12. TRANSFORMATION OF AXES

1. Find the transformed equation of $x\cos\theta + y\sin\theta = p$, when the axes are rotated through an angle θ **[AP EAMCET 17-09-20_Shift-1]**
   1. X=p
   2. Y=p
   3. X+Y=p
   4. X-Y=p

2. If the axes are rotated through an angle 45^0, then the co-ordinates of the point $\left(4\sqrt{2},-6\sqrt{2}\right)$ in the new system are ___ **[AP EAMCET 17-09-20_Shift-2]**
   1. (-10, -2)
   2. (-2, -10)
   3. (10, 10)
   4. (-2, 10)

3. When the origin is shifted to (2,3) the transformed equation $x^2+3xy-2y^2+17x-7y-11=0$ then the original equation of curve is **[AP EAMCET 18-09-20_Shift-2]**
   1. $x^2-2y^2-3xy+4x-y+20=0$
   2. $x^2-2y^2+3xy+4x-y-20=0$
   3. $x^2-2y^2-3xy-4x-y+20=0$
   4. $x^2-2y^2-3xy+4x-y-20=0$

4. The point to which the origin should be shifted so that the equation $y^2-6y-4x+13=0$ is transformed in the form $y^2+Ax=0$ is **[AP EAMCET 21-09-20_Shift-1]**
   1. (3,1)
   2. (-1,-1)
   3. (1,3)
   4. (-1,3)

5. Find the coordinates of M in the original system if the point M changes to (4,-3) when the axes are rotated through an angle of 135^o **[AP EAMCET 22-09-20_Shift-2]**
   1. $\left(\frac{-1}{2},\frac{7}{2}\right)$
   2. $\left(\frac{1}{2},\frac{7}{2}\right)$
   3. $\left(\frac{-1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$
   4. $\left(\frac{1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$

6. The point to which the origin should be shifted so that the equation $y^2-6y-4x+13=0$ will not contain term in y and the constant term, is ______ **[AP EAMCET 23-09-20_Shift-1]**
   1. (1, 1)
   2. (1, 2)
   3. (2, 1)
   4. (1, 3)

7. Which of the following statement is false?
   1. The area of a triangle is invariant under the translation of the Axes
   2. The slope of a straight line is invariant under the translation of the Axes
   3. The shifting of origin to another point, while changing the direction of the axes, is called translation of axes.
   4. If f(x.y)=0 is transformed equation of a curve when the axes are translated to the point (h,k) then the original equation of the curve is f(x-h,k-k)=0

   1. 1
   2. 2
   3. 3
   4. 4

8. The transformed equation of $3x^2-4xy=r^2$, when the coordinate axes are rotated through an angle $\mathrm{Tan}^{-1}(2)$ is **[TS EAMCET 09-09-20_Shift-1]**
   1. $X^2-4Y^2=r^2$
   2. $2XY+r^2=0$
   3. $4Y^2-X^2=r^2$
   4. $XY=r^2$

9. By shifting the origin to the point (2,3) and then rotating the coordinate axes through am angle θ in the counter clockwise direction, if the equation $3x^2+2xy+3y^2-18x-22y+50=0$ is transformed to $4X^2+2Y^2-1=0$, then the angle $\theta=$ **[TS EAMCET 09-09-20_Shift-2]**
   1. $\frac{\pi}{6}$
   2. $\frac{\pi}{2}$
   3. $\frac{\pi}{4}$
   4. $\frac{\pi}{3}$

10. When the origin is shifted to the point $\left(\frac{3}{2},\frac{3}{2}\right)$ by the translation of coordinate axes,then the transformed equation of $32x^2+8xy+32y^2-108x-108y+99=0$ is **[TS EAMCET 10-09-20_Shift-1]**
    1. $72X^2+56Y^2-63=0$
    2. $X^2-14XY-7Y^2-2=0$
    3. $32X^2-16XY+32Y^2-225=0$
    4. $32X^2+8XY+32Y^2-63=0$

