

Statement II is false.

$$262. \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 3 & 4 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \bar{i}(-3-4) - \bar{j}(-2-4) + \bar{k}(2-3)$$

$$= -7\bar{i} + 6\bar{j} - \bar{k}$$

$$\text{dr's of line } (-7, 6, -1)$$

$$\text{Equation of line is } \frac{x-16}{-7} = \frac{y+9}{6} = \frac{z-0}{-1} = \lambda$$

$$x = -7\lambda + 16, y = 6\lambda - 9, z = -\lambda$$

$$\therefore \bar{r} = (-7\lambda + 16)\bar{i} + (6\lambda - 9)\bar{j} + (-\lambda)\bar{k}$$

$$263. \overline{OA} = \bar{a}, \overline{OB} = \bar{b}, \overline{OC} = \bar{c}$$

$$\overline{OD} = \frac{3\bar{c} + \bar{b}}{4}$$

$$\overline{OE} = \frac{4\left(\frac{3\bar{c} + \bar{b}}{4}\right) + 1(\bar{a})}{4+1}$$

$$\overline{OE} = \frac{\bar{a} + \bar{b} + 3\bar{c}}{5}$$

$$\overline{OE} = \frac{3(\overline{OF}) + 2(\overline{OB})}{3+2}$$

$$\frac{\bar{a} + \bar{b} + 3\bar{c}}{5} = \frac{3(\overline{OF}) + 2(\overline{OB})}{5}$$

$$\overline{OF} = \frac{1}{3}(\bar{a} - \bar{b} + 3\bar{c})$$

$$264. \text{ Given } \left(\frac{7}{3} + \beta\right)\bar{i} - \bar{j} + (\alpha + \gamma)\bar{k} = \bar{i}\left(\frac{5}{3}\alpha + 1\right) + \bar{j}\left(\frac{5}{3} + 2\beta + \gamma\right) + \bar{k}\left(\frac{-5}{3} + \beta + 3\right)$$

$$\frac{7}{3} + \beta = \frac{5}{3}\alpha + 1$$

$$5\alpha - 3\beta - 4 = 0 \rightarrow (1)$$

$$-1 = \frac{5}{3} + 2\beta + \gamma$$

$$6\beta + 3\gamma + 8 = 0 \rightarrow (2)$$

$$\alpha + \gamma = \frac{-5}{3} + \beta + 3$$

$$3\alpha - 3\beta + 3\gamma - 4 = 0 \rightarrow (3)$$

$$\text{From (1), (2) and (3) We get } \alpha = 0, \beta = \frac{-4}{3}, \gamma = 0$$

$$\text{Consider } 5\alpha - 9\beta + 13\gamma = 5(0) - 9\left(\frac{-4}{3}\right) + 13(0)$$

$$= 12$$

$$265. \text{ Given } \bar{r} = (2\bar{i} + \bar{j} + 2\bar{k}) + \lambda(-\bar{i} - 3\bar{k}) + \mu(\bar{i} - \bar{j} + 2\bar{k})$$

$$\text{Which is of the form } \bar{r} = \bar{a} + \lambda\bar{b} + \mu\bar{c}$$

$$\text{Let } \bar{r} = x\bar{i} + y\bar{j} + z\bar{k}, \bar{a} = 2\bar{i} + \bar{j} + 2\bar{k},$$

$$\bar{b} = -\bar{i} - 3\bar{k}, \bar{c} = \bar{i} - \bar{j} + 2\bar{k}$$

$$\text{Required equation is } (\bar{r} - \bar{a}) \cdot (\bar{b} \times \bar{c}) = 0$$

$$\begin{vmatrix} x-2 & y-1 & z-2 \\ -1 & 0 & -3 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

$$(x-2)(-3) - (y-1)(-2+3) + (z-2)(1) = 0$$

$$\Rightarrow 3x + y - z = 5$$

$$266. |\bar{a}| = \sqrt{6}, |\bar{b}| = \sqrt{11}$$

$$\text{W.K.T. } \bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}|\cos\theta$$

$$\bar{x} = \left(\frac{\sqrt{6}\sqrt{11}\cos\theta}{11}\right)(\bar{i} - \bar{j} + 3\bar{k})$$

$$= \sqrt{\frac{6}{11}}\cos\theta(\bar{i} - \bar{j} + 3\bar{k})$$

$$\bar{y} = \left(\frac{\sqrt{6}\sqrt{11}\cos\theta}{6}\right)(2\bar{i} + \bar{j} - \bar{k})$$

$$= \sqrt{\frac{11}{6}}\cos\theta(2\bar{i} + \bar{j} - \bar{k})$$

$$\text{Consider } x^2 + y^2$$

$$\frac{6}{11}\cos^2\theta(1+1+9) + \frac{11}{6}\cos^2\theta(4+1+1)$$

$$= \cos^2\theta(6+11)$$

$$= 17\cos^2\theta$$

$$267. \cos\theta = \frac{|0+5+0|}{\sqrt{1+4}\sqrt{25+16}}$$

$$\cos\theta = \frac{5}{\sqrt{5}\sqrt{41}}$$

$$\cos\theta = \frac{\sqrt{5}}{\sqrt{41}}$$

$$\theta = \cos^{-1}\left(\sqrt{\frac{5}{41}}\right)$$

268. $Volume = Base\ area \times Height$

$$[\vec{a} \ \vec{b} \ \vec{c}] = |\vec{a} \times \vec{b}| \times h$$

$$h = \frac{[\vec{a} \ \vec{b} \ \vec{c}]}{|\vec{a} \times \vec{b}|}$$

269. $x + 2y + 3 = 4x + 6y + 10$

$$3x + 4y + 7 = 0 \dots (1)$$

$$5x - y + 2 = 2x - 10y - 4$$

$$3x + 9y + 6 = 0 \dots (2)$$

$$\text{From (1) and (2)} \quad x = \frac{-13}{5} \quad y = \frac{1}{5}$$

270. ΔABC is equilateral

$\Rightarrow orthocentre = centroid$

271. $\pi_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 0 & 3 & -2 \end{vmatrix} = -4\vec{i} + 2\vec{j} + 3\vec{k}$

