distance from origin to normal is $=a$

19. length of the subnormal= $|ym|$

$$=\left|\frac{x^n}{a^{n-1}}\frac{1}{a^{n-1}}nx^{n-1}\right|$$

$$\Rightarrow\left|\alpha^{2n-1}\frac{n}{a^{2n-2}}\right| \textit{proportional to } \alpha^2$$

$$2n-1=2$$

$$n=\frac{3}{2}$$

20. $2x=\frac{2xy}{a^2}(mx+y)$

$$mx+y=\frac{a^2}{y}$$

$$m=\frac{\frac{a^2}{y}-y}{x}\Rightarrow m=\frac{a^2-y^2}{xy}$$

$$|ym|=\frac{a^4}{x^3}\Rightarrow\frac{k}{\alpha^3}=\frac{a^4}{\alpha^3}\Rightarrow k=a^4$$

21. *Given* $P(x,y)=(a(\theta+\sin\theta),a(1-\cos\theta))$

$$at\,\theta=\frac{\pi}{2};\ p(x,y)=\left[a\left(\frac{\pi}{2}+1\right),a\right]$$

$$now,\ x=a(\theta+\sin\theta),\ y=a(1-\cos\theta)$$

$$\frac{dx}{d\theta}=a(1+\cos\theta),\ \frac{dy}{d\theta}=a(\sin\theta)$$

$$\textit{slope of tangent } m_T=\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}}=1$$

$$\textit{slope of normal } m_N=-1$$

the point at which tangent meets $X-axis$ *is*

$A\left(\frac{a\pi}{2},0\right)$ *and normal meets* $X-axis$

is $B\left(2a+\frac{a\pi}{2},0\right)$

Now area of $\Delta PAB=a^2$

22. Given $y=x^4-6x^3+13x^2-10x+5$

Also,

$Also\ (y^1)_p = 2 \Rightarrow 4x^3 - 18x^2 + 26x - 10 = 2$

$\Rightarrow 4x^3 - 18x^2 + 26x - 12 = 0$

$on\ simplifying\ we\ get\ x = 1\ or\ \frac{3}{2}\ or\ 2$

$for\ x_1 = 1, y_1 = 3 \quad i.e, (x_1, y_1) = (1,3)$

$x_2 = 2, y_2 = 5 \quad i.e, (x_2, y_2) = (2,5)$

$now, x_1x_2 + y_1y_2 = 2 + 15 = 17$

23. $x = 0 \Rightarrow y = 1$

$\therefore$ At point $(0,1)$

Slope (m) = 2

Equation of normal $y - y_1 = \frac{-1}{m}(x - x_1)$

$\Rightarrow x + 2y - 2 = 0$

Perpendicular distance from origin to normal

$= \frac{|c|}{\sqrt{a^2 + b^2}} = \frac{2}{\sqrt{5}}$

24. Let $P(x_1, y_1)$ be the point of intersection of curves.

$\frac{x^2}{a^2} + \frac{y^2}{4} = 1 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = m_1 = \frac{-4x_1}{a^2 y_1}$

$y^3 = 16x \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = m_2 = \frac{16}{3{y_1}^2}$

$m_1 m_2 = -1 \Rightarrow a^2 = \frac{4}{4}$

25. Slope of tangent at $(3,6) = \left(\frac{dy}{dx}\right)_{(3,6)} = 4$

Given curve $y^2 = ax^4 + b$

$\frac{dy}{dx} = \frac{4ax^3}{2y}$

$\Rightarrow$slope (m) $= \frac{4a \times 3^3}{2 \times 6} = 9a$

$9a = 4 \Rightarrow a = \frac{4}{9}$

$(3,6)$ lies on the curve $\Rightarrow b = 0$

26. $\frac{dy}{dx} = \frac{1 - x^2}{(x^2 + 1)^2}$

$\left(\frac{dy}{dx}\right)_{x=-4} = \frac{-15}{289}$

Slope of normal $= \frac{289}{15}$

27. Equations of tangent at (a,a) is

$\frac{x}{-a/5} + \frac{y}{a/6} = 1$

$\alpha = \frac{-a}{5}, \beta = \frac{a}{6}$

Substitute α & β in $\alpha^2 + \beta^2 = 61$

We get $|a| = 30$

28. $\frac{dx}{d\theta} = a(1 - \cos\theta)$

$\frac{dy}{d\theta} = a \sin\theta$

$\Rightarrow \frac{dy}{dx} = \cot\frac{\theta}{2}$

$m = \left(\frac{dy}{d\theta}\right)_{\theta = \frac{\pi}{6}} = \sqrt{3}$

Sum of sub tangent and subnormal

$= \left| y\left(m + \frac{1}{m}\right) \right| = \frac{2a}{\sqrt{3}}$

29. $x^2 + y^2 = 4x$

$\left(\frac{dy}{dx}\right)_{(2,2)} = m_1 = 0$

$x^2 + y^2 = 8$

$\left(\frac{dy}{dx}\right)_{(2,2)} = m_2 = -1$

$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = 1 \Rightarrow \theta = \frac{\pi}{4}$