11. The point (4,1) undergoes the following transformations successively:

i) reflection in the line x-y=0

ii) shifting through a distance of 2 units along the positive X-axis

iii) projection on X-axis

[TS EAMCET 10-09-20_Shift-2]

1. (3,4) 2. (4,3)
3. (3,0) 4. (4,0)

12. Let C be a curve

$ax^2+2hxy+by^2+2gx+2fy+c=0$

in a Cartesian plane, by rotating the coordinate axes through an angle $\frac{\pi}{4}$ in the positive direction, if the transformed equation of C is

$Y^2+XY-X=0$, then $(h^2-ab)-2gf=$

[TS EAMCET 11-09-20_Shift-1]

1. 0 2. 2
3. 1 4. -1

13. When the coordinate axes are rotated through an angle θ in anticlockwise direction, if the transformed equation of

$x^2+y^2+2xy+2x+6y+1=0$ *is*

$(2+\sqrt{3})X^2+2XY+(2-\sqrt{3})Y^2+aX+bY+2=0$

Then 3a-b= **[TS EAMCET 11-09-20_Shift-2]**

1. 10 2. $2(1+2\sqrt{3})$
3. 20 4. $2(3+\sqrt{3})$

14. If the axes are rotated through an angle 45^0, the coordinates of the point $(2\sqrt{2},-3\sqrt{2})$ in the new system are

[AP EAMCET 19-08-2021_Shift-1]

1. $(3\sqrt{3},-5)$ 2. $(-1,-5)$
3. $(5\sqrt{3},-7)$ 4. $(7,-\sqrt{3})$

15. When the coordinate axes are rotated through an angle 135^0, the coordinates of a point P in the new system are known to be $(4,-3)$. Then find the coordinates of P in the original system

[AP EAMCET 19-08-2021_Shift-2]

1. $\left(\frac{1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$ 2. $\left(\frac{-1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$
3. $\left(\frac{1}{\sqrt{2}},\frac{-7}{\sqrt{2}}\right)$ 4. $\left(\frac{-1}{\sqrt{2}},\frac{-7}{\sqrt{2}}\right)$

16. When the axes are rotated through an angle 45^0, the new coordinates of a point P are $(1,-1)$. The coordinates of P in the original system are ______ **[AP EAMCET 20-08-2021_Shift-1]**

1. $(\sqrt{2},\sqrt{2})$ 2. $(\sqrt{2},0)$
3. $(0,\sqrt{2})$ 4. $(-\sqrt{2},0)$

17. The point to which the origin should be shifted in order to eliminate the x and y terms from the equation $9x^2+4y^2+10x+12y+1=0$ is

[AP EAMCET 20-08-2021_Shift-2]

1. $\left(\frac{5}{9},\frac{3}{2}\right)$ 2. $\left(\frac{-5}{2},\frac{-3}{9}\right)$
3. $\left(\frac{-5}{9},\frac{-3}{2}\right)$ 4. $\left(\frac{-3}{2},\frac{-5}{9}\right)$

18. The transformed equation $3x^2+3y^2+2xy=2$, when the coordinate axes are rotated through an angle 45^0 is

[AP EAMCET 23-08-2021_Shift-1]

1. $x^2+2y^2=1$ 2. $2x^2+y^2=1$
3. $x^2+y^2=1$ 4. $x^2+3y^2=1$

19. If a square *ABCD* where $A(0,0),B(2,0)$, $C(2,2),D(0,2)$ undergoes the following transformations successively, then the final figure would be a ______

(i) $f_1(x,y)\to(y,x)$

(ii) $f_2(x,y)\to(x+3y,y)$

(iii) $f_3(x,y)\to\left(\frac{x-y}{2},\frac{x+y}{2}\right)$

[AP EAMCET 23-08-2021_Shift-1]

1. Square 2. Rhombus
3. Rectangle 4. Parallelogram

20. Find the transformed equation of the curve $x^2+2\sqrt{3}xy-y^2=8$ when the axes are rotated through an angle $\frac{\pi}{3}$.