$$\pi_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 2 \\ -2 & 0 & 3 \end{vmatrix} = 3\vec{i} - 4\vec{j} + 2\vec{k}$$

now find angle between π_1 and π_2

272. $\vec{a} \times (\vec{b} \times \vec{c})$ is $\perp$ to $\vec{a}$ and $\vec{b} \times \vec{c}$

Required vector $= \vec{a} \times (\vec{b} \times \vec{c})$

273. $\vec{OG} = \frac{\vec{OA} + \vec{OB} + \vec{OC}}{3} = -2\vec{j}$

$$BC^2 + CA^2 + AB^2 + 9(OG^2)$$

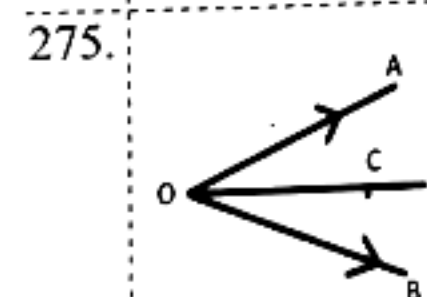
274. $\vec{OA} = 7\vec{i} - 4\vec{j} + 5\vec{k}$

$G_1 = Centroid\ of\ BCD$

$G = Centroid\ of\ ABCD$

$$\vec{OG}_1 = -\vec{i} + 4\vec{j} - 3\vec{k}$$

$$\vec{OG} = \frac{3\vec{OG}_1 + \vec{OA}}{4}$$



$$\hat{a} = \frac{\vec{i} + 2\vec{j} - 2\vec{k}}{3}$$

$$\hat{b} = \frac{-2\vec{i} - 3\vec{j} + 6\vec{k}}{7}$$

$$\vec{OC} = \sqrt{42} \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$$

$$= \vec{i} + 5\vec{j} + 4\vec{k}$$

276. $\vec{a} \cdot \vec{b} = -7$

$$\cos \theta = \frac{-7}{14} = \frac{-1}{2} \Rightarrow \theta = 120^\circ$$

$$\frac{|\vec{a} \times \vec{b}|}{|\vec{a} \cdot \vec{b}|} = \frac{14 \cdot \frac{\sqrt{3}}{2}}{7} = \sqrt{3}$$

277.

$$\vec{a} = 2\vec{i} + 3\vec{j} + 6\vec{k}$$

$$\cos \theta = \frac{4}{21}$$

$$\frac{4}{7|\vec{b}|} = \frac{4}{21}$$

$$|\vec{b}| = 3$$

$$|\vec{a} + \vec{b}| = \sqrt{16}$$

Now take option verification

278. $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ plane is $\vec{r} \cdot \vec{m} = q \Rightarrow 2x + 6y - 9z = -1$

$$p = \frac{|2 + 12 - 27 + 1|}{11} = \frac{12}{11}$$

$$q = \frac{1}{11}$$

$$p - q = 1$$

279. $\vec{OA} = 2\vec{i} - 3\vec{j} + 2\vec{k}$

$$\vec{OB} = 3\vec{i} - \vec{j} - 2\vec{k}$$

$$\vec{OC} = \frac{2(\vec{OB}) + 3(\vec{OA})}{2 + 3}$$

$$\vec{OP} = 2\vec{i} - \vec{j} + \vec{k}$$

Then find $|\overline{CP}|$

$$280. \frac{2x+y}{2} = \frac{3}{1} = \frac{9}{y-x}$$

$$2x+y=6$$

$$3y-3x=9$$

by solving we get $x=1, y=4$

$$281. (\overline{a} \times \overline{b}) \cdot \overline{c} = (\overline{a} \times \overline{b}) \cdot \overline{d}$$

$$282. \overline{a} = \overline{i} + 2\overline{j} + 3\overline{k}$$

$$\overline{b} = \overline{i} - 2\overline{j} - 3\overline{k}$$

A1 is the area of the quadrilateral having $\overline{a}, \overline{b}$

as its diagonals $= |\overline{a} \times \overline{b}| = 2\sqrt{13}$

$$\overline{a} \times \overline{b} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 1 & 2 & 3 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= 2(3\overline{j} - 2\overline{k})$$

A2 is The are of the Parallelogram having $\overline{a}, \overline{b}$ as its two adjacent sides

$$\frac{1}{2} |\overline{a} \times \overline{b}| = \sqrt{13}$$

$$283. \frac{1}{\lambda} \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 3 & -1 & \lambda \\ \lambda & 1 & -3 \end{vmatrix} = \frac{\sqrt{195}}{\lambda}$$

$$\Rightarrow \lambda^2 = -24 \text{ (Not possible) (or) } \lambda^2 = 4$$

$\therefore$ '2' distinct values for λ

$$284. \overline{r} = \frac{-(3\overline{a} - 2\overline{b})|\overline{r}|}{|3\overline{a} - 2\overline{b}|}$$

$$285. \overline{OG} = \overline{i}$$

$$\overline{OA} = 2\overline{i} + \overline{j} + \overline{k}, \overline{OB} = 2\overline{i} + 4\overline{j} - 4\overline{k}$$