$\sin^{-1}(a)=\frac{\pi}{4}\Rightarrow a=\frac{1}{\sqrt{2}}$

30. Slope of tangent = -1

$2x-3=-1$

$x=1$

$\Rightarrow y=0$

$\therefore$ Required point is $(1,0)$

31. Parallel to x-axis $\Rightarrow$ slope = 0

$3x^2=0\Rightarrow x=0$

$\Rightarrow y=0$

$\therefore$ Required point is $(0,0)$

32. Area of triangle formed by x-axis, tangent and normal is $\frac{y^2}{2}\left|m+\frac{1}{m}\right|$

33. L.S.N = ym

L.S.T $=\frac{y}{m}$

$p(ym)=q\left(\frac{y}{m}\right)^2$

$p(ym)=q\frac{y^2}{m^2}\Rightarrow pm^3=qy$

$p\left[\frac{3(x+a)^2}{2by}\right]^3=qy$

$\Rightarrow \frac{p}{q}=\frac{8b}{27}$

34. Tangent is parallel to x-axis

$\Rightarrow$slope = 0

$\frac{dy}{dx}=\frac{-x}{y}=0$

$\Rightarrow x=0$

$\Rightarrow y=a\ [\because y\geq 0]$

$\therefore$ point on the curve is $(a,0)$

35. (a^2,a) is the point of intersection of curves

$m_1=\frac{1}{2a}, m_2=\frac{1}{a}$

$m_1m_2=-1\Rightarrow a^2=\frac{1}{2}$

36. Given $y=\frac{16}{x}-x^2$.....(1)

$y'=\frac{-16}{x^2}-2x$

Tangent as horizontal line $m=0\Rightarrow y'=0$

$\Rightarrow x=-2$

From (1), $y=-12$

$\therefore P(-2,-12)$

37. Given $y=\pi e^{\frac{-x}{\pi}}$.......(1)

$y'=-e^{\frac{-x}{\pi}}$

Tangent crosses y-axis $\Rightarrow x=0$

From (1), $y=\pi$ $m=(y')_P=-1$

$\therefore P(0,\pi)$

Equation of tangent $y-y_1=m(x-x_1)$

$x+y=\pi$

38. Given $y=4x^4+x$

$y'=16x^3+1$

$m_1=\left(\frac{dy}{dx}\right)_{P(x_1,y_1)}=16x_1^3+1$

$m_2=\left(\frac{dy}{dx}\right)_{O(0,0)}=1$

Since $m_1m_2=-1$

$16x_1^3+1=-1$

$\Rightarrow x_1=-\frac{1}{2}$

Since P lies on the curve $y_1=-\frac{1}{4}$

$\therefore P\left(-\frac{1}{2},-\frac{1}{4}\right)$

39. Given curves

$2x^2+y^2=20....(1)$

$4y^2-x^2=8....(2)$

P.O.I of $(1)\&(2)$ we get $y=\pm 2$

If $y=2$ then $x=\pm 2\sqrt{2}$

If $y=-2$ then $x=\pm 2\sqrt{2}$

But point lies in Q_4

$\therefore P(2\sqrt{2},-2)$

From curve (1)

$2x^2+y^2=20$

$4x+2yy'=0$

$y'=-\frac{2x}{y}$

$m_1=\left(\frac{dy}{dx}\right)_P=2\sqrt{2}$

From curve (2)

$8yy'-2x=0$

$y'=\frac{x}{4y}$

$m_2=\left(\frac{dy}{dx}\right)_P=-\frac{1}{2\sqrt{2}}$

$\therefore m_1m_2=-1$

$\therefore$ angle between the curves is $\frac{\pi}{2}$

40. $xy^5+2x^2y-x^3+y+1=0...(1)$

Put $x=0$, $y+1=0 \Rightarrow y=-1$

Diff (1) w.r.t x

$\Rightarrow y'=\frac{3x^2-4xy-y^5}{5xy^4+2x^2+1}$

$\Rightarrow (y')_{(0,-1)}=m=1$

Equation of tangent at $(0,-1)$ is :