[AP EAMCET 24-08-2021_Shift-2]

1. $x^2+y^2+2\sqrt{3}xy=8$
2. $x^2+y^2-2\sqrt{3}xy=8$
3. $x^2-y^2+2\sqrt{3}xy=8$
4. $x^2-y^2-2\sqrt{3}xy=8$

21. The equation obtained by transforming $x^2+y^2-6x+10y-2=0$ to the parallel axis through (3, –5) is _______

[AP EAMCET 25-08-2021_Shift-1]

1. $x^2+y^2=16$ 2. $x^2+y^2=9$
3. $x^2+y^2=25$ 4. $x^2+y^2=36$

22. If the axes are transformed to the point $(-1,1)$ then the equation $3x^2+y^2+2x+4y+15=0$ would transform to

[AP EAMCET 25-08-2021_Shift-2]

1. $3x^2+2y^2-4x+6y+23=0$
2. $3x^2+y^2-4x+6y+21=0$
3. $3x^2+y^2+4x-6y-21=0$
4. $3x^2+y^2+4x+6y+21=0$

23. If $P(a,b)$ is the point to which the origin is to be shifted by translation of axes so as to remove the first degree terms from the equation $4x^2+2xy+y^2-8x-4y-12=0$ and θ is the angle through which the axes are to rotated about the origin so as to remove the xy-term from the above equation, then $a+b+3\tan 2\theta=$ **[TS EAMCET 04-08-2021_Shift-2]**

1. 2 2. 4
3. 8 4. 6

24. The transformed equation of the curve $2x^2+y^2-3x+5y-8=0$ when the origin is translated to the point (-1,2) is

[TS EAMCET 04-08-2021_Shift-1]

1. $2x^2+y^2-7x+9y+11=0$
2. $2x^2+y^2+7x+9y+11=0$
3. $2x^2+y^2-x+y+11=0$
4. $2x^2+y^2+7x-9y+11=0$

25. When the origin is shifted to $(-1,2)$ by the translation of axes, the transformed equation of $x^2+y^2+2x-4y+1=0$ is

[TS EAMCET 05-08-2021_Shift-1]

1. $X^2+Y^2=4$ 2. $X^2+Y^2=16$
3. $X^2+2X+Y^2=4$ 4. $X^2-2X+Y^2=16$

26. The angle by which axes are to be rotated without changing the origin so that the transformed equation of $x^2+4xy-y^2=0$ in new coordinates (X,Y) does not contain XY term is

[TS EAMCET 05-08-2021_Shift-2]

1. $\frac{1}{2}Tan^{-1}(2)$ 2. $Tan^{-1}(2)$
3. $\frac{\pi}{8}$ 4. $\frac{\pi}{4}$

27. The equation of a curve C is transformed to $X^2+Y^2-6X+8Y+21=0$ by the rotation of coordinate axes about the origin through an angle of $\frac{\pi}{4}$ in the positive direction of X-axis. If $ax^2+by^2+cx+dy+e=0$ is the equation of the curve C before the transformation, then $\left(a+b+c^2+d^2-5e\right)^2=$

[TS EAMCET 06-08-2021_Shift-2]

1. 4 2. 9
3. 16 4. 25

28. If the coordinate axes are rotated in positive direction by 45^0 without changing the origin, then the transformed equation of $3x^2+3y^2+2xy-2=0$ is

[TS EAMCET 06-08-2021_Shift-1]

1. $2X^2+Y^2=1$ 2. $X^2+2Y^2=1$
3. $X^2-2Y^2=1$ 4. $2X^2-Y^2=1$

29. When the coordinate axes are rotated about the origin in the positive direction through an angle $\frac{\pi}{4}$, if the equation $49x^2+25y^2=1225$ is transformed to $px^2+qxy+ry^2=t$ and the G.C.D of p,q,r, t is 1, then **[TS EAMCET 18-07-2022_Shift-1]**