$$\text{wkt } \overline{OA} + \overline{OB} + \overline{OC} = 3\overline{OG}$$

$$\Rightarrow \overline{OC} = -\overline{i} - 5\overline{j} + 3\overline{k}$$

$$\text{Now } AG^2 + BG^2 + CG^2$$

$$\Rightarrow 3 + 33 + 38 = 74$$

$$286. \overline{a} + \overline{b} = (t-2)\overline{i} + 7\overline{j}$$

$$|\overline{a} + \overline{b}| = 25$$

$$t_1 = 26, t_2 = -22$$

Now

$$t_1 + t_2 = 26 - 22$$

$$= 4$$

287. By using formula.

$$\overline{a} - \frac{(\overline{a} \cdot \overline{b})\overline{b}}{|\overline{b}|^2} \text{ (req. vector)}$$

$$= -\overline{i} + 7\overline{j} + 10\overline{k}$$

$$288. \overline{r} = x\overline{a} + y\overline{b}$$

$$\overline{r} = (2x+3y)\overline{i} + (-3x+2y)\overline{j} + (-5x-5y)\overline{k}$$

$$\text{and } \overline{r} \cdot \overline{c} = 0 \Rightarrow x - 4y = 0$$

$$|\overline{r}| = \sqrt{94} \Rightarrow 846y^2 = 94$$

$$y = \frac{1}{3}$$

$$\text{Now } |\overline{r} \cdot \overline{b}| = 138.y$$

$$= 138 \cdot \frac{1}{3}$$

$$= 46$$

$$289. \overline{a} + \overline{b} + \overline{c} + \overline{d} = m\overline{d} - \overline{c} + \overline{c} + \overline{d}$$

$$\overline{a} + \overline{b} + \overline{c} + \overline{d} = n\overline{d} - \overline{d} + \overline{a} + \overline{d}$$

$$\overline{a}(n+1) = (m+1)\overline{d}$$

$$n = -1, m = -1$$

$$\therefore \overline{a} + \overline{b} + \overline{c} + \overline{d} = 0 \rightarrow 1$$

$$\text{Consider } 3\overline{a} + 2\overline{b} + 2\overline{c} + \overline{d}$$

$$= \overline{a} + \overline{a} + \overline{b} + \overline{c} + \overline{a} + \overline{b} + \overline{c} + \overline{d}$$

290.

$$\overline{AB} = -\overline{i} - 4\overline{j} + 6\overline{k}$$

$$|AB| = \sqrt{53}$$

$$\overline{BC} = -4\overline{i} + 7\overline{j} - \overline{k}$$

$$|\overline{BC}| = \sqrt{66}$$

$$\overline{CA} = 5\overline{i} - 3\overline{j} - 5\overline{k}$$

$$|CA| = \sqrt{59}$$

$\therefore ABC$ is scalene triangle

291.

$$\bar{r} = (1-t)(\bar{i} - 9\bar{k}) + t(7\bar{j} + \bar{k})$$

$$\bar{r} = (1-t)\bar{i} + 7t\bar{j} + (t-9+9t)\bar{k}$$

$$L \equiv \bar{r} = (1-t)\bar{i} + 7tj + (10t-9)\bar{k}$$

$$\pi \equiv (r - \bar{a}) \cdot \hat{n} = 0$$

$$\pi \equiv (r - (6\bar{i} + \bar{j})) \cdot (\bar{i} + \bar{j} + \bar{k}) = 0$$

$$((x-6)\bar{i} + (y-1)\bar{j} + z\bar{k}) \cdot (\bar{i} + \bar{j} + \bar{k}) = 0$$

$$x-6+y-1+z=0$$

$$x + y + z - 7 = 0$$

$$(a,b,c)=(1,1,1)$$

$$\frac{x-1}{-1} = t, \quad \frac{y}{7} = t, \quad \frac{z+9}{10} = t$$

$$(l, m, n) = (-1, 7, 10)$$

$$\sin \theta = \frac{|-1+7+10|}{\sqrt{1+1+1}\sqrt{1+49+100}}$$

$$\sin \theta = \frac{16}{\sqrt{3} \cdot 5\sqrt{6}} = \frac{8\sqrt{2}}{15}$$

292.

$$\text{Required vector} = \frac{\overline{AB} \times \overline{AC}}{|\overline{AB} \times \overline{AC}|}$$

293.

Given $\bar{a} + \bar{b} + \bar{c} = 0$

$$\bar{a} = -(\bar{b} + \bar{c})$$

$$\bar{a} = -4\bar{i} + 3\bar{j}$$

Clearly $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = 3\vec{i} + 4\vec{j} - \vec{k}$

$$\begin{aligned} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| &= |3(3\vec{i} + 4\vec{j} - \vec{k})| \\ &= 3\sqrt{26} \\ &= \sqrt{234} \end{aligned}$$

294.

$$\overrightarrow{OA} = 2\vec{i} + 3\vec{j} - \vec{k} \text{ and } \overrightarrow{OB} = \vec{i} - \vec{j} + 2\vec{k}$$