$y-y_1=m(x-x_1)$

$\Rightarrow x-y-1=0$

41. $x=y^2, xy=k$

Let $P(x,y)$ be the point of intersection

$x=y^2 \Rightarrow \frac{k}{y}=y^2$

$\Rightarrow y=k^{1/3}$

Now, $x=y^2=k^{2/3}$

$\therefore P\left(k^{2/3},k^{1/3}\right)$

Now, $x=y^2$ $xy=k$

$\Rightarrow y'=\frac{1}{2y} \Rightarrow y'=\frac{-y}{x}$

$\Rightarrow m_1=\frac{1}{2.k^{1/3}} \Rightarrow m_2=-k^{-1/3}$

Given : $m_1m_2=-1 \Rightarrow k=\frac{1}{2\sqrt{2}}$.

42. Given curve $xy+ax+by=0$ at $(1,1)$

$1+a+b=0.....(1)$

$xy+ax+by=0$

Diff w.r.to x

$x\frac{dy}{dx}+y+a+b\frac{dy}{dx}=0$

$(x+b)\frac{dy}{dx}=-(a+y)$

$\frac{dy}{dx}=\frac{-(a+y)}{x+b} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)}=\frac{-(a+1)}{1+b}$

$\tan^{-1}2=\theta \Rightarrow 2=\tan\theta$

$2=\frac{-(a+1)}{1+b} \Rightarrow 2+2b+a+1=0 \Rightarrow a+2b+3=0...(2)$

From (1) & (2), $a=1, b=-2$

$\frac{a+b}{ab}=\frac{1-2}{1(-2)}=\frac{1}{2}$

43. $y^2=(2x+1)^3$

$2yy'=3(2x+1)^2.2=6(2x+1)^2$

$y'=\frac{3(2x+1)^2}{y}$

LST $=\left|\frac{y_1}{m}\right|=\left|\frac{y_1}{\frac{3(2x+1)^2}{y_1}}\right|=\left|\frac{y_1^{\,2}}{3(2x+1)^2}\right|$

LSN $=|y_1m|=3.(2x+1)^2$

$\frac{LSN}{(LST)^2}=27$

44. $2x+2y\frac{dy}{dx}=0.....(1)$, $2x-2y\frac{dy}{dx}=0....(2)$

$m_1=\frac{dy}{dx}=\frac{-x}{y}$, $m_2=\frac{dy}{dx}=\frac{x}{y}$

$x^2+y^2=2020\sqrt{2}$, $x^2-y^2=2020$

P.O.I (1) and (2),

$x=\sqrt{100(\sqrt{2}+1)},y=\sqrt{1010(\sqrt{2}-1)}$

$m_1=-\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}},m_2=\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}$

Tan $\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=1$

$\theta=\frac{\pi}{4}$

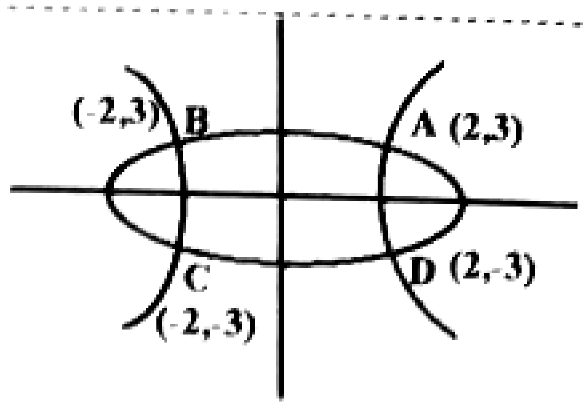
$\frac{\sin\theta+\cos\theta}{\operatorname{Tan}\theta}=\frac{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{1}=\sqrt{2}$

45. $P(x_1,y_1)=\left(\frac{\pi}{4},\frac{1}{\sqrt{2}}\right)$ and normal slope $=\frac{\sqrt{2}}{3}$

Equation of normal is $y-\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{3}\left(x-\frac{\pi}{4}\right)$

$\therefore y=\frac{\sqrt{2}}{3}\left(x+\frac{6-\pi}{4}\right)$

46.