1. $(p-q+r-32)^2=4t$ 2. $(p-q-r+12)^2=t$
3. $(p+q+r-15)^2=t$ 4. $(-p-q+r+13)^2=t$

30. The transformed equation of $3x^2+4xy+y^2-8x-4y-4=0$ is $f(X,Y)=aX^2+2hXY+bY^2+c=0$ when the origin is shifted to a new point by the translation of axes. Then $f(1,1)=$

[TS EAMCET 18-07-2022_Shift-2]

1. 0
2. 1
3. -1
4. -8

31. The point to which the origin is to be shifted by translation of axes so that the transformed equation of $y^2+4y+8x-2=0$ will not contain y term and constant term is

[TS EAMCET 19-07-2022_Shift-I]

1. $\left(\frac{3}{4},-2\right)$
2. $\left(\frac{-3}{4},-2\right)$
3. $\left(2,\frac{3}{4}\right)$
4. $\left(-2,\frac{-3}{4}\right)$

32. If $x^2=8ay$ is the transformed equation of $x^2-4y+6x+15=0$ when the origin is shifted to the point (α,β) by translation of axes, then $2\alpha+8\beta^2=$

[TS EAMCET 19-07-2022_Shift-2]

1. 8
2. 18
3. 12
4. 16

33. By rotating the axes through an angle of 30^0 in the anti-clockwise direction about the origin, the equation $4x^2+12xy+9y^2+6x+9y+2=0$ becomes $ax^2+2hxy+by^2+2gx+2fy+c=0$ then **[TS EAMCET 20-07-2022_Shift-2]**

1. $a=21-6\sqrt{3}$
2. $g/f=\frac{3+2\sqrt{3}}{3\sqrt{3}-2}$
3. $b=31+6\sqrt{3}$
4. $c=6$

34. The transformed equation of $2x^2+3y^2-z^2-8x+18y+2z+9=0$ when the axes are translated to the point (2, -3, 1) is **[18th May2023 shift -1]**

1. $2x^2+3y^2-z^2=25$
2. $2x^2+3y^2+z^2=25$
3. $2x^2-3y^2-z^2=25$
4. $2x^2+3y^2-z^2=50$

35. The angle by which the coordinate axes are to be rotated about the origin so that the transformed equation of $\sqrt{3}x^2+\left(\sqrt{3}-1\right)xy-y^2=0$ would be free from xy term is

[12TH MAY 2023 SHIFT-1]

1. 45^o
2. 22.5^o
3. 15^o
4. 7.5^o

36. Let P be the point to which origin has to be shifted by the translation of axes so as to remove the first degree terms from the equation $3x^2+y^2-6x+4y+4=0$ if the origin is shifted to P by the translation of axes, then the transformed equation of $2x^2+3xy-5y^2+2x-23y-24=0$

[13TH MAY 2023 SHIFT-1]

1. $x^2+4xy-3y^2-4x+20y+23=0$
2. $2x^2-3xy+5y^2=0$
3. $2x^2+3xy-5y^2=0$
4. $2x^2+3xy-5y^2-13=0$

37. When the origin is shifted to the point (h,k) by translating the coordinate axes, the equation $S\equiv 2x^2-xy+y^2+2x+3y+1=0$ is changed to $S^1\equiv ax^2+2hxy+by^2-3=0$. Again by rotating the coordinate axes about the new origin through the angle θ in the positive direction, $S^1=0$ is changed to $Ax^2+By^2+C=0$. Then $h+k+\tan 2\theta=$

[EAPCET 13-05-23 SHIFT-2]

1. -4
2. 0
3. 1
4. -1

KEY

1)	1	2)	2	3)	2	4)	3	5)	3
6)	4	7)	3	8)	3	9)	3	10)	4
11)	3	12)	1	13)	3	14)	2	15)	2
16)	2	17)	3	18)	2	19)	4	20)	4
21)	4	22)	2	23)	2	24)	1	25)	1
26)	1	27)	2	28)	1	29)	3	30)	1
31)	1	32)	3	33)	2	34)	1	35)	4
36)	3	37)	1						