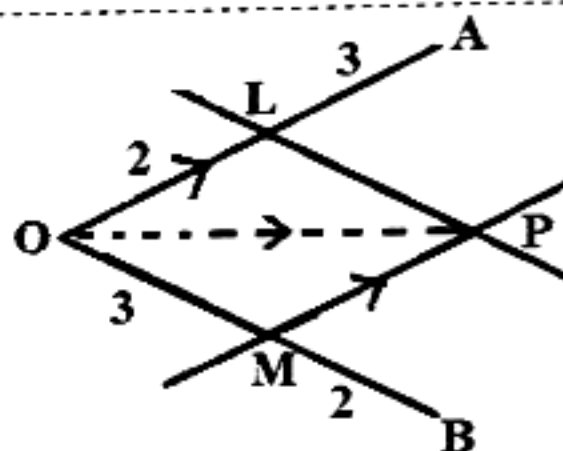
$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -\vec{i} - 4\vec{j} + 3\vec{k}$$

$$|\overrightarrow{AB}| = \sqrt{1+16+9} = \sqrt{26}$$

unit vector along $\overline{BA}$ in the direction $\overline{AB}$ is

$$\frac{1}{\sqrt{26}}(-\bar{i} - 4\bar{j} + 3\bar{k}).$$

295.



$$\overrightarrow{OA} = -3\vec{i} - 3\vec{j} + 4\vec{k}, \overrightarrow{OB} = 4\vec{i} - 4\vec{j} - 3\vec{k}$$

$$\overrightarrow{OL} = \frac{2}{5}(-3\vec{i} - 3\vec{j} + 4\vec{k})$$

$$\overrightarrow{OM} = \frac{3}{5}(4\vec{i} - 4\vec{j} - 3\vec{k})$$

$$\overline{OP} = \overline{OL} + \overline{OM} (OLMP \parallel \text{gm})$$

$$= \frac{1}{5}(6\vec{i} - 18\vec{j} - \vec{k})$$

$$|\overline{OP}| = \frac{1}{5} \sqrt{36 + 324 + 1}$$

$$= \frac{1}{5} \sqrt{361} = \frac{19}{5}$$

$$296. \vec{i} \times (\vec{a} \times \vec{i}) + \vec{j} \times (\vec{a} \times \vec{j}) + \vec{k} \times (\vec{a} \times \vec{k}) = 2\vec{a}$$

$$\text{Given} \Rightarrow [2\vec{a}]^2 = 440$$

$$\Rightarrow |\vec{a}|^2 = 110$$

$$\Rightarrow \lambda^2 + 61 = 110$$

$$\Rightarrow \lambda^2 = 110 - 61 = 49$$

$$\Rightarrow \lambda = 7$$

297. The vector equation of the plane is

$$\Rightarrow \vec{r} \cdot \vec{q} = \vec{p} \cdot \vec{q}$$

$$\Rightarrow \vec{r} \cdot \frac{\vec{q}}{|\vec{q}|} = \vec{p} \cdot \frac{\vec{q}}{|\vec{q}|} \rightarrow (1)$$

$\therefore$ Perpendicular distance from $(0,0,0)$ to plane (1)

$$\begin{aligned} \text{is } (4\vec{i} - \vec{j} + \vec{k}) \cdot \frac{9\vec{i} - 2\vec{j} + 6\vec{k}}{\sqrt{81 + 4 + 36}} \\ = \frac{36 + 2 + 6}{11} \\ = \frac{44}{11} = 4. \end{aligned}$$

$$298. \vec{a} - \vec{b} + \vec{c} = 11\vec{i} + (y-6)\vec{j} - 2\vec{k}$$

$$|\vec{a} - \vec{b} + \vec{c}| = \sqrt{141}$$

$$\Rightarrow 121 + (y-6)^2 + 4 = 141$$

$$\Rightarrow y = 10, 2$$

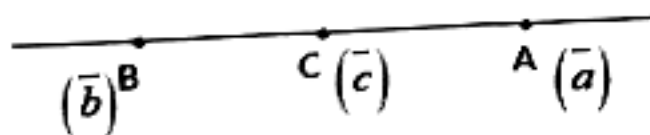
$$\therefore |y_1 - y_2| = |10 - 2| = 8$$

$$299. \vec{a} = i + 2j - 3k, \vec{b} = 2i - 3j - 5k$$

$$\vec{a} - \vec{b} = i + 5j + 2k$$

$$\therefore |\vec{a} - \vec{b}| > |\vec{b}| - |\vec{a}|$$

300.



$$\text{Given points } \vec{OA} = i - j + 2k = \vec{a}$$

$$\vec{OB} = i + 2j - 2k = \vec{b}$$

$$\vec{AB} = 3j - 4k$$

$$\text{Let } \vec{OC} = \vec{c}$$

$$\begin{aligned} \text{Equation of lines AB is } \vec{r} &= \vec{b} + t(\vec{AB}) \\ \vec{r} - \vec{b} &= t\vec{AB} \end{aligned}$$

$$\text{Take } \vec{r} = \vec{c} \Rightarrow \vec{c} - \vec{b} = t\vec{AB}$$

$$\vec{BC} = t\vec{AB}$$

$$|\vec{BC}| = |t||\vec{AB}|$$

$$10 = |t|(5)$$

$$|t| = 2$$

$$t = \pm 2$$

The P.V of C is

$$\vec{c} = \vec{b} + t\vec{AB}$$

$$= i + 2j - 2k \pm 2(3j - 4k)$$

$$= i + 8j - 10k \text{ or } i - 4j + 6k$$

301.

$$\text{volume} = 2 \Rightarrow \frac{1}{6} [\vec{a} \vec{b} \vec{c}] = 2$$

$$\Rightarrow \begin{vmatrix} 1 & -\lambda & 1 \\ \lambda & -1 & -1 \\ 1 & 1 & \lambda \end{vmatrix} = 12$$

$$\Rightarrow \lambda^3 + \lambda - 10 = 0 \Rightarrow \lambda = 2 (\lambda \in \mathbb{Z})$$

$$|\lambda\vec{i} - 3\lambda\vec{j} + 3\vec{k}| = |2\vec{i} - 6\vec{j} + 3\vec{k}| = 7$$