$\theta_1=\theta_2=\theta_3=\theta_4$

47. Let $P(x,y)$

$y=x^3-3x^2-8x-4$

$m_1=\frac{dy}{dx}=3x^2-6x-8\rightarrow(1)$

$y=3x^2+7x+4$

$m_2+\frac{dy}{dx}=6x+7\rightarrow(2)$

$m_1=m_2$

$3x^2-6x-8=6x+7$

$3x^2-12x-15=0$

$x^2-4x-5=0$

$(x-5)(x+1)=0$

$x=5\ x=-1\ y=0$

48. conceptual

49. $x\cos\alpha+y\sin\alpha=p....(1)$

$\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2\quad\left(\frac{dy}{dx}\right)_{(a,b)}=\frac{-b}{a}$

Slope of eq (1) is $\frac{-\cos\alpha}{\sin\alpha}$

$b\sin\alpha=a\cos\alpha.....(2)$

Put (a,b) in eq.(1) then

$a\cos\alpha+b\sin\alpha=p....(3)$

On solving (2) & (3) we get

$\sin\alpha=\frac{p}{2b}$

$\cos\alpha=\frac{p}{2a}$

Substitute in $\cos^2\alpha+\sin^2\alpha=1$

50. $m_1 m_2 = -1$

51. conceptual

52. The line joining of two points $(0,3),(5,-2)$ is

$y = -x+3....(1)$

$y = \frac{c}{x+1}.....(2)$

On solving (1) and (2)

$\frac{c}{x+1} = 3-x$

$c = (3-x)(x+1)$

$\Delta = 0$

$c = 4$

53. $Tan\alpha = \left|\frac{m_1 - m_2}{1+m_1 m_2}\right|$

54. Area = $\frac{C^2}{2|ab|}$

55. Equation is $y = mx$, curve is $y = x^2+x+16$

$\frac{dy}{dx} = 2x+1$ at $P(\alpha,\beta)$

$M = 2\alpha+1$

$\alpha^2+\alpha+16 = (2\alpha+1)\alpha$

$\alpha^2 = 16$

$\alpha = 4$

$M = 2\alpha+1 = 9$

$M+4 = 9+4 = 13$

56. $xy = 1$

$x\frac{dy}{dx}+y = 0$

$\frac{dy}{dx} = -\frac{y}{x}$

at $P(x_1\ y_1)\ M = -\frac{y_1}{x_1}$

Equation of Normal is

$y - y_1 = \frac{x_1}{y_1}(x-x_1)$

$y_1 y - y_1^2 = xx_1 - x_1^2$

$x_1 x - y_1 y + y_1^2 - x_1^2 = 0$

$ax+by+c=0$

$a = x_1$ $\quad b = -y_1$

$a > 0$ $\quad b < 0$

57. Find $\frac{dy}{dx}$ at k = 1

58. $\frac{dy}{dx}$ is not defined

59. At $P(x_1,y_1)$

$x_1 = y_1 \Rightarrow y_1 = \sqrt{9-2x_1^2}$

$x_1 = \sqrt{9-2x_1^2}$

$x_1 = \sqrt{3}\ P(\sqrt{3},\sqrt{3})$

60. Find M_1, M_2 and using $\tan\theta = \left|\frac{m_1 - m_2}{1+m_1 m_2}\right|$

61. Find Normal equation and verify options

62. $y = ax^3+bx^2+cx+5 \to (1)$

$P(-2,0)$ lies on (1)

$0 = -8a+4b-2c+5$

Slope = 0 $\quad \frac{dy}{dx} = 0 \quad 12a-4b+c = 0$

63. Find m_1, m_2 and verify options

64. $\frac{dy}{dx} = 1-\frac{8}{x^3} = 0$

$x = 2$

$y = 2+1 = 3$

Equation of Tangent is y = 3

65. $\frac{dy}{dx} = f'(x),\ m = f'(3)$

Normal slope = $-\frac{1}{f'(3)} = -1 \quad f'(3) = 1$

66. Find $\frac{dy}{dx}$ (a, b)

And Find equation of Tangent

67. $x=0,\ 2y=e^{-x/2}$

$y=1/2$

$M=\frac{dy}{dx}=\frac{-1}{4}$

$Tan\alpha=-\frac{1}{4}$

$\theta=\frac{\pi}{2}-\alpha$

$Tan\theta=\tan\left(\frac{\pi}{2}-\alpha\right)$

$=|\cot\alpha|=4$

$\theta=Tan^{-1}(4)$

68. Find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{3}$

69. $y^2=4x$, $y=e^{\frac{-x}{2}}$

$2yy'=4$ $y'=e^{\frac{-x}{2}}\left(\frac{-1}{2}\right)$

$y'=\frac{2}{y}$

$m_1m_2=\left(\frac{2}{y}\right)e^{\frac{-x}{2}}\left(\frac{-1}{2}\right)$

$=\frac{-e^{\frac{-x}{2}}}{y}=-1$

$\theta=\frac{\pi}{2}$

70. Given curve $y^2=4x$, point P(1, 2)

Equation of Tangent is

$x-y+1=0$ meet y–axis B(0,1)