SOLUTIONS

1. Sub
$x = X\cos\theta - Y\sin\theta$
$y = X\sin\theta + Y\cos\theta$
In given equation

2. Given
$(x,y) = \left(4\sqrt{2}, -6\sqrt{2}\right)$ *and* $\theta = 45^0$
$X = x\cos\theta + y\sin\theta$
$= 4\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) - 6\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = -2$
$Y = -x\sin\theta + y\cos\theta$
$= -4\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) - 6\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = -10$
$\therefore (x,y) = (-2,-10)$

3. x=X+2 and y=Y+3
T.E is
$X^2 - 2Y^2 + 3XY + 4X - Y - 20 = 0$

4. Given $y^2 - 6y - 4x + 13 = 0$
$(y-3)^2 - 4(x-1) = 0$
req $(h,k) = (1,3)$

5. $\theta = 135^0, (X,Y) = (4,-3)$
$x = \frac{-1}{\sqrt{2}}, y = \frac{7}{\sqrt{2}}$
original coordinates $= \left(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

6. $(y+k)^2 - 6(y+k) - 4(x+h) + 13 = 0$
$y^2 + 6yk + k^2 - 6y - 6k - 4x - 4h + 13 = 0$
$y^2 + (6k-6)y - 4x + (k^2 - 6k - 4h + 13) = 0$
$6k - 6 = 0 \Rightarrow k = 1$
$k^2 - 6k - 4h + 13 = 0 \Rightarrow h = 2$
Shifting point =(2, 1)

7. Conceptual
Statement (3) is false

8. Given original equation $3x^2 - 4xy = r^2$
$\theta = \tan^{-1}(2) \Rightarrow \sin\theta = \frac{2}{\sqrt{5}}, \cos\theta = \frac{1}{\sqrt{5}}$
$x = X\cos\theta - Y\sin\theta = \frac{X - 2Y}{\sqrt{5}}$
$y = X\sin\theta + Y\cos\theta = \frac{2X + Y}{\sqrt{5}}$
Substitute x, y in (1)
$4y^2 - x^2 = r^2$

9. $\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$

10. $(h,k) = \left(\frac{3}{2}, \frac{3}{2}\right)$
$x = X + \frac{3}{2}, y = Y + \frac{3}{2}$
$x = \frac{2X+3}{2}, y = \frac{2Y+3}{2}$
$32\left(\frac{2X+3}{2}\right)^2 + 8\left(\frac{2X+3}{2}\right)\left(\frac{2Y+3}{2}\right) +$
$32\left(\frac{2Y+3}{2}\right)^2 - 108\left(\frac{2X+3}{2}\right) - 108\left(\frac{2Y+3}{2}\right) + 99 = 0$
$\therefore 32X^2 + 32Y^2 + 8XY - 63 = 0$

11. $(i)\ \frac{h-4}{1} = \frac{k-1}{-1} = \frac{-2(4-1)}{2}$
$(h,k) = (1,4)$
$(ii)\ (1+2,4) = (3,4)$
$(iii)\ (3,0)$

12. Given curve
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
w.k.t
$x = X\cos\theta - Y\sin\theta = \frac{X-Y}{\sqrt{2}}$
$y = X\sin\theta + Y\cos\theta = \frac{X+Y}{\sqrt{2}}$
On substituting and simplifying, we get
$\left(\frac{a}{2} + h + \frac{b}{2}\right)X^2 + \left(\frac{a}{2} - h + \frac{b}{2}\right)Y^2 + (-a+b)XY +$
$\sqrt{2}(g+f)X + \sqrt{2}(f-g)Y + C = 0$
compare above equation with
$Y^2 + XY - X = 0$
We get
$h = \frac{-1}{2}, a = 0, b = 1, g = \frac{-1}{2\sqrt{2}}, f = \frac{-1}{2\sqrt{2}}$
Now, $\left(h^2 - ab\right) - 2gf = 0$