302.

$$\begin{aligned} (\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) &= \begin{vmatrix} \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \end{vmatrix} \\ &= \begin{vmatrix} -23 & 31 \\ 50 & -58 \end{vmatrix} \\ &= -216 \end{aligned}$$

303.

$$\begin{aligned} \text{Normal to } \pi_1 \text{ is } \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} \\ = i(1-0) - j(1-0) + k(1) \\ = i - j + k = \vec{n}_1 \end{aligned}$$

$$\begin{aligned} \text{Normal to } \pi_2 \text{ is } \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & 1 & -1 \end{vmatrix} \\ = i + j + 2k = \vec{n}_2 \end{aligned}$$

$$\therefore \vec{a} \parallel \vec{n}_1 \times \vec{n}_2$$

$$\vec{a} = t(\vec{n}_1 \times \vec{n}_2)$$

$$= t(-3\vec{i} - \vec{j} + 2\vec{k})$$

$$|\vec{a}| = |t|\sqrt{9+1+4}$$

$$t = \pm 1$$

$$\therefore \vec{a} = \pm(-3\vec{i} - \vec{j} + 2\vec{k})$$

304. Since $t < 0$

$$\vec{a} \longrightarrow$$

$$\vec{b} \longleftarrow$$

Unlike vectors

$$\text{Clearly, } |\vec{a}| \leq |\vec{b}| \text{ or } |\vec{a}| > |\vec{b}|$$

305. $\vec{a}, \vec{b}, \vec{c}$ lies on same plane

$$\therefore \vec{c} = x\vec{a} + y\vec{b} \quad (\because L.C)$$

$$306. 2(\vec{a} \times \vec{b}) + 12(\vec{a} \times \vec{b}) = xi + yj + zk$$

$$14(\vec{a} \times \vec{b}) = xi + yj + zk$$

$$\vec{a} \times \vec{b} = 25\vec{i} + 50\vec{j} + 25\vec{k}$$

$$\therefore x + y + z = 14(25 + 50 + 25) = 1400$$

$$307. |\vec{a} - \vec{b}| = 1$$

S.O.B.S

$$|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} = 1$$

$$\Rightarrow (\vec{a}, \vec{b}) = 60^\circ$$

$$\text{W.K.T. } |\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2[|\vec{a}|^2 + |\vec{b}|^2]$$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = 4 - 1 = 3$$

$$\Rightarrow |\vec{a} + \vec{b}| = \sqrt{3}$$

$$\text{Now, } 2|\vec{a} + \vec{b}| \cdot \cos \frac{\theta}{2}$$

$$= 2\sqrt{3} \frac{\sqrt{3}}{2} = 3$$

$$308. \begin{vmatrix} 2 & 3 & -1 \\ 4 & -1 & 3 \\ p & 1 & -1 \end{vmatrix} = 0$$

$$p = -\frac{1}{2}$$

$$|\vec{a} \times \vec{c}| = \frac{3\sqrt{10}}{2}$$

309. Diagonals are bisect each other
Mid pt of AC = Mid pt of BD

$$\overleftrightarrow{AC} \quad \overleftrightarrow{BD}$$

Then $C = (1, 0, 1)$

Let P and Q are trisection pts of AC

P divides AC in the ration 1:2

$$P = \left(\frac{1+4}{3}, \frac{2}{3}, \frac{1}{3} \right)$$

$$\therefore \vec{OP} = \frac{1}{3}(5\vec{i} + 2\vec{j} + \vec{k})$$

$$310. \text{Let } \vec{OA} = \vec{a} + \vec{b} + \vec{c}, \vec{OB} = \vec{a} - \vec{b} + 3\vec{c}$$

$$\vec{OC} = 2\vec{a} - \vec{b} + \vec{c} \text{ and } \vec{OD} = \vec{a} - 2\vec{b} + 4\vec{c}$$

$$\text{equation of } \vec{AB} \text{ is } \vec{r} = \vec{a} + \vec{b} + \vec{c} + t(-2\vec{b} - 2\vec{c}) \rightarrow (1)$$

$$\text{equation of } \vec{CD} \text{ is } \vec{r} = 2\vec{a} - \vec{b} + \vec{c} + s(-\vec{a} - \vec{b} + 3\vec{c}) \rightarrow (2)$$

at the point of inter section the value of $\vec{r}$ is equal

$$\Rightarrow \vec{a} + \vec{b} + \vec{c} + t(-2\vec{b} - 2\vec{c}) = 2\vec{a} - \vec{b} + \vec{c} + s(-\vec{a} - \vec{b} + 3\vec{c})$$

compare b.s a coefficients

$$1 = 2 - s \Rightarrow s = 1$$

$$\vec{r} = 2\vec{a} - \vec{b} + \vec{c} + 1(-\vec{a} - \vec{b} + 3\vec{c})$$

$$= \vec{a} - 2\vec{b} + 4\vec{c}$$

311. The equation of the req plane

$$\vec{r} \cdot (i - 2j + 3k + \lambda(3i + j - 2k)) = 5 + 7\lambda$$

$$\Rightarrow (xi + yj + zk) \cdot (3\lambda + 1)i + (\lambda - 2)j + (-2\lambda + 3)k$$

$$= 7\lambda + 5$$

$$\Rightarrow (3\lambda + 1)x + (\lambda - 2)y + (3 - 2\lambda)z = 7\lambda + 5$$