Equation of Normal is $x+y-3=0$ meet at y-axis at A(0, 3)

Area of $\Delta^{le}\ PAB$ is = 1

71. Find m_1 and m_2 after check it

72. $y^2=4x, x^2+y^2=5$

Solving we get (1, 4)

Find m_1, m_2 and apply

$Tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

73. $4x^2+9y^2=36$

$\frac{x^2}{9}+\frac{y^2}{4}=1$

$a=3\ \ b=2$

Equation of Normal is

$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

74. $x^2+y^2=4,\ y^2=3x$ solving we get $(1,\sqrt{3})$ and Find m_1, m_2

$Tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

75. $x=0 \Rightarrow y=0$

Find $\frac{dy}{dx}$ at (0, 0)

Equation of Normal is $y=-\frac{1}{m}x$

76. Cut orthogonally $m_1m_2=-1$

77. $y=x^2\ \ y=(x-a)^2$

Solve we get $\left(\frac{a}{2},\frac{a^2}{4}\right)$

Find slope m_1, m_2 and apply $Tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

78. Find $M=\frac{dy}{dx}$ at (1, 10)

Equation of Tangent is $y-10=m(x-1)$

79. $x^2-y^2=4, y^2=3x$ Solve we get $(4,2\sqrt{3})$

Find Slopes m_1, m_2 and use $Tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

80. $f'(x) > 0 \Rightarrow 3x^2 - 2ax + 1 > 0$

$$\Rightarrow b^2 - 4ac < 0$$

$a^2 - 3 < 0 \Rightarrow aE\left(-\sqrt{3}, \sqrt{3}\right)$

81. Slope of Tangent $= \frac{4t+3}{3t^2 - 8t - 3}$

Normal is parallel to X-Axis

$\therefore 3t^2 - 8t - 3 = 0$

$t = 3, -\frac{1}{3}$

$\therefore$ No of points=2

82. Given curves are.

$y = e^{2(1+x)-4} = e^{2x-2}$

$\frac{dy}{dx} = e^{2x-2}(2)$

$\left(\frac{dy}{dx}\right)_{(1,1)} = 2(m_1)$

$x^2 y = 1$

$x^2 \frac{dy}{dx} + 2xy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-2y}{x} \quad \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = -2(m_2)$

$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{2+2}{1-4}\right| = \frac{4}{3}$

$|\sin\theta| + |\cos\theta| = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$

83. *Given curve is* $y = x^2 + 3x + 4$

let $P(x_1, y_1)$ *be the point of contact*

$\therefore y_1 = x_1^2 + 3x_1 + 4 \rightarrow (1)$

Also $\frac{dy}{dx} = 2x + 3$

$\left(\frac{dy}{dx}\right)_p = 2x_1 + 3$

Equation of tangent *at p is*

$y - y_1 = m(x - x_1)$

$= (2x_1 + 3)(x - x_1)$

it passes through $(0,0)$, *then*

$-y_1 = -x_1(2x_1 + 3)$

$\Rightarrow x_1^2 + 3x_1 + 4 = 2x_1^2 + 3x_1$

$\Rightarrow x_1^2 = 4$

$\Rightarrow x_1 = \pm 2$

If $x_1 = 2$ *then* $y_1 = 14$

If $x_1 = -2$ *then* $y_1 = 2$

$\therefore (\alpha, \beta) = (2, 14) \,\&\, (\gamma, \delta) = (-2, 2)$

Hence $\beta + \delta = 14 + 2 \Rightarrow 16$

84. Given $x = 1 + \frac{1}{y^2}$ *at* $A(2,1)$

$1 = \frac{-2}{y^2}\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-y^2}{2}$

$\left(\frac{dy}{dx}\right)(2,1) = -\frac{1}{2}$

$B(\alpha, \beta)$

Slope of $AB = \frac{\beta - 1}{\alpha - 2} = -\frac{1}{2}$

$\alpha + 2\beta - 4 = 0 .....(1)$

$\alpha = 1 + \frac{1}{\beta^2} .....(2)$

Solve (1) & (2)

85. Given

$y^2 = x + \sin x$

$2y\frac{dy}{dx} = 1 + \cos x$

$1 + \cos x = 0$

$\cos x = -1$

$x = \pi$

$y^2 = x$

86. (α,β) lies on y=f(x) and px+my+n=0

$\therefore$ px+my+n=0 intersect curve when slope of line $\neq$ slope of tangent