13. Given equation $x^2 + y^2 + 2xy + 2x + 6y + 2 = 0$

if the axes are rotated an angle 'θ' in anti clockwisedirection, transformed equation

$$\left(2+\sqrt{3}\right)x^2+2xy+(2-\sqrt{3})y^2+ax+by+2=0$$

$$x=X\cos\theta-Y\sin\theta,\ y=X\sin\theta+Y\cos\theta$$

$$\Rightarrow (X\cos\theta-Y\sin\theta)^2+(X\sin\theta+Y\cos\theta)^2$$
$$+2(X\cos\theta-Y\sin\theta)(X\sin\theta+Y\cos\theta)$$
$$+2(X\cos\theta-Y\sin\theta)+6(X\sin\theta+Y\cos\theta)$$
$$+2=0$$

$$2(1+\sin 2\theta)X^2+2(1-\sin 2\theta)Y^2$$
$$+2\left(2\cos\theta+6\sin\theta\right)X$$
$$+2\left(6\cos\theta-2\sin\theta\right)Y+2=0$$

$$\Rightarrow 2+2\sin 2\theta=2+\sqrt{3}\Rightarrow \sin 2\theta=\frac{\sqrt{3}}{2}$$

$$\theta=\frac{\pi}{6}$$

Compare

$$a=4\cos\theta+12\sin\theta=4.\frac{\sqrt{3}}{2}+12.\frac{1}{2}=2\sqrt{3}+6$$

$$b=12\cos\theta-4\sin\theta=12.\frac{\sqrt{3}}{2}-4.\frac{1}{2}=6\sqrt{3}-2$$

$$3a-b=6\sqrt{3}+18-6\sqrt{3}+2=20$$

14. $\theta=45^0$, $(x,y)=\left(2\sqrt{2},-3\sqrt{2}\right)$

$$X=x\cos\theta+y\sin\theta,\ Y=-x\sin\theta+y\cos\theta$$

$$X=-1,Y=-5 \quad \therefore (X,Y)=(-1,-5)$$

15. $\theta=135^0,(X,Y)=(4,-3)$

$$x=X\cos\theta-Y\sin\theta,\ y=X\sin\theta+Y\cos\theta$$

$$x=\frac{-1}{\sqrt{2}},y=\frac{7}{\sqrt{2}} \qquad \therefore (x,y)=\left(\frac{-1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$$

16. $\theta=45^0$, $(X,Y)=(1,-1)$

$$x=X\cos\theta-Y\sin\theta,\ y=X\sin\theta+Y\cos\theta$$

$$x=\sqrt{2},y=0 \quad \therefore (x,y)=\left(\sqrt{2},0\right)$$

17. $9x^2+4y^2+10x+12y+1=0$

The point to which the origin should be shifted in order to eliminate the x and y terms

$$=\left(\frac{-g}{a},\frac{-f}{b}\right)=\left(\frac{-5}{9},\frac{-3}{2}\right)$$

18. $\theta=45^0$,

Original equation is $3x^2+3y^2+2xy=2...(1)$

$$x=\frac{X-Y}{\sqrt{2}},\ y=\frac{X+Y}{\sqrt{2}}$$

From (1), $2X^2+Y^2=1$

19. $A(0,0),B(2,0),C(2,2),D(0,2)$

$$f_1(x,y)=(y,x)$$
$$=A(0,0),B(0,2),C(2,2),D(2,0)$$
$$f_2(x,y)=(x+3y,y)$$
$$=A(0,0),B(6,2),C(8,2),D(2,0)$$
$$f_3(x,y)=\left(\frac{x-y}{2},\frac{x+y}{2}\right)$$
$$=A(0,0),B(2,4),C(3,5),D(1,1)$$
$$AB=\sqrt{20},BC=\sqrt{2},CD=\sqrt{20},DA=\sqrt{2}$$
$$AC=\sqrt{34},BD=\sqrt{10}$$
$$\therefore AB=CD,\ BC=DA, AC\neq BD$$