$$\Rightarrow 6\lambda + 2 - 3\lambda + 6 + 3 - 2\lambda = 7\lambda + 5$$

$$\Rightarrow \boxed{\lambda = 1}$$

req. plane is

$$\vec{r} \cdot (4i - j + k) = 12$$

$$312. \vec{a} \cdot \vec{b} = 0, \vec{b} \cdot \vec{c} = 0$$

$$|\vec{a} + \vec{b} + \vec{c}| = 4\sqrt{3}$$

s.o.b.s

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a} = 48$$

$$\Rightarrow 4 + 9 + 25 + 0 + 0 + 2|\vec{c}||\vec{a}|\cos(\vec{a}, \vec{c}) = 48$$

$$\Rightarrow 2 \cdot 2 \cdot 5 \cdot \cos(\vec{a}, \vec{c}) = 10$$

$$\Rightarrow \cos(\vec{a}, \vec{c}) = \frac{1}{2}$$

$$\boxed{(\vec{a}, \vec{c}) = \frac{\pi}{3}}$$

$$313. \vec{d} = x(\vec{b} \times \vec{c}) - \frac{7}{9}(\vec{c} \times \vec{a}) + z(\vec{a} \times \vec{b})$$

taking dot product with $\vec{a}$

$$\vec{d} \cdot \vec{a} = x(\vec{b} \times \vec{c}) \cdot \vec{a} - \frac{7}{9}(\vec{c} \times \vec{a}) \cdot \vec{a} + z(\vec{a} \times \vec{b}) \cdot \vec{a}$$

$$\vec{d} \cdot \vec{a} = x[a \ b \ c] - 0 + 0$$

$$x = \frac{\vec{d} \cdot \vec{a}}{[a \ b \ c]} = \frac{2}{9}$$

$$314. \vec{OP} = \vec{i} + 2\vec{j} - 7\vec{k}$$

$$\vec{OQ} = 5\vec{i} - 3\vec{j} + 4\vec{k}$$

$$\vec{PQ} = \vec{OQ} - \vec{OP} = 4\vec{i} - 5\vec{j} + 11\vec{k}$$

let $\vec{OC} = \vec{k}$ is vector on Z-axis

$$\cos \theta = \frac{\vec{PQ} \cdot \vec{OC}}{|\vec{PQ}| |\vec{OC}|}$$

$$= \frac{11}{\sqrt{16 + 25 + 121} \sqrt{1}}$$

$$\cos \theta = \frac{11}{\sqrt{162}}$$

$$315. |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$\vec{a}$ perpendicular $\vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$

$$(\vec{c}, \vec{a}) = \alpha, (\vec{c}, \vec{b}) = \beta$$

$$\text{Given } |\vec{a} + \vec{b} + \vec{c}| = 1$$

squaring on both sides

$$\vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 1$$

$$1 + 1 + 1 + 2[0 + \cos \beta + \cos \alpha] = 1$$

$$\therefore \cos \alpha + \cos \beta = -1$$

$$316. \text{Let}$$

$$\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{a} \times \vec{i} = \vec{j} + \vec{k}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix} = \vec{j} + \vec{k}$$

$$\vec{i}(0-0) - \vec{j}(0-z) + \vec{k}(0-y) = \vec{j} + \vec{k}$$

$$z\vec{j} - y\vec{k} = \vec{j} + \vec{k}$$

$$z = 1, y = -1$$

$$\vec{a} \cdot \vec{i} = 1$$

$$\rightarrow x = 1$$

$$\therefore \vec{a} = \vec{i} - \vec{j} + \vec{k}$$

vector equation of the line is

$$\vec{r} = \vec{i} + \vec{j} + \vec{k} + t\vec{a}$$

$$\vec{r} = \vec{i} + \vec{j} + \vec{k} + t(\vec{i} - \vec{j} + \vec{k})$$

$$= (t+1)\vec{i} + (1-t)\vec{j} + (t+1)\vec{k}$$

$$317. \text{Given } \vec{OA} = \vec{a}, \vec{OB} = \vec{b}, \vec{OC} = \frac{\vec{a}}{2} + \frac{\vec{b}}{3}$$

$$\text{Now, } \vec{OC} = \frac{3\vec{a} + 2\vec{b}}{6} < \frac{3\vec{a} + 2\vec{b}}{5}$$

$$318. |\vec{a} - \vec{b}|^2 + |\vec{a} + 2\vec{b}|^2 = 20$$

$$\cos(\vec{a}, \vec{b}) = \frac{-1}{2} = \cos \frac{2\pi}{3}$$

$$\therefore (\vec{a}, \vec{b}) = \frac{2\pi}{3}$$

$$319. \quad \vec{a} \cdot \vec{b} = 0, |\vec{a}| = 8, |\vec{b}| = 3$$

$$|\vec{a} - 2\vec{b}|^2 = |\vec{a}|^2 + 4|\vec{b}|^2 - 8\vec{a} \cdot \vec{b}$$

$$|\vec{a} - 2\vec{b}| = 10$$

$$320. \quad (\lambda - 1)\vec{a} + 2\vec{b} \text{ and } 3\vec{a} + \lambda\vec{b}$$

are collinear vectors

$$\frac{\lambda - 1}{3} = \frac{2}{\lambda} \Rightarrow \lambda^2 - \lambda = 6$$