$\therefore \frac{-p}{m} \neq f^1(\alpha)$

$\Rightarrow p + m.f^1(\alpha) \neq 0$

87. Let $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ slope $m = \frac{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2}{2\frac{1}{2}\frac{\sqrt{3}}{2}}$

$= \frac{\frac{3}{4} - \frac{1}{4}}{\frac{\sqrt{3}}{2}} = \frac{2}{4} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

Equation of normal

$y - \frac{\sqrt{3}}{2} = -\sqrt{3}\left(x - \frac{1}{2}\right)$

$\Rightarrow \sqrt{3}x + y = \sqrt{3}$

88. $y = Tan^{-1}(\sin\sqrt{x})$

$\frac{dy}{dx} = \frac{1}{1 + \sin^2\sqrt{x}} \times \cos\sqrt{x} \times \frac{1}{2\sqrt{x}}$

Tangent is parallel to X – axis

$\frac{dy}{dx} = 0$

$\Rightarrow \cos\sqrt{x} = 0$

$\Rightarrow \sin\sqrt{x} = \pm 1$

$y = \tan^{-1}(\pm 1) = \pm\frac{\pi}{4}$

89. $(a^2 - 1)x + ay + (3 - a) = 0$

Slope of above curve $m_1 = \frac{-(a^2 - 1)}{a}$

Slope of the given curve $\frac{-1}{x^2}$

Slope of the normal $m_2 = x^2$

$m_1 \geq 0 \qquad (\because m_1 = m_2)$

$\frac{-(a^2 - 1)}{a} \geq 0$

$\frac{a^2 - 1}{a} \leq 0$

$a(a^2 - 1) \leq 0$

$a(a + 1)(a - 1) \leq 0$

$\therefore a \in (-\infty, -1] \cup (0, 1]$

90. $x = e^t \cos t,\ y = e^t \sin t\ \ AT\ \ x = 1 \Rightarrow t = 0$

$m = \frac{e^t(\sin t + \cos t)}{e^t(\cos t - \sin t)} \ \therefore (m)_{t=0} = 1$

Slope of tangent = 1,

Slope of normal $= -1$

$\therefore$ inclination of tangent = $\frac{\pi}{4}$

Inclination of normal = $\frac{3\pi}{4}$

91. $y = \sin x$ locus point $P(h,k)$ p (h ,k)

$\frac{dy}{dx} = \cos x$

Slope of normal $= -\frac{1}{\cos h}$

Equation of normal is

$$y-k=-\frac{1}{\cos h}(x-h)$$

Which is passes through (0,0)

$$-k=\frac{h}{\cos h} \Rightarrow \cos h=\frac{-h}{k}$$

$\because (h,k)$ *lies on* $y=\sin x$

$$\Rightarrow k=\sin h$$

$$\sin^2 h+\cos^2 h=1$$

$$k^2+\frac{h^2}{k^2}=1$$

$$k^4+h^2=k^2 \Rightarrow h^2=k^2-k^4$$

locus is $x^2=y^2-y^4$

92. $y=x^3,\ p(\alpha,\beta)=(t,t^3), Q(\alpha_1,\beta_1)=(t_1,t_1^3)$

Slope of $P\overline{Q}=\frac{t_1^3-t^3}{t_1-t}=t_1^2+tt_1+t^2$

$$\because y=x^3 \Rightarrow \frac{dy}{dx}=3x^2 \quad \therefore (m)_p=3t^2$$

$$\therefore t_1^2+t^2+t\,t_1=3t^2 \quad \Rightarrow t_1^2+t\,t_1-2t^2=0$$

$$\Rightarrow (t_1+2t)(t_1-t)=0 \quad t_1=-2t$$

$$\therefore Q(-2t,-8t^3) \quad \therefore \frac{\beta_1}{\beta}=\frac{-8t^3}{t^3}=-8$$

93. Tangent parallel to x-axis $\Rightarrow \frac{dy}{dx}=0$

Differentiating given equation

$$3x^2y^2+x^3.2y.\frac{dy}{dx}+\frac{2xy-x^2\frac{dy}{dx}}{y^2}=0$$

$$\frac{dy}{dx}=0 \Rightarrow 3x^2y^2+\frac{2x\cancel{y}}{y^{\cancel{2}}}=0 \quad \Rightarrow 3x^2y^3+2x=0$$