$\therefore$ parallelogram

20. $\theta=\frac{\pi}{3}=60^0$

Original equation is $x^2+2\sqrt{3}xy-y^2=8.....(1)$

$$x=X\cos\theta-Y\sin\theta,\ y=X\sin\theta+Y\cos\theta$$

$$x=\frac{X-\sqrt{3}Y}{2},\ y=\frac{\sqrt{3}X+Y}{2}$$

From (1), $X^2-Y^2-2\sqrt{3}XY=8$

21. $(h,k)=(3,-5)$,

$$x^2+y^2-6x+10y-2=0.....(1)$$

$$x=X+h,\ y=Y+k$$
$$\Rightarrow x=X+3,\ y=Y-5$$

From (1), $X^2+Y^2=36$

22. $(h,k)=(-1,1)$,

$$3x^2+y^2+2x+4y+15=0.....(1)$$

$$x=X+h,\ y=Y+k$$
$$\Rightarrow x=X-1,\ y=Y+1$$

From (1), $3X^2+Y^2-4X+6Y+21=0$

23. $4x^2+2xy+y^2-8x-4y-12=0$

$$\frac{\partial f}{\partial x}=8x+2y-8=0$$
$$\frac{\partial f}{\partial y}=2x+2y-4=0$$
$$6x=4\Rightarrow x=2/3$$
$$y=4/3$$

$$P(a,b)=\left(\frac{2}{3},\frac{4}{3}\right)$$
$$\theta=\frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)=\frac{1}{2}\tan^{-1}\left(\frac{2}{4-1}\right)$$
$$\tan(2\theta)=\frac{2}{3}\ a+b+3\tan(2\theta)=\frac{2}{3}+\frac{4}{3}+2=4$$

---

24. $x=X-1, y=Y+2$

$$2x^2+y^2-3x+5y-8=0$$
$$2\left(X^2+1-2X\right)+\left(Y^2+4+4Y\right)-3X+3+5Y+10-8=0$$
$$2X^2+Y^2-7X+9Y+11=0$$

---

25. $(h,k)=(-1,2)$

$$x=X+h\Rightarrow x=X-1$$
$$y=Y+k\Rightarrow y=Y+2$$

Given equation $x^2+y^2+2x-4y+1=0$

$$(X-1)^2+(Y-2)^2+2(X-1)-4(Y+2)+1=0$$
$$X^2+Y^2=4$$

---

26. Given equation $x^2+4xy-y^2=0$

$$\theta=\frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$$
$$\theta=\frac{1}{2}\tan^{-1}\left(\frac{4}{2}\right)=\frac{1}{2}\tan^{-1}(2)$$

---

27. Conceptual

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28. Given angle $\theta=45^0$

$$x=X\cos\theta-Y\sin\theta=\frac{X-Y}{\sqrt{2}}$$
$$y=X\sin\theta+Y\cos\theta=\frac{X+Y}{\sqrt{2}}$$

Given original equation is

$$3x^2+3y^2+2xy-2=0$$

Required transformed equation is

$$3\left(\frac{X-Y}{\sqrt{2}}\right)^2+3\left(\frac{X+Y}{\sqrt{2}}\right)^2+2\left(\frac{X-Y}{\sqrt{2}}\right)\left(\frac{X+Y}{\sqrt{2}}\right)-2=0$$
$$\Rightarrow 2X^2+Y^2=1$$

---

29. $49^2+25y^2=1225,\theta=\frac{\pi}{4}$

$$x=\frac{x-y}{\sqrt{2}},y=\frac{x+y}{\sqrt{2}}$$

The transformed eq. is

$$37x^2-24xy+37y^2=1225$$
$$p=37,q=-24,r=37,t=1225$$
$$(p+q+r-15)^2=35^2=t$$

---

30. $3x^2+4xy+y^2-8x-4y-4=0$

Origin shifted to eliminate x, y terms is

$$\left(\frac{hf-bg}{ab-h^2},\frac{gh-af}{ab-h^2}\right)$$

The transformed eq. is

F(x, y) = $3x^2+4xy+y^2-8$

$$f(1,1)=0$$

---

31. $y^2+4y+8x-2=0$

Let $x=X+h$

$y=Y+k$

The transformed equation is

$Y^2+(2k+4)Y+8X+(k^2+4k+8h-2)=0$ to eliminate y term & constant,

$$2k+4=0,\quad k^2+4k+8h-2=0$$
$$k=-2,\ h=\frac{3}{4}$$
$$\therefore(h,k)=(3/4,-2)$$