$$\lambda^2 - \lambda - 6 = 0$$

$$(\lambda + 2)(\lambda - 3) = 0$$

$$\therefore \lambda = \{-2, 3\}$$

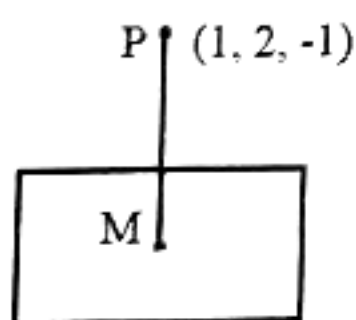
$$321. \quad \text{Required plane}$$

$$4(x - 3) + 7(y + 2) + (-4)(z - 1) = 0$$

$$4x - 12 + 7y + 14 - 4z + 4 = 0$$

$$4x + 7y - 4z + 6 = 0$$

Distance from $(1, 2, -1)$



$$= \frac{|4 + 14 + 4 + 6|}{\sqrt{16 + 49 + 16}} = \frac{28}{9}$$

$$322. \quad p = q + r$$

$$ai + bj + ck = (d + 3)i + 4j + 2k$$

$$a = d + 3, b = 4, c = 2$$

$$AB \times AC = \begin{vmatrix} i & j & k \\ d & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix} = -10i + (2d + 12)j + (d - 9)k$$

$$\frac{1}{2} |AB \times AC| = \frac{1}{2} \sqrt{100 + (2d + 12)^2 + (d - 9)^2} = 5\sqrt{6}$$

$$d = 5, -11$$

$$a = 8, b = 4, c = 2$$

$$|a| + |b| + |c| = 14$$

$$323. \quad (\vec{c}\vec{c})(\vec{a}\vec{c}) = (\vec{c}\vec{c})$$

$$(\vec{a}\vec{c}) = 1$$

$$(\vec{a}\vec{c})\vec{b} - (\vec{a}\vec{b})\vec{c} + (\vec{a}\vec{b})\vec{b} = (4 - 2\beta - \sin \alpha)\vec{b} + (\beta^2 - 1)\vec{c}$$

$$(1 + \vec{a}\vec{b})\vec{b} - (\vec{a}\vec{b})\vec{c} = (4 - 2\beta - \sin \alpha)\vec{b} + (\beta^2 - 1)\vec{c}$$

$$\vec{a}\vec{b} = 1 - \beta^2 \dots\dots\dots (1)$$

$$1 + \vec{a}\vec{b} = 4 - 2\beta - \sin \alpha \dots\dots\dots (2)$$

From (1) & (2),

$$\beta^2 - 2\beta + 2 = \sin \alpha$$

$$(\beta - 1)^2 + 1 = \sin \alpha$$

Which is possible only when $\beta = 1, \alpha = \frac{\pi}{2}$

$$324. \quad \vec{OP} = \frac{2\vec{OB} + \vec{OA}}{2 + 1}$$

$$\vec{OP} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{PC} = \vec{OC} - \vec{OP} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$l = \frac{3}{7}, m = \frac{2}{7}, n = \frac{6}{7}$$

$$l + 3m + 2n = 3$$

$$325. \quad A(1, -1, 0), B(0, 1, -1)$$

The equation of line AB is

$$\frac{x - 1}{-1} = \frac{y + 1}{2} = \frac{z}{-1}$$

$$P(2,1,0) \quad Q(0,2,-1) \quad R(1,0,2)$$

The equation of plane PQR is

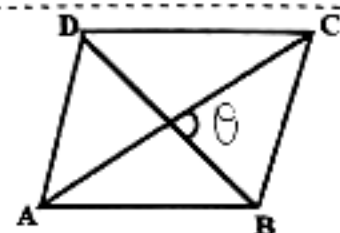
$$\begin{vmatrix} x-2 & y-1 & z \\ -2 & 1 & -1 \\ -1 & -1 & 2 \end{vmatrix} = 0$$

$$x + 5y + 3z - 7 = 0$$

By option verification

$$(x, y, z) = \left(\frac{-5}{6}, \frac{16}{6}, \frac{-11}{6} \right) \text{ satisfies}$$

326.



$$\overrightarrow{BC} = 2\vec{i} + \vec{j} + \vec{k} = a$$

$$\overrightarrow{CD} = \vec{i} + 2\vec{j} - 2\vec{k} = b$$

$$\overrightarrow{AC} = \vec{i} - \vec{j} + 3\vec{k}$$

$$\overrightarrow{BD} = 3\vec{i} + 3\vec{j} - \vec{k}$$

$$\cos \theta = \frac{\overrightarrow{AC} \cdot \overrightarrow{BD}}{|\overrightarrow{AC}| |\overrightarrow{BD}|}$$

$$\cos \theta = \frac{-3}{\sqrt{209}}$$

$$\tan \theta = \frac{-10\sqrt{2}}{3}$$

327. $(a, b, c) = (1, 2, -3)$

$$(x_1, y_1, z_1) = (3, -4, 5)$$

$$\Rightarrow a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$x + 2y - 3z + 20 = 0$$

328. $[\vec{a} \vec{b} \vec{c}] = 61$

$$\begin{vmatrix} \lambda & 3 & 4 \\ 3 & -1 & \lambda \\ \lambda & 1 & -3 \end{vmatrix} = 61$$