By option verification $\left(-2,\frac{1}{\sqrt[3]{3}}\right)$ is required

point

94. $x=e^{\sin y} \Rightarrow \log x=\sin y$

$$\frac{1}{x}=\cos y\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{1}{x\cos y}$$

$$m=\frac{1}{1.\cos 0}=1$$

$$\boxed{\therefore m=1}$$

equation : $y-0=\frac{-1}{1}(x-1)$

$$x+y-1=0$$

Area of Δle $=\frac{c^2}{2|ab|}=\frac{1}{2}$

95. $y=\frac{x}{x^2+1}+3$

Horizontal tangent $\Rightarrow \frac{dy}{dx}=0$

$$\frac{1(x^2+1)-x(2x)}{(x^2+1)^2}+0=0 \Rightarrow x^2=1 \Rightarrow x=\pm 1$$

Horizontal tangent $\Rightarrow$ vertical normal

$\Rightarrow$ equation is $x=\pm 1$

96. $m=\sqrt{2}e$

Equation of normal:

$$y-e=\frac{-1}{\sqrt{2}e}\left(x-\sqrt{2}\right)$$

$$x+\sqrt{2}ey=\sqrt{2}\left(1+e^2\right)$$

$$\frac{x}{\sqrt{2}(1+e^2)}+\frac{ey}{1+e^2}=1$$

$$\frac{b}{a}=\frac{\frac{e}{\cancel{(1+e^2)}}}{\frac{1}{\sqrt{2}\cancel{(1+e^2)}}}=\sqrt{2}e$$

97. Let (x1, y1) be the locus point

Given curve y = sin x

$$\frac{dy}{dx} = \cos x$$

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \cos x_1 = (m)$$

Eqn of tangent y – y1 = cos x1 (x – x1) → (1)

(1) passes through (0, □)

$$\pi - y_1 = \cos x_1 (0 - x_1)$$

$$y_1 - \pi = x_1 \cos x_1$$

$$\boxed{\frac{y_1 - \pi}{x_1} = \cos x_1} \rightarrow (2)$$

(x_1, y_1) lies on $y = \sin x$

$$\boxed{y_1 = \sin x_1} \rightarrow (3)$$

$$(2)^2 + (3)^2 \Rightarrow \frac{(y_1 - \pi)^2}{x_1^2} + y_1^2 = 1$$

$$(y_1 - \pi)^2 + x_1^2 y_1^2 = x_1^2$$

$$(y_1 - \pi)^2 = x_1^2 (1 - y_1^2)$$

Locus of (x1,y1) is

$$\boxed{(y - \pi)^2 = x^2 (1 - y^2)}$$

98. Given curve y = x2 + 5

$$\frac{dy}{dx} = 2x$$

$$\left(\frac{dy}{dx}\right)_{(-2,9)} = 2(-2) = -4 = (m_1)$$

"C" value verify from options

Option(1) take C = (-5, 30)

Slope of BC = $\frac{30 - 6}{-5 - 1} = \frac{24}{-6} = -4 = (m_2)$

$$\boxed{m_1 = m_2}$$

99. Given y = $\frac{1 + 3x^2}{3 + x^2} \rightarrow (1)$ Given y = 1 → (2)

Solve (1) & (2) $x = \pm 1$

Point of intersection (1, 1) (-1, 1)

$$\left(\frac{dy}{dx}\right)_{(1,1)} = 1 \qquad \left(\frac{dy}{dx}\right)_{(-1,1)} = -1$$

Eqn of normal at (1, 1)	*Eqn of normal at* (–1,1)
y – 1 = - 1 (x – 1)	y – 1 = 1 (x + 1)
y – 1 = -x + 1	x – y = - 2 → (3)
x + y = 2 → (4)	

solve (3) & (4) x = 0, y = 2

(α, β) = (0, 2)

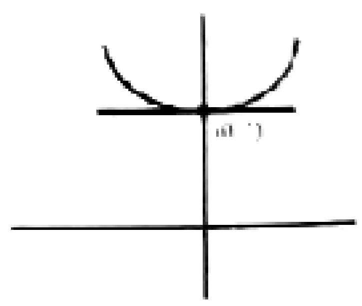
$$3\alpha + 2\beta = 3(0) + 2(2) = 4$$

100. y = cosh x

normal at (0, 1) is

y – axis whose eqn is

x = 0



101. conceptual

102. $px + my + n = 0$ is a tangent to y=f(x) at $x = \alpha$

$$\Rightarrow Slope = f^1(x) = \frac{-p}{m}$$

$$f^1(\alpha) = \frac{-p}{m}$$

Consider $\frac{d}{dx}\left[f\left(\alpha.e^{2x}\right)\right]=f'\left(\alpha.e^{2x}\right).\left(\alpha.e^{2x}\right)(2)$