---

32. $x^2=8ay,\quad x^2-4y+6x+15=0\rightarrow(1)$

The transformed equation of (1) is

$$x^2+x(2\alpha+6)-4y+\alpha^2-2\beta+6\alpha+15=0$$

Compare with $x^2=8ay$

$$a=\frac{1}{2},\alpha=-3,\beta=3/2$$
$$2\alpha+3\beta^2=12$$

---

33. $4x^2+12xy+9y^2+6x+9y+2=0\rightarrow(1)$

$$\theta=30^o$$
$$x=\frac{\sqrt{3}X-Y}{2},y=\frac{X+\sqrt{3}Y}{2}$$

The transformed equation of (1) is

$$\left(21+12\sqrt{3}\right)X^2+\left(10\sqrt{3}+24\right)XY+\left(31-12\sqrt{3}\right)Y^2+$$
$$\left(12\sqrt{3}+18\right)X+\left(18\sqrt{3}-12\right)Y+8=0$$

$a = 21 + 12\sqrt{3}$

$b = 31 - 12\sqrt{3}$

$g = 6\sqrt{3} + 9$, $f = 9\sqrt{3} - 6, c = 8$

$$\frac{g}{f} = \frac{2\sqrt{3}+3}{3\sqrt{3}-2}$$

34. Constant term=S(2, -3, 1)=-25

Required equation is $2x^2 + 3y^2 - z^2 - 25 = 0$

35.
$$\theta = \frac{1}{2} Tan^{-1}\left(\frac{2h}{a-b}\right)$$
$$= \frac{1}{2} Tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$$
$$= \frac{1}{2} Tan^{-1}\left(\frac{(\sqrt{3}-1)^2}{2}\right)$$
$$= \frac{1}{2} Tan^{-1}\left(2-\sqrt{3}\right)$$
$$= 7.5^\circ$$

36. $3x^2 + y^2 - 6x + 4y + 4 = 0$

$a = 3, b = 1, h = 0, g = -3, f = 2, c = 4$

$$\left[\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right] = \left[\frac{\cancel{3}}{\cancel{3}}, \frac{-\cancel{6}}{\cancel{3}}\right] = (1,-2) = (h,k)$$

$x = X + 1, y = Y - 2$

$2x^2 + 3xy - 5y^2 + 2x - 23y - 24 = 0$

$$\begin{pmatrix} 2(x+1)^2 + 3(x+1)(y-2) - 5(y-2)^2 \\ +2(x+1) - 23(y-2) - 24 = 0 \end{pmatrix}$$

$\boxed{2x^2 - 5y^2 + 3xy = 0}$

37. S= 2x2-xy+y2+2x+3y+1=0

a=2 , b=1,c=1,h= $\frac{-1}{2}$ g=1, f= $\frac{3}{2}$

$$\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right) = \left(\frac{-3/4-1}{2-\frac{1}{4}}, \frac{-\frac{1}{2}-3}{2-\frac{1}{4}}\right)$$

$$= \left(\frac{-7/4}{\frac{7}{4}}, \frac{-\frac{7}{4}}{\frac{7}{4}}\right) = (-1,-1) = (h,k)$$

S1 = ax2 + 2hxy+by2-3 =0

= 2x2-2xy+y2-3=0

$$\tan 2\theta = \frac{2h}{a-b}$$
$$= \frac{-2}{2-1} = -2$$

$\therefore$ h + k + tanθ = -1 - 1 - 2= -4