$$2\lambda^2 + 7\lambda - 22 = 0$$

$$(\lambda - 2)(2\lambda + 11) = 0$$

$$\lambda = 2, \lambda = -11/2 \text{ (is not integer)}$$

$$\therefore \lambda = 2$$

one possible value

329. Given $|\vec{a}| = 4, |\vec{b}| = 5, |\vec{a} - \vec{b}| = 3$

S.O.B.S

$$16 + 25 + 2(4)(5)\cos(\vec{a}, \vec{b}) = 9$$

$$40\cos\theta = -32$$

$$\cos\theta = \frac{-8}{10}$$

$$\cos\theta = -\frac{4}{5}$$

$$\cot^2\theta = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

330. $A = (1, 2, 3), B = (3, 7, -2), C = (6, 7, 7), D = (-1, 0, -1)$

Centroids of $\triangle ABD = (x_1, y_1, z_1) = (1, 3, 0)$

Centroid of $\triangle ACD = (x_2, y_2, z_2) = (2, 3, 3)$

Equation of required line segment is

$$\vec{r} = (1-t)\vec{a} + t\vec{b}$$

$$= (1-t)(\bar{i} + 3\bar{j}) + t(2\bar{i} + 3\bar{j} + 3\bar{k})$$

$$= (1+t)\bar{i} + 3\bar{j} + 3t\bar{k}$$

331. Given $|\bar{a}| = 3, |\bar{b}| = 5, |\bar{c}| = 7$

Given $\bar{a} + \bar{b} + \bar{c} = 0$

$$\bar{a} + \bar{b} = -\bar{c}$$

S.O.B.S

$$9 + 25 + 2(3)(5)\cos(\bar{a}, \bar{b}) = 49$$

$$30 \cos(\bar{a}, \bar{b}) = 15$$

$$\cos(\bar{a}, \bar{b}) = \frac{1}{2}$$

$$(\bar{a}, \bar{b}) = 60^\circ = \frac{\pi}{3}$$

332. $2\bar{i} - \bar{j} + 3\bar{k}, -12\bar{i} - \bar{j} - 3\bar{k}, -\bar{i} + 2\bar{j} - 4\bar{k}$

$\lambda\bar{i} + 2\bar{j} - \bar{k}$ are coplanar

$$[\overline{AB} \ \overline{AC} \ \overline{AD}] = 0$$

$$\begin{vmatrix} -14 & 0 & -6 \\ -3 & 3 & -7 \\ \lambda - 2 & 3 & -4 \end{vmatrix} = 0$$

$$-14(-12 + 21) - 0 - 6(-9 - 3\lambda + 6) = 0$$

$$-126 + 18 + 18\lambda = 0$$

$$\Rightarrow \lambda = 6$$

333. $\bar{a} = \bar{i} + 2\bar{j} - 2\bar{k}, \bar{b} = 2\bar{i} - \bar{j} - 2\bar{k}$

$$x = \frac{(\bar{a}\bar{b})\bar{b}}{|\bar{b}|^2} = \frac{2-2+4}{(3)^2}(2\bar{i} - \bar{j} - 2\bar{k}) = \frac{4}{9}(2\bar{i} - \bar{j} - 2\bar{k})$$

$$y = \frac{(\bar{b}\bar{a})\bar{a}}{|\bar{a}|^2} = \frac{2-2+4}{9}(\bar{i} + 2\bar{j} - 2\bar{k}) = \frac{4}{9}(\bar{i} + 2\bar{j} - 2\bar{k})$$

$$|x - y| = \left| \frac{4}{9}(\bar{i} - 3\bar{j}) \right|$$

$$= \frac{4}{9}\sqrt{10}$$

334. Given vectors are coplanar

$$[\overline{AB} \ \overline{AC} \ \overline{AD}] = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 2 & \lambda - 1 \end{vmatrix} = 0 \Rightarrow \lambda = \frac{1}{3}$$

$$\overline{OE} = 6\lambda\bar{i} - 3\bar{j} + 6\bar{k}$$

$$= 2\bar{i} - 3\bar{j} + 6\bar{k}$$

$$\text{Now } |\overline{OE}| = \sqrt{4 + 9 + 36} = 7$$

335. $\bar{\gamma} = (1-t)(\bar{a} - \bar{b} + \bar{c}) + t(\bar{b} - \bar{c})$
 $\bar{\gamma} = (1-t)\bar{a} + (2t-1)\bar{b} + (1-2t)\bar{c} \dots (1)$

$$\bar{\gamma} = (1-l-m)(2\bar{a} - \bar{b}) + l(2\bar{b} - \bar{c}) + m(2\bar{c} - \bar{a})$$

$$\bar{\gamma} = (2-2l-3m)\bar{a} + (3l+m-1)\bar{b} + (-l+2m)\bar{c} \dots (2)$$

From (1) & (2), we get

$$t=0$$

$$\therefore p.o.I = (\bar{a} - \bar{b} + \bar{c})$$

$$336. \quad |\bar{a}| = 3$$

$$\text{and } \bar{a} \cdot \bar{b} = -4, \quad \bar{a} \cdot \bar{c} = -13$$

$$\bar{p} = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}|^2} (\bar{a}) = \frac{-4}{9} (\bar{i} - 2\bar{j} + 2\bar{k})$$

$$\bar{q} = \frac{(\bar{a}) (\bar{a} \cdot \bar{c})}{|\bar{a}|^2} = \frac{-13}{9} (\bar{i} - 2\bar{j} + 2\bar{k})$$

$$\text{Then } 13\bar{p} = 4\bar{q}$$

$$337. \quad \bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}$$

$$\bar{b} = 3\bar{i} - \bar{j} + 5\bar{k}$$

$$\bar{c} = \bar{i} - 4\bar{j} - 2\bar{k}$$

and

$$\bar{r} \cdot \bar{b} = 0$$

$$\bar{r} \cdot \bar{c} = 0$$

$$\bar{r} \cdot \bar{a} = 11$$

$$\Rightarrow 3x - y + 5z = 0$$

$$x - 4y - 2z = 0$$

$$x + 2y + 3z = 11$$

Solving we get

$$x = 22, y = 11, z = -11$$

$$\bar{r} = 22\bar{i} + 11\bar{j} - 11\bar{k}$$

By option verification

$$\bar{i} - \bar{j} + \bar{k} \text{ is } \perp \text{ to } \bar{r}$$