Put $x=0$ then

$=f'(\alpha).(\alpha.(1)(2))$

$=\frac{-p}{m}(2\alpha)=\frac{-2p\alpha}{m}$

103. x=2sint, y=2cost

x=2, y=0

(x,y)=(2,0)

$x=2\sin t$ $y=2\cos t$

$\frac{dx}{dt}=2\cos t$ $\frac{dy}{dt}=-2\sin t$

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=-\frac{2\sin t}{2\cos t}$

$=-\tan t$

$m=\left(\frac{dy}{dt}\right)_{t=\frac{\pi}{2}}=\frac{-1}{0}$

slope of normal $=0$

Equation of Normal $y-0=0(x-2)$

$\Rightarrow y=0$

104. Given $y=e^{a+bx^2}\rightarrow(1)$ & $P(1,1)$, m = −2

$\frac{dy}{dx}=e^{a+bx^2}(2bx)$

$2be^{a+b}=-2 \Rightarrow be^{a+b}=-1\Rightarrow b=-1$

& from(1) $\Rightarrow e^{a+b}=1\Rightarrow e^{a-1}=1\Rightarrow a-1=\log_e 1$

$a=1$

Now $2a-3b=2+3=\underline{5}$

105. $y=\cos(x+y)$

$\frac{dy}{dx}=-\sin(x+y)\left(1+\frac{dy}{dx}\right)$

$-\frac{1}{2}=-\sin(x+y)\left(1-\frac{1}{2}\right)$

$1=\sin(x+y)$

$x+y=\frac{\pi}{2}\Rightarrow y=\cos\frac{\pi}{2}=0$

$\therefore\left(\frac{\pi}{2},0\right)$

$x+2y=k$

$\therefore \boxed{k=\frac{\pi}{2}}$

106. $y=x^3-10x^2+31x-30$

$\frac{dy}{dx}=3x^2-20x+17=0$

$x=1, x=\frac{17}{3}$

If $x=1,\Rightarrow y=-8$

$p=(1,-8), m=14$

$y+8=14(x-1)$

$14x-4=22$

$\frac{x}{22/14}-\frac{y}{22}=1$

x inter cept$=\frac{11}{7}$

107. Conceptual

108. $x=t^2-7t+7,\quad y=t^2-4t-10$

$x-y=-3t+17\Rightarrow t=\frac{x-y-17}{-3}$

$4x-7y=4t^2-28t+28-7t^2+28t+70$

$4x-7y=-3t^2+98$

$4x-7y=-3\left(\frac{x-y-17}{-3}\right)^2+98$

$4x-7y=-3\left(\frac{x-y-17}{9}\right)^2+98$

$4-7\frac{dy}{dx}=-\frac{2}{3}(x-y-17)\left(1-\frac{dy}{dx}\right)$

Slope of tangent at (1, 2)

$$4-7\frac{dy}{dx}=-\frac{2}{3}(-18)\left(1-\frac{dy}{dx}\right)$$

$$4-7\frac{dy}{dx}=12-12\frac{dy}{dx}$$

$|slope\ of\ normal|$

$$5\frac{dy}{dx}=8 \qquad m=-\frac{5}{8}$$

$$\left(\frac{dy}{dx}\right)_p=8/5$$

Eqn of normal is $y-2=-\frac{5}{8}(x-1)$

$$8y-16=-5x+5$$

$$5x+8y-21=0$$

$$a=5, b=8, c=-21$$

$$m(\text{a+b+c}) = \frac{-5}{8}(-8)=5$$

109. $x^2y-x^3=8\rightarrow(1)$

$y^3-xy^2=32\rightarrow(2)$

$x^2(y-x)=8 \quad y^2(y-x)=32$

$x^3=8 \qquad y^2.\frac{8}{x^2}=32$

$x=2 \qquad \frac{y}{x}=2\Rightarrow y=2x$

$\boxed{x=2} \qquad \boxed{y=4}$

Point of intersection p(2,4)

Diff w r t x by (1)

$x^2y^1+y.2x-3x^2=0$

$y^1=-1$

$m_1=-1$

Diff w r t 'x' by(2)

$3y^2y^1-x.2yy^1-y^2=0$

$12y^1-4y^1-4=0$

$m_2=y^1=\frac{1}{2}$

$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=3$

110. $m=3x^2-5$

$\frac{dy}{dx}=3x^2-5$

Take integration

$y=3.\frac{x^3}{3}-5x+c$

$\therefore at=x=1, y=2$

$2=1-5+c$

$c=6$

$\therefore y=x^3-5x+6$

By verification

$opt(3), (-2,8)$

$8=-8+10+6$

$8=8$

Satisfied